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Anne Tweed

Sunday 25 May

Addressing Student Misconceptions (Preconceptions) in Maths and Science Classrooms

Session 3

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A message from Hawker Brownlow Education

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Addressing Student Misconceptions (Preconceptions)

Designing Effective Science Instruction

Developing Student Understanding

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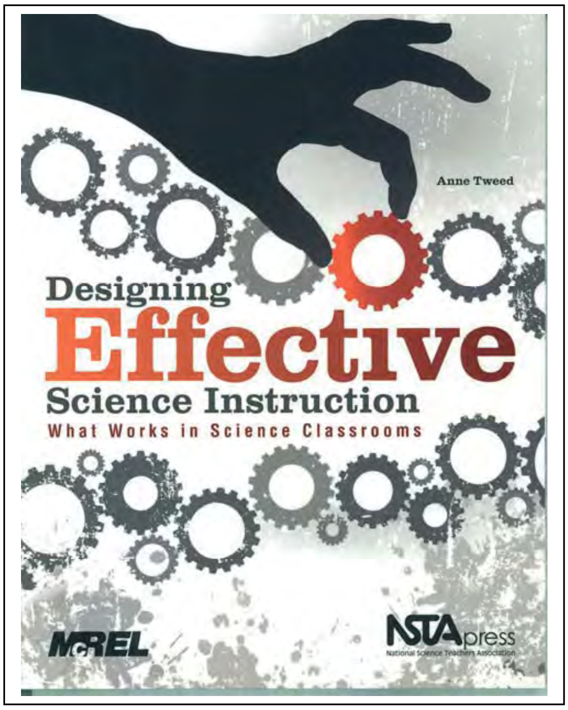

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Conceptual Change Model

Stage One: Students *become aware of their own preconceptions* about a concept by thinking about it and making predictions (*committing to an outcome)* before any activity, problem-solving or investigation begins.

Stage Two: Students *reveal their ideas and beliefs* by sharing them, initially in small groups and then with the entire class.

Stage Three: Students *confront their ideas and beliefs* by making observations, gathering data, by testing their ideas and then discussing them in small groups.

Stage Four: Students work toward *resolving conflicts* (if any) between their ideas (based on the revealed preconceptions and class discussion) and their observations, thereby *accommodating the new concepts* and revising their thinking to align with a scientific explanation.

Stage Five: Students *extend the concept* by trying to *make connections* between the concept learned in the classroom and other situations, including their daily lives.

Stage Six: Students are encouraged to *go beyond*, pursuing additional questions and problems related to the concept.

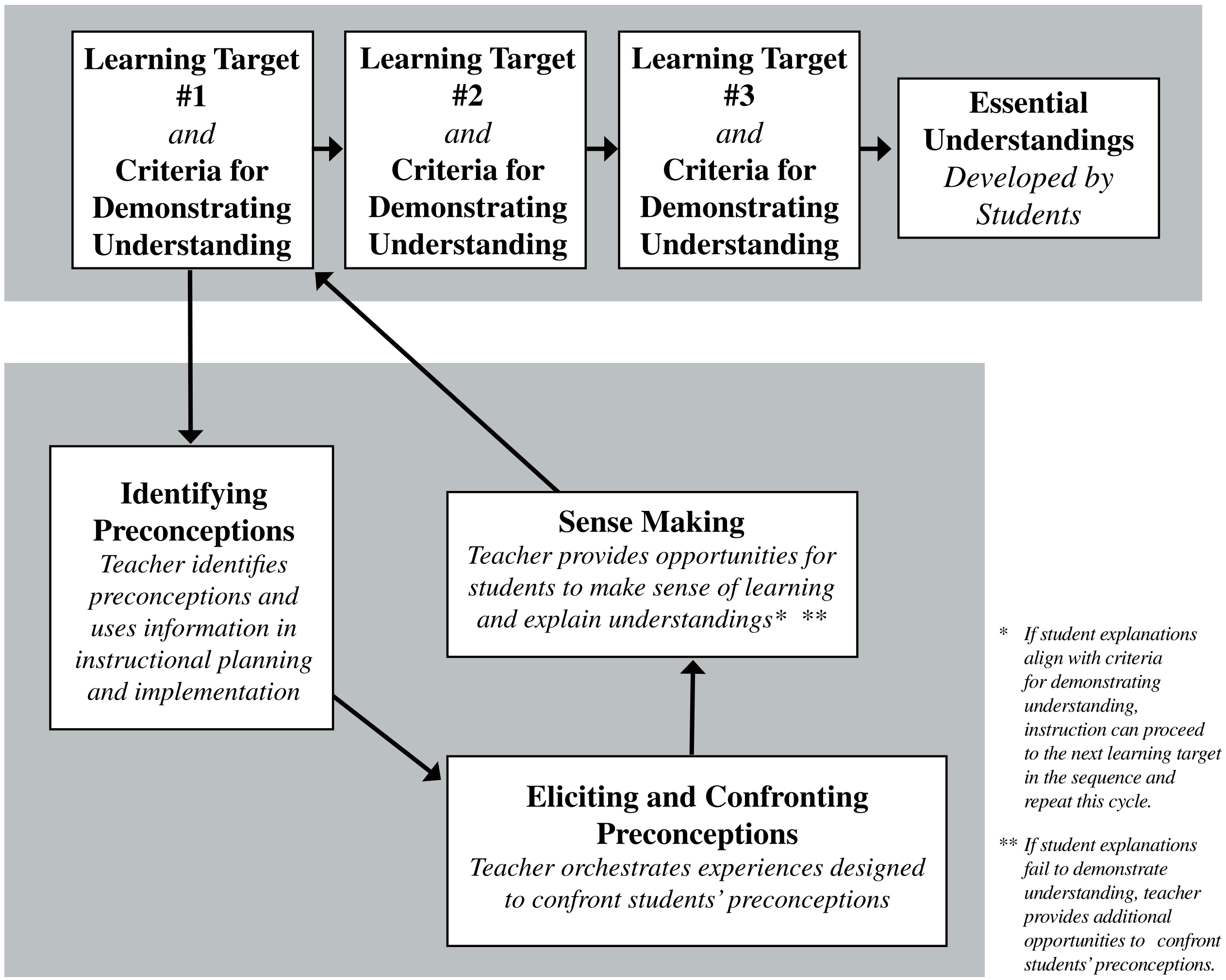
Based on the research of Posner, Strike, Hewson & Gertzog (1982) and Strike and Posner (1985) and revised from the work provided in *Targeting Students' Science Misconceptions: Physical Science Concepts Using the Conceptual Change Model,* Joseph Stepans, 2003.

Predictive Phase

The teacher determines the lesson's essential understandings, the sequence of learning targets that lead toward those understandings, and the criteria by which understanding is determined.

Responsive Phase

Building on the foundation of the predictive phase, the teacher plans for and implements instruction during the responsive phase, one learning target at a time.



* *If student explanations align with criteria for demonstrating understanding, instruction can proceed to the next learning target in the sequence and repeat this cycle.*

** *If student explanations fail to demonstrate understanding, teacher provides additional opportunities to confront students' preconceptions.*

Drawn from the first chapter of a new NSTA Press book to be released in June, 2009 (Hard-to-Teach Biology Concepts: A Framework to Deepen Student Understanding, by Susan Koba with Anne Tweed). For further information contact Judy Cusick at NSTA Press (jcusick@nsta.org).

Steps for a Misconceptions Literature Review

1. For science misconceptions, review Benchmarks for Science literacy (AAAS, 1993) for misconceptions discussed in each section. Chapter 15 of the book includes research finding organized by benchmark. If you do not have a copy of the book, you can read it online at www.project2061.org/publications/bsl/online/index.php?txtRef=&txtURIold=%2Fpublications%2Fonline%2Findex. Another source of information about students ideas is the Probing for Student Understanding books by Page Keeley and available at NSTA Press.
2. For math misconceptions, review the MisMath article provided by G Donald Allen from Texas A & M, TX.
3. Complete a web search for misconceptions on the selected topic. Simply run a search for your topic and misconceptions (e.g., "photosynthesis + misconceptions"). If you run your search at Google Scholar (http://scholar.google.com), you will gain access to numerous resources. In some cases you will only access an abstract, but in others you will find entire documents. This process is more time-consuming than step #1 and step #2 but yields additional resources.
4. An excellent source for a summary of students' misconceptions is *Making Sense of Secondary Science: Research into Children's Ideas (*Driver et al. 1994). The book is outlined by science topic and provides a rich summary of research on children's ideas about these topics.
5. For Physical Science misconceptions, there is a book entitled *Targeting Students' Science Misconceptions: Physical Science Concepts Using the Conceptual Change Model* by Joseph Stephans, 2003. This book is organized by topic and provides background information and guidance for teachers for using a conceptual change model to address student misconceptions.

Based on *Hard-to-Teach Biology Concepts: A Framework to Deepen Student Understanding,* by Susan Koba with Anne Tweed, 2009, NSTA Press.

Common Earth Science Misconceptions [(top)](#)

Phillips, W.C. 1991, Earth Science Misconceptions, Science Teacher Feb'91 pp 21-23.

1. The Earth is sitting on something.
2. The Earth is larger than the Sun.
3. The Earth is round like a pancake
4. We live on the flat middle of a sphere.
5. Astrology is able to predict the future.
6. Gravity increases with height.
7. Gravity cannot exist without air.
8. There is a definite up and down in space.
9. Any crystal that scratches glass is a diamond.
10. Coral reefs exist throughout the Gulf of Mexico and the North Atlantic.
11. Dinosaurs and caveman lived at the same time.
12. Rain comes from holes in clouds.
13. Rains comes from clouds' sweating.
14. Rain falls from funnels in the clouds
15. Rain occurs when clouds are shaken.
16. God and angles cause thunder and lightning.
17. Clouds move because we move.
18. Clouds come from somewhere above the sky.
19. Empty clouds are refilled by the sea.
20. Clouds are formed by vapors from kettles
21. The sun boils the sea to create water vapor.
22. Clouds are made of cotton, wool, or smoke.
23. Clouds are bags of water.

Hapkiewicz, A. (1999). Naïve Ideas in Earth Science. MSTA Journal, 44(2) (Fall'99), pp.26-30. http://www.msta-mich.org

Geosphere

24. Dirt is not the same as soil. Soil comes from rivers, result of volcanic action, or was there since Earth formed.
25. Fossils are actual preserved animals or plant parts.
26. All substances expand when heated.
27. Students are unaware of micro-organisms role as decomposers and recyclers of carbon, nitrogen, water and minerals.

Hydrosphere

28. Water disappears when it evaporates.

29. Clouds are "sponges" that hold water or bags of water than rain when they are shaken by wind or perhaps when they become cold or hot.
30. Ice is smaller than liquid water. There is a loss of mass when water freezes.
31. Groundwater refers to actual lakes and rivers under the surface of the earth.

Common Physical Science Misconceptions (top)

Hapkiewicz, A. (1999). Naïve Ideas in Earth Science. MSTA Journal, 44(2) (Fall'99), pp.26-30. http://www.msta-mich.org

1. When things dissolve they "disappear."
2. Materials can only exhibit properties of one state of matter.
3. Melting and dissolving are confused.
4. Dew formed on the outside of glass comes from the inside of the glass.
5. Expansion of matter is due to the expansion of particles rather than the increased particle spacing.
6. Molecules of a gas "just float" rather than being kept in the gaseous state by their motion.
7. There is not empty space between molecules, rather students believe there is dust, germs or "air' between the particles of air.
8. Particles of solids have no motion.
9. Relative particle spacing among solids, liquids, and gases is incorrectly perceived and not generally related to the densities of the states.
10. Frequent disregard for particle conservation and orderliness when describing physical changes.
11. Gases are not matter because most are invisible.
12. Absence of conservation of particles during a chemical change.
13. Failure to perceived that individual substances and properties correspond to a certain type of particle. Formation of a new substance with new properties is seen as simply happening rather than as a result of particle rearrangement.
14. The temperature of an object drops when it freezes.
15. Mass and volume, which both describe an "amount of matter," are the same property.
16. "Steam" is the visible cloud of water vapor over boiling water.
17. Energy is a "thing." and object or something that is tangible.
18. The chemistry of biological systems does not follow all the same rules of thermodynamics as other systems.
19. "Cold" can be transferred.
20. Energy is truly lost in many energy transformations.

Common Misconceptions about Sound (top)

Hapkiewicz, A. (1992). Finding a List of Science Misconceptions. MSTA Newsletter, 38(Winter'92), pp.11-14.

1. Sounds can be produced without using any material objects.
2. Hitting an object harder changes the pitch of the sound produced.
3. Human voice sounds are produced by a large number of vocal cords that all produce different sounds.
4. Loudness and pitch of sounds are the same things.
5. You can see and hear a distinct event at the same moment.
6. Sounds can travel through empty space (a vacuum).
7. Sounds cannot travel through liquids and solids.
8. Sounds made by vehicles (like the whistle of a train) change as the vehicles move past the listener because something (like the train engineer) purposely changes the pitch of the sound.
9. In wind instruments, the instrument itself vibrates (not the internal air column).
10. Music is strictly an art form; it has nothing to do with science.
11. Sound waves are transverse waves (like water and light waves).
12. Matter moves along with water waves as the waves move through a body of water.
13. When waves interact with a solid surface, the waves are destroyed.
14. In actual telephones, sounds (rather than electrical impulses) are carried through the wires.
15. Ultrasounds are extremely loud sounds.
16. Megaphones create sounds.
17. Noise pollution is annoying, but it is essentially harmless.

Common Misconceptions about LIGHT (top)

Hapkiewicz, A. (1992). Finding a List of Science Misconceptions. MSTA Newsletter, 38(Winter'92), pp.11-14.

1. Light is associated only with a source and/or its instantaneous effects. Light is not considered to exist independently in space. Light is not conceived as moving from one point to another with a finite speed.
2. An object is seen whenever light shines on it, with no recognition that light must move between the object and the observer's eye.
3. A shadow is something that exists on its own. Light pushes the shadow away from the object to a wall, the ground, or other surface where the shadow lies. Shadows are "dark reflections" of objects.

4. Light is not necessarily conserved. It may disappear or be intensified.
5. Lines drawn outward from a light bulb in a sketch represent the "glow" surrounding the bulb. Light from a bulb only extends outward a certain distance and then stops. How far it extends depends on the brightness of the bulb.
6. An observe can see more of his or her mirror image by moving further back from the mirror.
7. The mirror image of an object is located on the surface of the mirror. The image is often thought of as a picture on a flat or curved surface. To be seen in a mirror, the object must be directly in front of the mirror or in the line-of-sight from the observer to the mirror.
8. Light is reflected away from shiny surfaces, but light is not reflected from other surfaces.
9. Light always passes straight through transparent material (without changing direction).
10. When an object is viewed through a transparent material, the object is seen exactly where it is located.
11. An object gives off a "potential image," which travels through space. When the "potential image" reaches a mirror, the image is reversed. Also, the image is distorted by a curved mirror. When the "potential image" reaches a lens, the image may be turned upside down by the lens.
12. Blocking part of the lens surface would block the corresponding part of the image.
13. The purpose of the screen is to capture the image so that it can be seen. Without a screen, there is not image.
14. An image can be seen on the screen regardless of where the screen is placed relative to a lens. To see a larger image on the screen, the screen should be moved further back.
15. An image is always formed at the focal point of a lens.
16. The size of an image depends on the size (diameter) of the lens used to formed the image.
17. Gamma rays, X-rays, ultraviolet light, visible light, infrared light, microwaves, and radio waves are all very different entities.
18. Colors appearing on soap films and oil slicks are reflections of rainbows.
19. Polaroid sunglasses are just dark glass or dark plastic.
20. When waves or pulses interfere (as in a spring, rope, water wave, or light wave) they bounce off each other and go back in the opposite direction from which they came.
21. When a wave moves, particles move along with the wave from the point of transmission to the point of reception.
22. Color is a property of an object, and color is not affected by the eye-brain system or other receiving system.

Hapkiewicz, A. (1999). Naïve Ideas in Earth Science. MSTA Journal, 44(2) (Fall'99), pp.26-30. http://www.msta-mich.org

23. Light fills the room as water fills a bathtub. No mechanisms between the light, the object and the eye produces vision.
24. The primary colors for mixing colored lights are red, blue, and yellow.

Also see misconceptions for Vision and hearing and Color and Vision.

Common Misconceptions about Color and Vision (top)

Hapkiewicz, A. (1992). Finding a List of Science Misconceptions. MSTA Newsletter, 38(Winter'92), pp.11-14.

1. The pupil of the eye is a black spot on the surface of the eye.
2. The eye perceives upright images.
3. The lens is the only part of the eye responsible for focusing light.
4. The lens forms a picture on the retina. The brain then "locks" at this picture.
5. They eye is the only organ necessary for sight.
6. A white incandescent or fluorescent bulb produces light of only one color.
7. Sunlight is different from other sources of light because it is colorless and clear.
8. When light passes through a prism, color is added to the light.
9. When light passes through a filter, color is added to the light.
10. The rules for mixing color paints and crayons are the same as the rules for mixing colored lights.
11. The primary colors for mixing colored lights are red, blue, and yellow.
12. A Colored light striking an object produces a shadow behind it that is the same color as the light.
13. The shades of gray in a printed picture are produced with a gray ink for each shade.
14. The colors in a printed picture are produced with an ink for each color.
15. Colored light is darker than white light.
16. Color is a property of an object, and color is not affected by the illuminating light. The "true" color of an abject is seen in white light. When colored light illuminates a colored object, the color of the light mixes with the color of the object.

Also see misconceptions for Vision and hearing

Common Misconceptions about Astronomy (top)

Hapkiewicz, A. (1992). Finding a List of Science Misconceptions. MSTA Newsletter, 38(Winter'92), pp.11-14.

1. Stars and constellations appear in the same place in the sky every night.
2. The sun rises exactly in the east and sets exactly in the west every day.
3. The sun is always directly overhead or directly south at twelve o'clock noon.
4. The tip of a shadow always moves along an east-west line.
5. Changing distance between the earth and the sun causes seasonal changes (with the two closer in summer and father apart in winter).
6. The earth is the center of the solar system and is the largest object in the solar system. All stars are the same distance from the earth.
7. The moon can only be seen during the night, and its shape always appears the same.
8. The moon does not rotate on its axis as it revolves around the earth.
9. The phases of the moon are caused by shadows cast on its surface by other objects in the solar system, particularly the earth or the sun.
10. The solar system and galaxies are very "crowded." (Objects are relatively close together.)
11. The surface of the sun does not have any visible features.
12. Because all stars are the same size, the brightness of a star depends only on its distance from earth.
13. Stars are evenly distributed through a galaxy or throughout the universe.
14. All the stars in a particular constellation are near each other.
15. The constellations form patterns obviously resembling people, animals, or other objects.

Hapkiewicz, A. (1999). Naïve Ideas in Earth Science. MSTA Journal, 44(2) (Fall'99), pp.26-30. http://www.msta-mich.org

16. Moon and sun are about the same size. Stars are smaller than sun or moon.
17. The earth is the center of the solar system and is the largest object in the solar system.
18. Night occurs when sun covered by clouds, moon, or atmosphere.
19. Astronomical movements explain day and night: Sun goes around earth. Earth goes around sun. Sun moves up and down.
20. The sun is always directly overhead or directly south at noon.
21. The sun rises exactly in the east and sets exactly in the west everyday.
22. The moon can only be seen during the night, and its shape always appears the same.
23. The phases of the moon are caused by shadows cast on its surface by other objects in the solar system, particularly the earth and the sun.
24. All stars are the same size, the brightness of a star depends on its distance from earth.
25. One side of the moon is always dark
26. Stars and constellations appear in the same place in the sky every night.
27. Seasons are caused by changing distance between the earth and sun (the two are closer in the summer and further apart in the winter).
28. Days are shortest in the winter.

Common Misconceptions about Matter and Its Changes [(top)]

Hapkiewicz, A. (1992). Finding a List of Science Misconceptions. MSTA Newsletter, 38(Winter'92), pp.11-14.

1. Gases are not matter because most are invisible.
2. Gases do not have mass.
3. A "thick" liquid has a higher density than water.
4. Mass and volume, which both describes an "amount of matter," are the same property.
5. Air and oxygen are the same gas.
6. Helium and hot air are the same gas.
7. Expansion of matter is due to the expansion of particles, rather than the increased particle spacing.
8. Particles of solids have not motions.
9. Relative particle spacing among solids, liquids, and gasses is incorrectly perceived and not generally related to the densities of the states. (Microscopic model does not represent macroscopic properties.)
10. Materials can only exhibit properties of one state of matter.
11. Particles possess the same properties as the materials they compose. For example, atoms of copper are "orange and shiny," gas molecules are "transparent," and solid molecules are "Hard."
12. Melting/freezing and boiling/condensation are often understood only in terms of water.
13. Particles viewed as mini-versions of the substances they comprise: oxygen molecules are invisible, water molecules are tiny droplets, and diamond molecules are hard.
14. Particles misrepresented in sketches: no differentiation is made between atoms and molecules.
15. Particles misrepresented and undifferentiated in concepts involving elements, compounds, mixtures, solutions, and substances.
16. Frequent disregard for particle conservation and orderliness when describing changes.
17. Absence of conservation of particles during a chemical change.
18. Chemical, rather than interactive. After chemical change, the original substances are perceived as remaining even though they are altered.
19. Failure to perceived that individual substances and properties correspond to a certain type of particle ... formation of a new substance with new properties is seen as simply happening, rather than as a result of particle rearrangement.
20. The "smoke" seen with dry ice is carbon dioxide vapors.
21. The temperature of an object drops when it freezes.
22. The chemistry in biological systems does not follow all the same rules of thermodynamics as other systems.
23. Steam is visible water gas molecules.

Common Misconceptions about Energy (top)

Hapkiewicz, A. (1992). Finding a List of Science Misconceptions. MSTA Newsletter, 38(Winter'92), pp.11-14.

1. Energy is truly lost in many energy transformations.
2. There is no relationship between matter and energy.
3. If energy is conserved, why are we running out of it?
4. Energy can be changed completely from one form to another (no energy losses).
5. Things "use up" energy.
6. Energy is confined to some particular origin, such as what we get from food or what the electric company sells.
7. An object at rest has no energy.
8. The only type of potential energy is gravitational.
9. Gravitational potential energy depends only on the height of an object.
10. Doubling the speed of a moving object doubles the kinetic energy.
11. Energy is a "thing." This is a fuzzy notion, probably because of the way we talk about newton-meters or joules. It is difficult to imagine an "amount" of an abstraction.
12. The terms "energy" and "force" are interchangeable.
13. From the non-scientific point of view, "work" is synonymous with "labor." It is hard to convince someone that more "work" is probably being done playing football for one hour than studying an hour for a quiz.

Common Misconceptions about Force and Motion and Simple Machines(top)

Hapkiewicz, A. (1992). Finding a List of Science Misconceptions. MSTA Newsletter, 38(Winter'92), pp.11-14.

1. Time can be measured without establishing the beginning of the interval.
2. The location of an object can be described by stating its distance from a given point, ignoring direction.
3. The distance an object travels and its displacement are always the same.
4. An object's speed is the same as its velocity.
5. If an object is accelerating, then the object is speeding up.
6. An object's acceleration cannot change direction.
7. Acceleration always occurs in the same direction as an object is moving.
8. If an object has a speed of zero (even instantaneously), it has no acceleration.
9. The only "natural" motion is for an object to be at rest.
10. If an object is at rest, no forces are acting on the object.
11. A rigid solid cannot be compressed or stretched.

12. Only animate objects can exert a force. Thus, if an object is at rest on a table, no forces are acting on it.
13. Force is a property of an object. An object has force, and when it runs out of force it stops moving.
14. The motion of an object is always in the direction of the net force applied to the object.
15. Large objects exert a greater force than small objects.
16. A force is needed to keep an object moving with a constant speed.
17. Friction always hinders motion. Thus, you always want to eliminate friction.
18. Frictional forces are only due to irregularities in surfaces moving past one another.
19. Rocket propulsion is due to exhaust gases pushing on something behind the rocket.
20. When dropped in a vacuum, objects of different masses fall at different speeds.
21. When dropped in a vacuum, objects fall at constant speeds.
22. A simple machine with a mechanical advantage greater than one is easier to use than a simple machine with a mechanical advantage less than one.
23. Any force times any distance is work.
24. Machines put out more work than people put in.
25. Power is the same as force or work.
26. Work is any activity one gets tired doing, gets paid for doing, or doesn't like doing.

Hapkiewicz, A. (1999). Naïve Ideas in Earth Science. MSTA Journal, 44(2) (Fall'99), pp.26-30. http://www.msta-mich.org

1. Forces acting on bodies/objects are associated with living things.
2. Constant motion requries a constant force.
3. If a body is not moving, there is no force acting upon it.
4. Objects in a vacuum fall at a constant speed.
5. If a body is in mortion, there is a force acting upon it in the direction of motion.
6. There is no gracity in space.

Common Misconceptions about Measurement (top)

Hapkiewicz, A. (1992). Finding a List of Science Misconceptions. MSTA Newsletter, 38(Winter'92), pp.11-14.

1. Measurement is only linear.
2. Any quantity can be measured as accurately as you want.
3. Children who have used measuring devices at home already know how to measure.
4. The metric system is more accurate than other measurement systems (such as the English system).
5. The English system is easier to use than the metric system.

6. You can only measure to the smallest unit shown on the measuring device.
7. Some objects cannot be measured because of their size or inaccessibility.
8. The five senses are infallible.
9. An object must be “touched” to be measured.
10. A measuring device must be a physical object.
11. Mass and weight are the same and they are equal at all times.
12. Mass and volume are the same.
13. The only way to measure time is with a clock or a watch.
14. Time has an absolute beginning.
15. Heat and temperature are the same.
16. Heat is a substance.
17. Cold is the opposite of heat and is another substance.
18. There is only one way to measure perimeter.
19. Only the area of rectangular shapes can be measured in square units.
20. Surface area can be found only for two-dimensional objects.
21. Surface area is a concept used only for mathematics classes.
22. You cannot measure the volume of some objects because they do not have “regular” lengths, widths, or heights.
23. An object’s volume is greater in water than in air.
24. The density of an object depends only on its volume.
25. Density for a give volume is always the same.
26. The density of two samples of the same substance with different volumes or shapes cannot be the same.

Hapkiewicz, A. (1999). Naïve Ideas in Earth Science. MSTA Journal, 44(2) (Fall’99), pp.26-30. http://www.msta-mich.org

1. The location of an object can be described by stating its distance from a given point, ignoring direction.

Common Misconceptions about Electricity (top)

Hapkiewicz, A. (1992). Finding a List of Science Misconceptions. MSTA Newsletter, 38(Winter’92), pp.11-14.

1. Objects become positively charged because they have gained protons.
2. Objects become positively charged because their electrons have been destroyed.
3. All atoms are charged.
4. Larger magnets are stronger than smaller magnets.
5. Current flows from a battery (or other source of electricity) to a light bulb (or other item that consumes electricity), but not from the light bulb to the battery.
6. current flows out of both terminals of a dry cell or both connections in an electrical outlet.
7. Current flows around a complete circuit, but it is used u by objects like light bulbs so less current returns than leaves the source of the electricity.

8. All the electrons that make up a electrical current are initially contained in the battery or generator that is the source of the electricity.
9. Electricity is produced in the wall socket.
10. Electrons change into light when a lamp is turned on.
11. Wires are hollow like a water hose, and electrons move inside the hollow space.
12. A larger battery will make a motor run faster or a bulb burn brighter.
13. Pure water is a good conductor of electricity.
14. Electricity from a dry cell will shock or hurt if it is touched.
15. Insulation is used to keep electricity in the wire.
16. All wires are insulated.,
17. Birds can perch on bare wires without being hurt because birds have insulated feet.
18. A charge object can only affect other charged objects.
19. The electrostatic force between two charged objects in not affected by the distance between them.
20. Gravitational forces are stronger than electrostatic forces.

Common Misconceptions about Magnetism (top)

Hapkiewicz, A. (1992). Finding a List of Science Misconceptions. MSTA Newsletter, 38(Winter'92), pp.11-14.

1. All metals are attracted to a magnet
2. All silver colored items are attracted to a magnet.
3. All magnets are made of iron.
4. Larger magnets are stronger than smaller magnets.
5. The magnetic and geographic poles of the earth are located at the same place.
6. The magnetic pole of the earth in the northern hemisphere is a north pole, and the pole in the southern hemisphere is a south pole.
7. Only magnets produce magnetic fields.
8. A magnetic field is a pattern of lines (not a field of force) that surrounds a magnet.
9. In a magnet, the magnetic field lines exist only outside the magnet.

Common Misconceptions about Heat and Temperature (top)

Hapkiewicz, A. (1992). Finding a List of Science Misconceptions. MSTA Newsletter, 38(Winter'92), pp.11-14.

1. Ice cannot change temperature
2. When the temperature of a boiling substance remains constant, something is "wrong."

3. The bubbles in boiling water contain "air," Oxygen," or "nothing," rather than water vapor.
4. All liquids boil at 100°C (212°F) and freeze at 0° C (32°F).
5. Heat is a substance.
6. Heat is not energy.
7. Temperature is a property of a particular material or object (metal is naturally colder than plastic).
8. The temperature of an object depends on its size.
9. Heat and cold are different, rather than being opposite ends of continuum.
10. Boiling is the maximum temperature a substance can reach.
11. Objects of different temperatures which are in constant contact with each other, or in contact with air at a different temperature, do not necessarily move toward the same temperature.
12. Heat only travels upward.
13. Heat rises.
14. The kinetic theory does really explain heat transfer. (It is recited, but not believed.)
15. Objects which readily become ware (conductors of heat) do not readily become cold.
16. All solids expand at the same rate.

Common Misconceptions about Forces in Fluids (top)

Hapkiewicz, A. (1992). Finding a List of Science Misconceptions. MSTA Newsletter, 38(Winter'92), pp.11-14.

1. Objects float in water because they're "lighter" than water.
2. Objects sink in water because they're "heavier" than water.
3. Mass, volume, weight, heaviness, size, and density may be perceived as equivalent.
4. Wood floats and metal sinks.
5. All objects containing air float.
6. Liquids of high viscosity are also liquids with high density.
7. Adhesion is the same a cohesion.
8. Heating hair only makes it hotter.
9. Pressure and force are synonymous.
10. Pressure arises from moving fluids.
11. Moving fluids contain higher pressure.
12. Liquids rise in a straw because of "suction."
13. Fluid pressure only acts downward.

Common Misconceptions about Weather (top)

Hapkiewicz, A. (1999). Naïve Ideas in Earth Science. MSTA Journal, 44(2) (Fall'99), pp.26-30. http://www.msta-mich.org

1. Air as a substance is not well understood. Air has negative weight or not weight.
2. Days are shortest in winter.
3. Water vapor is held or soaked up by the air.
4. Humid air is "Heavier" than dry air.
5. A vacuum or low pressure area "pulls" object into it.
6. Wind speed is related to temperature of air - high speed means cold air and gentle or slow winds are warm.
7. Acid rain, ozone depletion and greenhouse effect are thought to be caused by same things and produce the same changes in the environment.
8. Air pollution is always caused by human activities.
9. Rain water should be neutral in pH.

Common Misconceptions in the Life Sciences (top)

Berthelsen, B. (1999). Students Naïve Conceptions in Life Science. MSTA Journal, 44(1) (Spring'99), pp. 13-19. http://www.msta-mich.org/

Organization of Living Things:

1. Plants, fungi, eggs and seeds are not living.
2. Young children do not recognize tress as living although they understand that seedlings are alive.
3. Only large land mammals are animals.
4. Penguins and turtles are amphibians because they are both in and out of water.
5. Whales, jellyfish, and starfish are all fish.
6. Behavior and habitat are criteria for classification.
7. Food is anything useful taken into the body including: water, minerals, carbon dioxide (plants), and sunlight.
8. Students concept of digestion is often confused both in the route and the process.
9. Digestion is the process that releases usable energy from food.
10. Respiration is synonymous with breathing.

Common Misconceptions about Plants - Obtaining and Using Energy (top)

Berthelsen, B. (1999). Students Naïve Conceptions in Life Science. MSTA Journal, 44(1) (Spring'99), pp. 13-19. http://www.msta-mich.org

1. Plants obtain their energy directly from the sun.
2. Plants have multiple sources of food (heterotrophic as well as autotrophic).
3. Carbon dioxide, water, and minerals are food.
4. Plants feed by absorbing food through their roots.
5. Plants use heat from the sun as a source of energy for photosynthesis
6. Sunlight is a food.
7. Sunlight is composed of molecules.
8. Sunlight is "consumed" in photosynthesis.
9. Plants absorb water through their leaves.
10. Plants produce oxygen for our benefit.

Common Misconceptions about Cells (top)

Berthelsen, B. (1999). Students Naïve Conceptions in Life Science. MSTA Journal, 44(1) (Spring'99), pp. 13-19. http://www.msta-mich.org

1. Students are unsure about the hierarchy of atoms, molecules and cells. Cells are described as the components of many things including carbohydrates and proteins
2. Students have difficulty discriminating between cell division, enlargement and differentiation. They may believe that living things grow because their cells get larger. The role of cell differentiation in growth is poorly understood.
3. Students think in terms of two kinds of cells - plant and animal.

Common Misconceptions about Ecosystems (top)

Berthelsen, B. (1999). Students Naïve Conceptions in Life Science. MSTA Journal, 44(1) (Spring'99), pp. 13-19. http://www.msta-mich.org

1. Stronger organisms have more energy.
2. There are more herbivores because they have more offspring.
3. A species high on the food web is a predator to everything below it.
4. Energy accumulates in an ecosystem so that a top predator has all the energy from the organisms below it.
5. Carnivores can exist in a plant free world if their prey reproduce enough.
6. The food that is eaten and used as a source of energy is part of the good chain; food that is synthesized into the body of the eater is now food for the next level.

Common Misconceptions about Heredity (top)

Berthelsen, B. (1999). Students Naïve Conceptions in Life Science. MSTA Journal, 44(1) (Spring'99), pp. 13-19. http://www.msta-mich.org

1. Daughters inherit most of their characteristics from their mothers. Boys inherit most of their characteristics from their fathers.
2. Variation between species is a result of adaptation to environment instead of inheritance.
3. Sexual reproduction occurs in animals but not in plants.
4. Students do not distinguish between sexual and asexual reproduction.
5. Asexual reproduction produces weak offspring. Sexual reproduction produces superior offspring.
6. Students believe that transmitted characteristics are acquired during the life time of the animal.
7. Individuals can adapt to a changing environment. These adaptations are heritable.
8. Students do not understand the relationship between DNA, genes, and chromosomes
9. Students can apply chance and probability to assigned genetics problems, but not to human situations in families.

Common Misconceptions about Evolution (top)

Berthelsen, B. (1999). Students Naïve Conceptions in Life Science. MSTA Journal, 44(1) (Spring'99), pp. 13-19. http://www.msta-mich.org

1. Students have difficulty relating an individuals adaptation to environment with changes in species phenotypes over long period of time due to selection.
2. Students believe that transmitted characteristics are acquired during the life time of the animal.
3. Individuals can adapt to a changing environment. These adaptations are heritable.

Common Misconceptions about Vision and Hearing (top)

Berthelsen, B. (1999). Students Naïve Conceptions in Life Science. MSTA Journal, 44(1) (Spring'99), pp. 13-19. http://www.msta-mich.org

1. Objects are seen because they are bathed in light.
2. Light travels from the eyes to the object.
3. We can see because light travels to your eyes and then from the eyes to the object.
4. Light to both our eyes and the object. There is o link between the two.

5. Light is not necessary to see since we can see a little in a dark room.
6. We see because we look, it has nothing to do with light.
7. We can hear because we concentrate on the source of the sound.
8. There is no similarity between light and sound.

Student Thinking

Lesson 1. Misconceptions in mathematics

Objectives

1. What is a misconception?
2. Why study misconceptions?
3. Misconeptions in algebra
4. Miconceptions about math - in general
5. References
6. Student thinking - TEKS connections
7. Discussion Questions

What is a misconception?

1. Student thinking consists of many things. Formulas, relevance, tedium, and enjoyment are part of their attitudes and thinking about mathematics. One problem that leads to very serious learning difficulties in mathematics is those misconceptions student may have from previous inadequate teaching, informal thinking, or poor remembrance. It may be best to begin with a definition. From the Encarta online dictionary, a misconception is “a mistaken idea or view resulting from a misunderstanding of something." Paraphrasing from the educational literature [Pines, 1985] , we offer,

> Certain conceptual relations that are acquired may be inappropriate within a certain context. We terms such relations as "misconceptions." A misconception does not exist independently, but is contingent upon a certain existing conceptual framework. Misconceptions can change or disappear with the framework changes.

Changing the conceptual framework of students is one of the keys goals in repairing mathematics and science misconceptions. That is to say, it is not usually successful to merely inform (e.g. lecture) the student on a misconception. The misconception must be changed internally partly through the student’s belief systems and partly through their own cognition.

In another misconceptions framework, we may say many students do not come to the classroom as "blank slates" (Resnick, 1983). Rather, they come with informal theories constructed from everyday experiences. These theories have been actively constructed. They provide an everyday functionality to make sense of the world but are often incomplete half-truths (Mestre, 1987). They are misconceptions.

In this module, we consider student misconceptions in mathematics, particularly those that impact algebra and algebraic thinking. Yet, misconceptions are but one facet of **faulty**, **inaccurate**, or **incorrect** thinking. These are all intertwined causing students unlimited trouble in grasping with mathematics from the most elementary concepts through calculus. In turn, student misconceptions cause teachers immense frustration about why their teaching isn’t "getting through."

Why study misconceptions?

Our first thought would be that misconceptions, once rooted in the student's memory, are hard to erase. The situation is somewhat more complex. Researchers' interest in student conceptions has been provoked by numerous studies indicating that

1. Before formal study, persons have firmly held, descriptive, and explanatory systems for scientific and logico-mathematical phenomena, that is, systems of belief about mathematics.
2. These systems of belief differ from what is incorporated into the standard curriculum.
3. Certain constellations of these belief systems show remarkable consistency across ages, abilities, and nationalities.
4. Belief systems are resistant to change through traditional instruction.
   See (Champagne,Gunstone, & Klopfer, 1983; Osborne & Wittrock, 1983)
   See also (Confrey, Jere, 1990).

This research also suggests that repeating a lesson or making it clearer will not help students who base their reasoning on strongly held misconceptions. (Champagne, Klopfer & Gunstone, 1982; McDermott, 1984; Resnick, 1983). Students tend to be emotionally and intellectually attached to their misconceptions, partly because they have actively constructed them and partly because they give ready methodologies for solving various problems. They definitely interfere with learning when students use them to interpret new experiences.

It is very important to recognize student misconceptions and to re-educate students to correct mathematical thinking.

Although the results apply more to children younger than high school age, Ginsburg [1977] offers a number of observations about errors:

1. Errors result from organized strategies and rules.
2. Faulty rules underlying errors have sensible origins.
3. Too often children see arithmetic as an activity isolated from their ordinary concerns. (As you will note below, many misconceptions and faulty thinking in algebra are related to misconceptions and faulty thinking with arithmetic (e.g. fractions).
4. Children often demonstrate a gap between formal and informal knowledge.

The last point on formal vs. informal knowledge requires definition. Usually, formal knowledge refers to that which is taught in an organized, structured, educational institution. It refers to a system of interrelated definitions and proofs, experiments and arguments. It usually is linked with written methods. On the other hand, informal knowledge refers to more tentative intuitive conjectures and mental strategies. Informal knowledge is generated or learned through one's personal actions. That is, informal knowledge refers to routines that are carried out mechanically, or by habit, or by tradition.

A body of research has also developed connections of misconceptions to math anxiety, as well contributions of acceptance of misconceptions about mathematics, mathematical self-concept, and arithmetic skills to mathematics anxiety. In a study of 92 adult students aged 18 to 57 with a median age of 27, (16 males and 76 females) taking a statistics course, results showed that acceptance of misconceptions and mathematical self-concept were significantly related to mathematics anxiety. The combination of misconceptions, mathematical self-concept and arithmetic skills was significantly related to statistics course performance. Older students returning to school after several years' absence were the ones most debilitated by negative attitudes toward mathematics. It was concluded that mathematics anxiety involves a mechanistic, nonconceptual approach to math, a low level of confidence and a tendency to give up easily when answers are not immediately apparent.

How to re-educate away from misconceptions?

As mentioned, misconceptions must be deconstructed, and teachers must help students reconstruct correct conceptions. Lochead & Mestre (1988) describe an effective inductive technique for these purposes. There are three steps.

1. Probe for and determine qualitative understanding.
2. Probe for and determine quantitative understanding.
3. Probe for and determine conceptual reasoning.

In addition, it is helpful to confront students with counterexamples to their misconceptions. A self-discovered counterexample will have a far stronger and lasting effect. Incorrect beliefs can be loosened somewhat when so confronted.

Misconceptions in Algebra

Many misconceptions apparent in algebra are rooted in misconceptions of arithmetic.

1. Arithmetic.
   a. Relations between percentages (%), decimals, and fractions is not well understood. Finding 9.5% or 123% of a number may be a problem for many students. Converting decimal to percent and the inverse are also misunderstood.
   b. The pseudo equation $-4 + \ ? \ = -10$ causes many students trouble early on. This misconception lies with the improper understanding of negative numbers. Some teachers use expressions as *owning* when the quantity is positive and *owing* if it negative. Is this wise? In algebra, there are numerous problems of solving equations for which this lack of understanding/misconception is the source of the difficulty.
2. Number sense. Students do not understand well the difference between rational and irrational numbers. Some think that $\frac{3\pi}{5\pi}$ is irrational, and many think that repeating decimals such as $23.4545\overline{45}$. are irrational.
3. Exact vs. approximate. If we compute on our calculator $\frac{\sqrt{3}+\sqrt{7}}{\sqrt{2}} = 3.09557356478$, many students will assume this answer is exact. Similarly calculating $\frac{1}{7}$ as 0.142857142857 will be interpreted as exact. This misunderstanding may well arise from overreliance of calculators or improper teaching of the meaning of numbers generated by the calculator.
4. Fractions. For most of mathematics through calculus, it happens that misconceptions about fractions provide the root source of many student difficulties. Many of these problems come from fractions not being properly understood.
   a. Example. Operations. Incorrect cancelling of
   $$\frac{ab+c}{b} \text{ to obtain } a+c$$
   b. Example. Working with large numerators and denominators rather than reducing: when multiplying fractions such as $\frac{13}{7} \cdot \frac{14}{65}$ to get $\frac{13 \cdot 14}{7 \cdot 65} = \frac{182}{455} = \frac{2}{5}$ Instead of cancelling common factors as $\frac{13}{7} \cdot \frac{14}{65} = \frac{1}{1} \cdot \frac{2}{5} = \frac{2}{5}$. This is a bad habit because

working with large numbers increases chances of making a mistake dramatically and, even more importantly, because not seeing numbers as products in arithmetic makes it hard to see the same in working with algebraic expressions. This example is one of faulty procedures leading to algebraic difficulties.

c. Example. Understanding the "invert and multiply" rule for dividing fractions escapes the comprehension of the vast majority of students. Explanation: The misconception here lies at the nature of the notation and exactly what the division means. In fact

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \left(\frac{c}{d}\right)^{-1}\left(\frac{a}{b}\right) = \frac{d}{c} \cdot \frac{a}{b} = \frac{ad}{bc}$$

So, the invert and multiply rule is little more than writing the multiplicative inverse in place of division and then noting that $\left(\frac{c}{d}\right)^{-1} = \frac{d}{c}$ because

$$1 = \left(\frac{c}{d}\right)^{-1}\left(\frac{c}{d}\right) = \left(\frac{d}{c}\right)\left(\frac{c}{d}\right) = \frac{cd}{cd}$$

d. Example. Addition of fractions generates a number of errors, most particularly with treating different denominators incorrectly. There is the notational conflict between adding multiplying fractions. In multiplying fractions the numerators and denominators, respectively, are multiplied to achieve the numerator and denominator of the product. It is natural to *want* to do this for addition. In a sense, the student wants to do something correct and proper, but not understanding the absolute need to determine a common denominator at the first step, uses the default, but incorrect, analog method for multiplying fractions.

e. Example. The very term "cancel" or "cancelling" can cause confusion because both the addition and division operations are involved while the results are respectively, say, zero or one.

5. Magnitudes for negative numbers. For example, which number is larger –8 or –5 ? The problem here is how the student perceives "large." Some teachers mistakenly use expressions such as the "larger negative value." This causes confusion to many students.
6. Order of operations. This is perhaps the most common misconception.
   a. Example. Many students will compute $4 + 3x^2$ as $7x^2$, effectively interpreting $4 + 3x$ as $7x$.
   b. Example. Students often misuse the distributive rule. We may see $x^2 - 2(x - 3)$ written as $x^2 - 2x - 6$. The –2 is not correctly distributed.
7. Powers. Students have trouble with precedence of operations.
   a. Example. Compute $-4^2 = -16$. Many will incorrectly compute $(-4)^2 = 16$.
   b. Example. Compute $16^{(-1/4)}$. Any number of answers may be reported including –2 and 2 rather than correctly as $\frac{1}{2}$.
   c. Example. Is $a \times a \times a$ the same as $3a$? Similarly is a^3 the same as $a \times 3$? Some students are not secure in correct mathematical meaning of various notations. This is partly a fault of there being multiple representations of the same thing. It is partly the misconceptions of not fully understanding and being functionally fluent in making computations.
8. Square roots - definition. Many students have trouble with the correct definition.

$$\left(\sqrt{a^2}\right) = |a|$$

More generally, students have trouble with

$$\sqrt[n]{a^n} = \begin{cases} a & \text{if } n \text{ is odd} \\ |a| & \text{if } n \text{ is even} \end{cases}$$

Even more generally, students have trouble when the definition of a mathematical object is divided by cases.

a. Example. $\left(\sqrt{(-8)}\right)^2 = |-8| = 8.$ Many will give –8 as the answer. In more generality many students will want to write $(x-5)$ as equal to $\sqrt{(x-5)^2}$ rather than correctly writing $|x-5|$.

9. Square roots - with sums. A typical error many students make is to write

$$\sqrt{a+b} = \sqrt{a} + \sqrt{b}$$
$$\sqrt{a-b} = \sqrt{a} - \sqrt{b}$$

This persists well into college.

10. Simplification/factorization of algebraic expressions. Students typically abandon the rules or misinterpret them in many types of simplification problems.
   a. Example. Simplifying a rational expression like $\frac{x+2}{x}$ ....students often think this expression simplifies to 2.
   b. Example. Expanding perfect square binomials. Many students compute $(x+3)^2$ as $x^2 + 9$ or even worse $x^2 + 6$.
   c. Example. More generally, we often see $(a+b)^n = a^n + b^n$.
   d. Factorization. Many students do not see a common factor in an expression such as $2 \cdot 3 \cdot 4 \cdot 25 + 5$. This could be why they have difficulty seeing the common factor in $2x^2y^3 + 3x$.

11. Using the definition of the absolute value function, particularly for negative numbers. The definition

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

just causes many students to pause. They understand the idea of the absolute value. However, it is here the actual mathematics notation that causes trouble. It is important for students to use this definition for several problem examples. It is particularly important in many algebra problems, particularly those with solving inequalities.

12. Inequalities. Students have trouble solving various types of inequalities. The worst offenders are inequalities involving quadratic terms and inequalities with an absolute value.
   a. Example. Solve $x^2 - 8 > 0$.
   b. Example. Solve $|x-7| > 6$.

students need to skilled at fundamental facts about factoring and what the terms actually mean. In the first example, the one procedure is factoring $\left(x-\sqrt{8}\right)\left(x+\sqrt{8}\right)$. Then, noting the product of two values is positive only if the multiplicands are either both positive or both negative, we solve $x - \sqrt{8} > 0$ and $x + \sqrt{8} > 0$. This also gives the solutions for the terms being negative. We could also write $x^2 > 8$ and take square roots. In this situation, the student must take both roots. This procedure also involves the expression $\sqrt{x^2}$ discussed above. The second example, Solve $|x-7| > 6$, requires the student to have a working knowledge of the definition of absolute value.

13. Expansion of algebraic expressions. In some views, the only things the students seem to be able to do correctly are FOIL and add/subtract a constant from both sides of an equation.
14. Exponentials - Properties.. Students are not firm with negative and fractional exponents. Students have trouble with correctly simplifying

$$\frac{a^m}{a^{-n}}, \ a^m a^{-n}, \ a^{-m}a^{-n}$$

15. Exponential functions. Students do not work well with exponents. Once misconceptions take root, they become hard to eliminate.
    a. Example. Students will often linearize exponential rules such as writing e^{a+b} as $e^a + e^b$.
    b. Similarly, we often see e^{ab} written as $e^a \ e^b$.
16. Logarithm - properties.
    a. Often students will write $\log x - \log y = \frac{\log x}{\log y}$ instead of the correct expression $\ln \frac{x}{y}$.
    b. Students will also linearize rules and produce such aslog $\ln(a+b) = \ln a + \ln b,$ and $\ln(2x) = 2\ln x$
17. Logarithms - solving equations. When solving a logarithmic equation, students forget to check if the answer is in the domain, or if they get two answers and the first one checks, they tend to automatically eliminate the second choice.
    a. Example. Solve $\log_2(x-4) = 3 - \log_2(x+3)$.
       Solution.

$$\begin{aligned} \log_2(x-4) &= 3 - \log_2(x+3) \\ \log_2(x-4) + \log_2(x+3) &= 3 \\ \log_2((x-4)(x+3)) &= 3 \\ (x-4)(x+3) &= 2^3 = 8 \\ x^2 - x - 12 &= 8 \\ x^2 - x - 20 &= 0 \\ (x-5)(x+4) &= 0 \\ x &= 5, \ -4 \end{aligned}$$

       The solution $x = 5$ is valid, but the solution $x = -4$ is not. Students often do not check this. This serves as the type of misconception or misbelief that if an algorithm is followed correctly, only correct answers will result.
    b. Example. Solve $\log(x^2 - 7) = \log(x-5)$.
       Solution.

$$\begin{aligned}
\log(x^2-7) &= \log(x-6)\\
\log(x^2-7)-\log(x-6) &= 0\\
\log\frac{(x^2-7)}{(x-6)} &= 0\\
\frac{(x^2-7)}{(x-6)} &= 1\\
(x^2-7)-(x-6) &= 1\\
x^2-x-2 &= 0\\
x &= 2,\ -1
\end{aligned}$$

Note when substituted into the original equation, both left and right side are undefined.

18. Functions. Finding the domain of a rational function when a common factor is present in the numerator and denominator.
    a. Example. What is the domain of

$$\frac{x-3}{x^2-4x+3}$$

Clearly, the domain is $\{x \mid x \neq 3 \text{ and } x \neq 1\}$. Students tend to cancel the common factor and work with what remains. (Cancellation only works when the factor cancelled is nonzero.)

19. Functions - asymptotes. Vertical and horizontal asymptotes are often confused because students poorly understand the reasons making which one vertical and which one horizontal. Also, when asked to count how many asymptotes a function has, the student will frequently count only the vertical ones.
    a. Example. The rational function $f(x) = \frac{x-3}{x^2-1}$ has how many asymptotes.

20. Translational errors. Consider the problem given to freshmen college engineering majors. (Clement, 1982).
    Example. Write an equation using the variables C and S to represent the following statement: "At Mindy's restaurant, for every four people who ordered cheesecake, there are five people who ordered strudel." Let C represent the number of cheesecakes and S represent the number of strudels ordered. The correct answer is $5C = 4S$. This question was missed answered correctly by only 27% of the students. The typical wrong answer was $4C = 5S$. Students were also given the hint: "Be careful. Some students put a number in the wrong place in the equation." This hint improved correctness of this answer by only 6%.

    The translational error here is the incorrect conversion of the words to symbols. Clearly this problem is rather nonstandard in appearance. One way to solve it is to consider the equation $\frac{S}{5} = \frac{C}{4}$, which equates groups of five strudel orders with groups of four cheesecake orders. The correct equation follows from this.

    For students, the misconception arises from two factors. First, students translate the words of the problem from left to right. Second, they confuse the idea of variables and labels. Using a left-to-right strategy, students interpret the C and the S in the equation as labels for the terms "cheesecake" and "strudel." They apparently fail to apply the idea that variables stand for numerical expressions.

21. Algebra word problems are interpreted through misconceptions.
    a. Example. Original and sale prices. Students often mistake the way in which an original price and a sale price relate to one another. They may incorrectly calculate the original price from a sale price by applying the discount to the known sale price, rather than to an unknown original price.

Online resources:

1. http://www.mathsyear2000.co.uk/resources/misconceptions/index.shtml
2. http://www.ericdigests.org/pre-9213/hispanic.htm

Misconceptions about math — in general

Society has many misconceptions about mathematics and its role in our world.

1. Mathematics is incorrectly viewed as a collection of rigid rules and mysterious procedures that seem to be unrelated to each other and require total mastery with little or no understanding.
2. Mathematics is perceived by many to be difficult and demanding and is considered to be a subject in which it is socially acceptable to do poorly.
3. Mathematical thinking is regarded as essentially unimportant to people that do not actually "do" mathematics in their employment capacity.
4. The pervasive role of mathematics is underestimated in the world of everyday living.

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Student Thinking and Misconceptions — TEKS Connections

In Algebra I,

1. Multiple Representation is a(3).
   The students are going to represent and model problem situations through analyzing and interpreting relationships between two quantities in the function.
2. Student Thinking (misconceptions) is a(4): students are supposed to use a variety of methods to solve equations. Different approaches in solving the problem is the way to look for any misconception that the students might have. The teacher should conquer this misconception for the students through exposing them to many methods in solving the same problem. Students work in many situations to set up equations and inequalities and use different methods to solve them.
3. Problem-solving is a(6): the students use the problem solving at the end to apply what have been learned in the class in the real life (Meaningful mathematics).
4. Algebraic thinking is a(5) &a(2): the students are solving the meaningful problems in algebra after their experience with different algebraic tools they have learned. Symbolic reasoning is powerful in expressing an algebraic thinking.

In Algebra II,

1. Multiple Representation is a(4).
   The students perceive the connection between Algebra and Geometry and try to use the tools of one to solve problems in the other. Also, the relationship between the function and equation is powerful tool in expressing generalization.
2. Student Thinking (misconceptions) is a(3):
   The students are supplied with more mathematical experiences to overcome any misconception in their algebraic thinking. Understanding and analyzing a broad variety of relationships between functions and equations is to express generalizations.
3. Problem-solving is a(6): the students use the problem solving at the end to apply what have been learned in the class in the real life.
4. Algebraic thinking is a(5) & a(2): students model mathematical situations to solve meaningful problems in Algebra.

In Pre-calculus,

1. In the introduction section b(1) & b(2), we can see all the items achieved in them.
2. Multiple representations are used to solve different real life problems in order to overcome

all students' misconceptions at that particular level of algebraic thinking.

Misconceptions in Math — Discussion Questions and Activities

1. Develop strategies to re-educate students for misconceptions about the solution of inequalities described in #13 above.
2. Determine counterexamples that can help students resolve misconceptions about cancellation of variables in algebraic quotients.
3. Discuss any misconceptions you may have discovered in your own algebraic thinking. What mistakes have you commonly made? Are they misconceptions or perhaps poorly understood ideas.?
4. Develop a brief diagnostic test to determine common misconceptions.
5. How can you determine whether students are using formal or informal knowledge in making algebraic calculations? How can reliance on informal knowledge lead to continued misconceptions? Make an argument for students having a solid formal knowledge for continued correct calculations.
6. Rank the algebra misconceptions in order of importance. Give reasons for your ranking.

Misconceptions in Math - Survey

Directions. For problems 1-4, please circle the answer that is best reflected by the Misconceptions in Math readings. For 5-7, give your impressions about the topic.
Use the back of this paper if you need more room.

1. Misconceptions are linked to
   A. The belief system. B. Incorrect thinking.
   C. Algebraic Thinking. D. Blank slates.

2. The best thing to do when you discover your students have a misconception is
   A. Re-teach the lesson. B. Re-teach the lesson in a clearer way.
   C. Inform the students of their misconception.
   D. Re-educate the belief causing the misconception.

3. Student misconceptions can cause
   A. Sleepless nights. B. Math anxiety.
   C. A stronger desire to learn correctly. D. Hostility toward mathematics.

4. Which of the following is not an observation about student errors in mathematics?
   A. Errors result from organized strategies and rules.
   B. Faulty rules underlying errors have sensible origins.
   C. Children often demonstrate a gap between formal and informal knowledge.
   D. Inaccurate concept maps for mathematical thinking.

5. When I teach mathematics and notice students have a misconception about some topic, my best strategy will be to

6. The most surprising aspects about student misconceptions I learned from this reading was

7. I found the reading very useful and plan to use this knowledge in my teaching.
   ☐ Strongly Agree ☐ Agree ☐ Neutral ☐ Disagree ☐ Strongly Disagree

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Table 4.2

Teacher Work Template: Responsive Phase

Lesson Topic—Flow of Energy and Matter: Photosynthesis	
Identifying Student Preconceptions	Two major activities are used to determine students' preconceptions in this lesson. The first targets the "big ideas" for the lesson and the second is used with Learning Target #1, which addresses a major misconception related to photosynthesis and plant growth. 1. Use a concept cartoon to probe students' understandings about carbon as the source of plant biomass. 2. Use student-developed, annotated drawings to determine students' ideas about the processes and resources a plant uses to grow from seed to mature plant. The intent is to determine students' current understandings about flow of matter and cycling of energy in a plant system.

Learning Sequence Targets

Learning Target #1	The vast majority of plants are able to convert inorganic carbon in CO_2 into organic carbon through photosynthesis. Carbon dioxide and water are used in the process to create biomass. The surrounding environment is the source of raw materials for photosynthesis.

Research-Identified Misconceptions Addressed

- When asked to describe a plant's needs, some students attribute anthropomorphic characteristics to plants, such as breathing, drinking, and eating (Barman et al. 2006; Ebert-May 2006).
- Some students of all ages are unaware that plants make their food internally, thinking instead that they take it in from the outside. They struggle to comprehend that plants make their food from water and air, and that this is their only source of food (AAAS 1993).
- Students think photosynthesis provides energy for uptake of nutrients through roots and building biomass and that no biomass is built through photosynthesis alone (Ebert-May 2006).
- Some students at all ages think plants get most of their food from the soil, through their roots. This is why some students will say that plants need fertilizer (Barker 1995; Barman et al. 2006; Driver et al. 1994; Köse 2008; Russell, Netherwood, and Robinson 2004).
- Many students know that water is absorbed through a plant's roots, but they assume that water is the primary growth material for the plant. Other studies show that students often think minerals are food for plants or that they directly contribute to photosynthesis (Driver et al. 1994).
- There is disbelief that weight increase in plants is due to a gas (CO_2), even if students know that the gas is absorbed by plants (Driver et al. 1994; Ebert-May 2006).
- Only a third of 15-year-olds understand gas exchange in plants or that green plants take in carbon dioxide. Forty-six percent of 16-year-olds do not understand that increased photosynthesis decreases the level of carbon dioxide in a closed system (Driver et al. 1994).

Table 4.2 (continued)

Initial Instructional Plan
Eliciting Preconceptions: Notice that each of the research-identified misconceptions revolves around the source of biomass in the plant or around what causes the plant to grow. Students attribute mass in the growing plant to almost anything but the carbon in CO_2, and the concept cartoon elicits our own students' preconceptions by choosing each student's individual "best answer" to the cartoon. ***Confronting Preconceptions:*** Groups of students now determine the group's "best answer" to the cartoon, confronting their individual conceptions as well as those of their peers. At the same time, teachers should support the metacognitive focus by using a strategy Claim/Support/Question, found at the Visible Thinking website (*www.pz.harvard.edu/vt*). It requires students to clarify claims of truth by making claims, identifying support for their claims, and further questioning their own claims. Have student groups consider the stance taken in their answers and ask them, "How would you test your claim?" There are many resources we can use to help students test their claims. We suggest these two options: *Option #1:* Students mass out three batches of radish seeds, each batch weighing 1.5g. Apply various experimental treatments to the seeds (e.g., seeds on moist paper towels in the light, seeds on moist paper towels in the dark, and seeds not moistened in the light). Grow the seeds for one week. Then dry them overnight in an oven and measure biomass in grams. Prior to revealing results, have students predict the biomass of the various treatments (Ebert-May 2003). *Option #2:* Students design experiments using Wisconsin Fast Plants (floating leaf discs) to test their claims. Information about use of Fast Plants as well as developed activities can be found at *www.fastplants.org*. ***Sense Making:*** Students write an explanation about the results once results are revealed. If there is not enough time to conduct the Option #1 experiment, the author's (Ebert-May 2003) results can be used (light no water,1.46g; light, water,1.63g; and no light, water, 1.20g). Conclude with a discussion comparing student results to the Van Helmont experiment and his conclusions. Have students discuss in small groups and then record their explanations in their science notebooks. Additional discussion can help students explore common research-identified misconceptions, including why no soil is required in hydroponics and why soil does not disappear from pots in which plants are growing.

Formative Assessment Plan (Demonstrating Understanding)
1. Student discussions and explanations in their science notebooks serve as formative assessments. 2. Revisit the concept cartoon and ask students to record their current responses in their science notebooks and justify their explanations. 3. Finally, ask students to propose equations for photosynthesis based on their current understanding, record these equations in their science notebooks, and write explanations of their thinking.

Table 4.2 (continued)

Learning Target #2	Photosynthesis captures the energy of sunlight that is used to create chemical bonds in the creation of carbohydrates. Chloroplasts in the cells of plant leaves contain compounds able to capture light energy. Photosynthesis uses CO_2 and the hydrogen from water to form carbohydrates, releasing oxygen.

Research-Identified Misconceptions Addressed

- Some students of all ages confuse energy with other concepts—including food, force and temperature—making it difficult to understand the importance of energy conversions in photosynthesis (AAAS 1993).
- There is confusion about what chlorophyll is and what its role is in plants. Few students understand its role in converting light to chemical energy (Driver et al. 1994).
- Many students believe that plants are green because they absorb green light (Russell, Netherwood, and Robinson 2004).
- Chlorophyll alone is insufficient for plant photosynthesis. Many other enzymes and organic compounds are required. "Chloroplasts" is a better requirement (Hershey 2004).

Initial Instructional Plan

Eliciting and Confronting Preconceptions: Students continue experiments begun in Learning Target #1 and compare the results with other activities that address this target. Provide students with secondary sources that show oxygen concentration around leaves over a 24-hour period. Ask them to find patterns in the data and make a claim, using the data as evidence.

Sense Making: Establish that the evidence supports photosynthesis occurring in the presence of daylight, and ask students to propose investigations that would further test their claims. Students then devise a way to measure photosynthesis under varying conditions, stressing light intensity. Have them make predictions, time the collection of fixed amount of oxygen or use an oxygen probe, graph results, and identify/explain anomalous results. One approach is to use floating leaf discs, an example of which is "Exploring Photosynthesis with Fast Plants," an activity in which students measure rates of photosynthesis by measuring oxygen produced (*www.fastplants.org/pdf/activities/exploring_photosynthesis.pdf*). An optional activity is to read and discuss historical experiments with radioactively tagged water to identify water as the source of oxygen. *Note*: There are difficulties with approaches using the freshwater plant Elodea that tend to produce erroneous data. Photosynthesis does not always cause the bubbles formed on submerged leaves. If you use cold water, bubbles form as the water warms and gases become less soluble. The gas is not always pure oxygen since, as photosynthetic oxygen dissolves, some nitrogen comes out of solution (Hershey 2004).

Close with an activity that requires students to compare their results with those of Joseph Priestley.

Formative Assessment Plan (Demonstrating Understanding)

Students expand or modify their equations for photosynthesis. Require that they explain their reasoning for changes they make to their equations

Table 4.2 (continued)

Learning Target #3	Carbohydrates produced during photosynthesis in leaves can be used immediately for energy in the plant, stored for future use, or converted to other macromolecules that help the plant grow and function.

Research-Identified Misconceptions Addressed

- Students have little understanding of energy transfers in plant metabolism, thinking that food accumulates in a plant as it grows and having little understanding that food provides energy for a plant's life processes (Driver et al. 1994).
- Glucose is not the major photosynthetic product. There is virtually no free glucose produced in photosynthesis. The most common product is starch or sucrose, and students often test leaves for starch (Hershey 2004).

Initial Instructional Plan

Eliciting Preconceptions: Have student groups brainstorm what they know about plant parts and their use as food sources, using one of the brainstorming webs (Instructional Tool 2.10, on page 72. Encourage them to think of all plant parts that might eventually lead them to plant products that include molecules other than glucose.

Confronting Preconceptions: Show students some variegated plants. Ask them to consider why only some parts of the leaves are green and what that might mean about photosynthesis. You can use traditional activities—with green leaves that have parts covered or with variegated leaves— to demonstrate that only green parts of plants make glucose and store it as starch. Have student groups conduct experiments and make sketches of their results. Summarize the results of all groups and discuss consistencies and inconsistencies. Revisit the idea that chlorophyll is necessary to absorb light.

Present students with a wide range of plant products (e.g., cellulose, fats, proteins, starches, sugars) and have them test some for a variety of moles (e.g., fats, proteins, starches, sugars).

Sense Making: Have students research the composition and role of the various plant products in the plant. Establish that these determine the plant's biomass, together with the glucose. A possible extension is to have students find out about the molecular structures of glucose, sucrose, and starch and the relationship among them. At this point, they have not been exposed to respiration in the plant, but this sense-making activity serves as a transition to Learning Target #4.

Formative Assessment Plan (Demonstrating Understanding)

Again have student revisit their equations for photosynthesis, having them modify their equations and write explanations for any modifications.

Table 4.2 (continued)

Learning Target #4	A variety of gases move into and out of plant leaves. Leaves use CO_2 and release O_2 during photosynthesis. When they respire, leaves use $O_2$and release CO_2. Other gases enter the leaf as well, but are excreted.

Research-Identified Misconceptions Addressed

- Photosynthesis is often seen as something that plants do for the benefit of animals and people (especially with gas exchange) and that it is not as important to the plant itself (Driver et al. 2004).
- Some students at various ages think that the main job of leaves is to give off carbon dioxide or give off oxygen (Köse 2008).
- Students often think that air is used in opposite ways in plants and animals or they think that plants don't use air (Driver et al. 1994).
- Plants carry on photosynthesis; animals respire (Cottrell 2004; Köse 2008).
- Photosynthesis and respiration function in an opposite and contrasting manner (Köse 2008).
- Plants carry on photosynthesis during the day and respiration during the night (Hershey 2008; Russell, Netherwood, and Robinson 2004). While photosynthesis in plants takes in CO_2 and gives off O_2 during the day, it takes in O_2 and gives off CO_2 at night (Köse 2008).
- Many students think plants require light to grow, including for the germination of seeds (Driver et al. 2004).

Initial Instructional Plan

Eliciting Preconceptions: Raise the question, "If plants produce oxygen, why don't oxygen levels continually rise in the atmosphere?" Students' likely response will be that respiration occurs in animals (since that topic has already been studied).

Confronting Preconceptions:

- Provide students with secondary data sources that indicate O_2 and CO_2 levels around leaves during daytime and nighttime. Ask students what happens at night and how this might be tested. You can again use floating leaf discs (Wisconsin Fast Plants) to have students design and conduct experiments that test their thinking. A wonderful resource explaining use of leaf discs is found *at www.elbiology.com/labtools/Leafdisk.html* (Williamson, n.d.). It will be helpful with Learning Targets #1 and #4 because it not only explains use of the leaf discs to test oxygen generation in the light, but also includes an extension with discs in the dark.
- You can also consider germinating pea seedlings in the dark over a period of about four weeks. Indeed, you might initiate germination at the beginning of this lesson/unit. O'Connell (2008) provides the steps of the process: (1) obtain uniform lots of peas and begin germination; (2) remove some seedlings from each lot on days 8, 15, and 22, leaving them to air dry; and (3) on days 25 and 26, when the seedlings should be completely dry (the author provides details on how to ensure this), mass out all peas and examine and explain data. Refer to O'Connell (2008) for more detailed information.

Sense Making: Student groups share experimental results. Consistencies and inconsistencies are discussed. Individual students record findings and explanations in their science notebooks.

Formative Assessment Plan (Demonstrating Understanding)

Students finalize their photosynthesis equations, once again justifying any changes.

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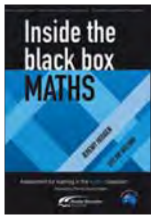

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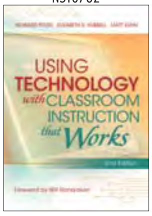

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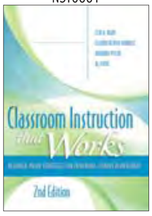

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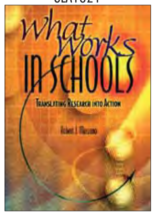

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