Hawker Brownlow EDUCATION

13th Annual

Thinking & Learning Conference

DR CHRIS WEBER

Saturday 21 May

Next Generation Mathematics

Session 2

MELBOURNE

Hawker Brownlow EDUCATION

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DR CHRIS WEBER

Dr Chris Weber is a consultant and administrative coach. He delivers trainings and presentations on pyramid response to intervention (RTI), a tiered approach that centres on professional learning community (PLC) concepts and strategies to ensure that every student receives the support necessary to succeed. Chris also offers workshops and presentations that provide the tools educators need to build and sustain PLCs.

As principal of RH Dana Elementary School in California, Chris was the leader of a highly effective PLC. Together with his staff, he lifted the school to remarkable levels of success, with gains over four years that were among the top 1 per cent in the state. He credits this achievement to the daily practise of key principles: focusing on student engagement, maximising instructional time, reallocating resources and developing systematic student support programs based on RTI.

Chris has held a number of teaching and leadership roles in both primary and secondary schools. He was director of instruction for the Garden Grove Unified School District in California, which was the 2004 winner of the prestigious Broad Prize for Urban Education.

A message from Hawker Brownlow Education

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Next Generation Mathematics

Chris Weber, Ed.D.
Senior Fellow
International Center for Leadership in Education
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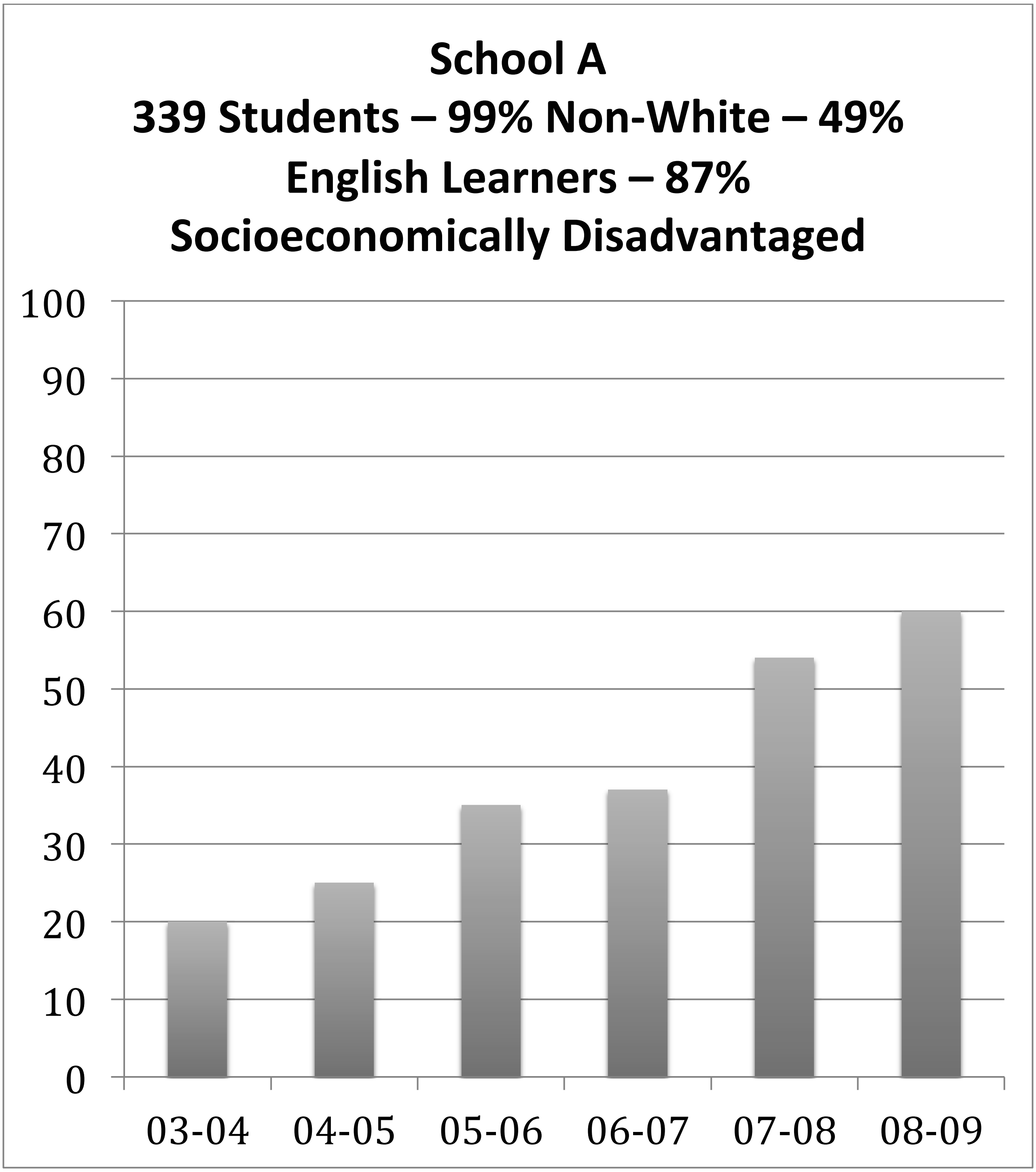
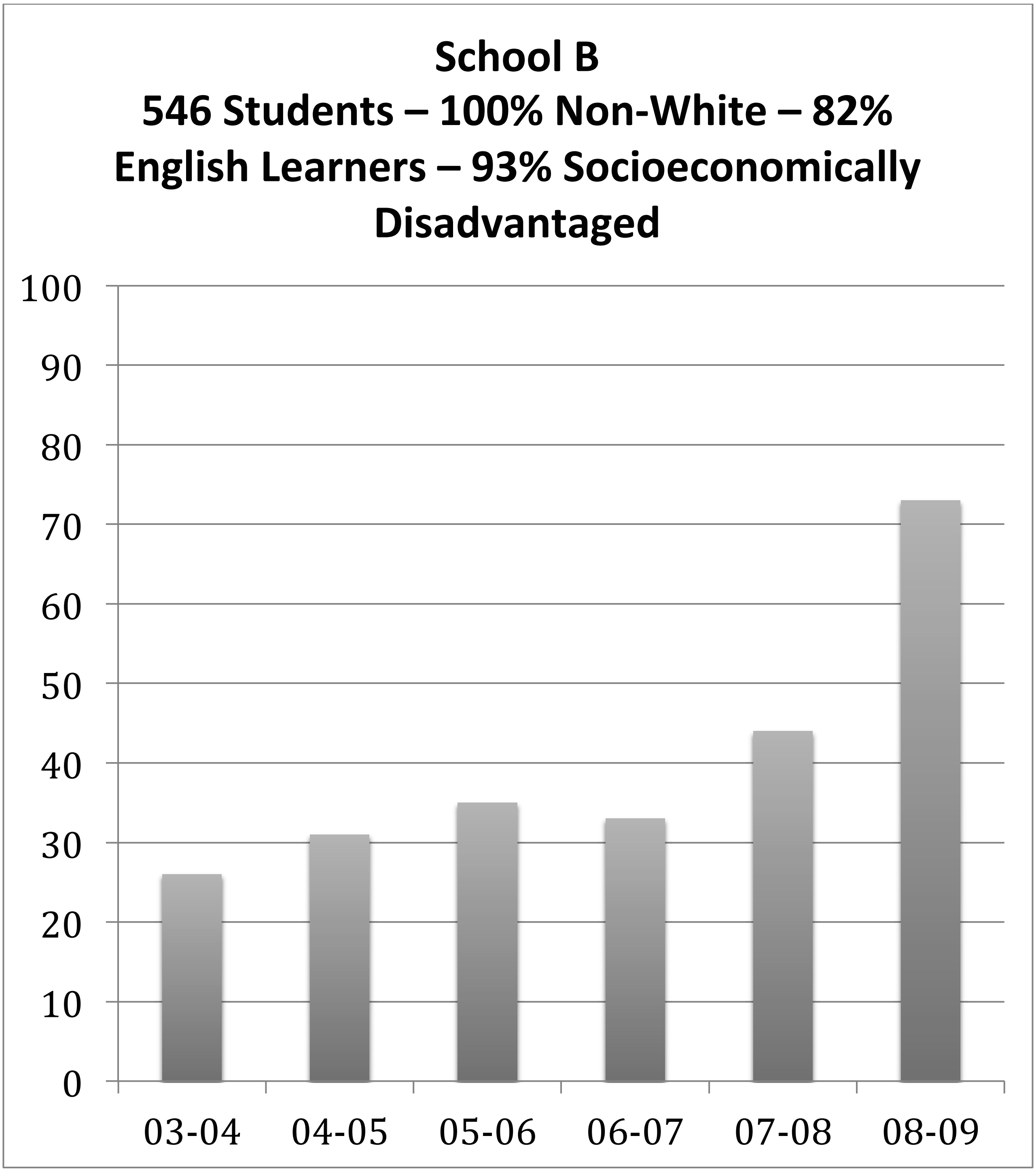
"There are only three ways to improve student learning a scale: You can raise the level of the content that students are taught. You can increase the skill and knowledge that the teachers bring to the teaching of that content. And you can increase the level of the students' active learning of the content. That's it…Schools don't improve through political and managerial incantation; they improve through the complex and demanding work of teaching and learning."

Richard Elmore

Evidence of the success of Next Generation Mathematics from one school system:




School B
546 Students – 100% Non-White – 82% English Learners – 93% Socioeconomically Disadvantaged
100
90
80
70
60
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0
03-04
04-05
05-06
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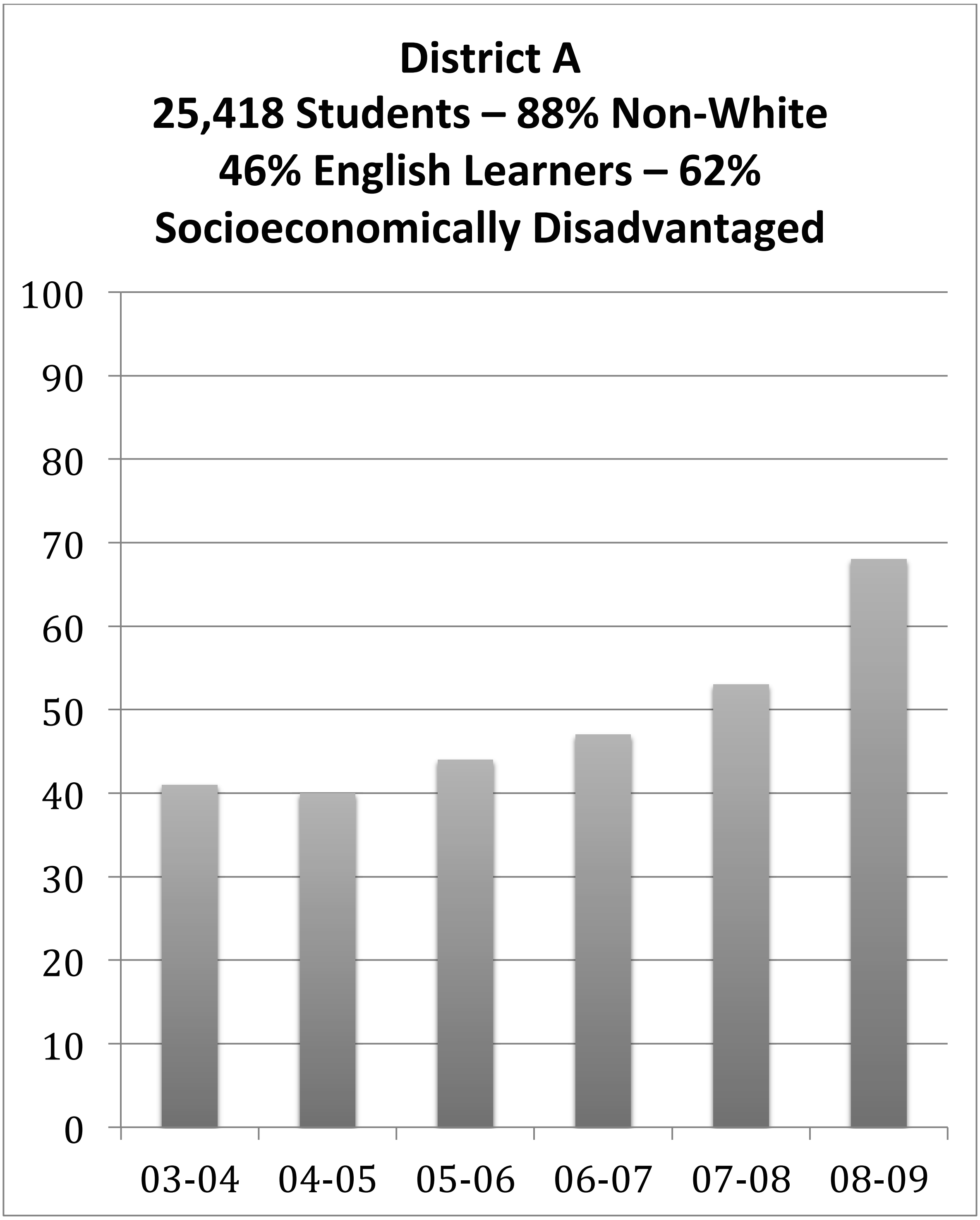
07-08
08-09


District A
25,418 Students – 88% Non-White
46% English Learners – 62%
Socioeconomically Disadvantaged
100
90
80
70
60
50
40
30
20
10
0
03-04
04-05
05-06
06-07
07-08
08-09

Why Next Generation Mathematics?

The value of math education can be found not only in its ability to help contribute to students' college & career readiness, it can also help develop individuals as thought leaders who can understand the world better because of their mathematics capabilities. Mathematics is a common language that can help students unlock complex problems & a lens of understanding by which to make applied & important connections to other fields, professions & disciplines. This project has been developed in order to give educational leaders in & out of the classroom additional tools to help more students recognize this common language so that all students develop skills to lead in the 21st century.

- Employers
  - Proficiency in mathematics
  - Mastery of key mathematics concepts
- Integrate mathematics with the 21st Century Skills
- More engaging, relevant, & rigorous teaching & learning
- A growth mindset around an aptitude in mathematics
- A need to support educators in the teaching and facilitating of mathematics, e.g.,
  - Multi-digit division
  - Visual representations of operations with fractions
- The lack of student success in Algebra

In today's world, economic access & full citizenship depend crucially on mathematics & science literacy.
—Robert Moses, civil rights leader

What are the characteristics of Next-Generation Mathematics?

- A focus on critical mathematics concepts.
- A blend of concrete, representational (or pictorial), and abstract representations of mathematics throughout the teaching and learning cycles.
- A blend of visual, auditory, and kinesthetic approaches as students make meaning of mathematics and demonstrating their understanding of key concepts.
- A greater emphasis on the use of mathematical practices and habits of mind.
- Greater opportunities for students to exercise agency, voice, and choice within the study of mathematics: to explore a personalized path through which students can to pursue their passions and find purpose in learning.
- A blend of teaching and learning on the concepts of mathematics, the procedures and algorithms that allow for efficient problem-solving, and the application of mathematics within predictable and unpredictable situations.

"Conceptual and procedural knowledge develop iteratively, with increases in one type of knowledge leading to increases in the other type of knowledge, which triggers new increases in the first" (p. 346).

Rittle-Johnson, Siegler, & Alibali (2001).

What do employment leaders seek in today's candidates?

1. Problem solving (50%)
2. Team-working (35%)
3. Communication (32%)
4. Critical thinking (27%)
5. Creativity (21%)
6. Leadership (18%)
7. Literacy (17%)
8. Digital literacy (16%)
9. Foreign language (15%)
10. Emotional intelligence (12%)

[The Economist's Intelligence Unit and Google's (Tabary, 2015) survey of business executives regarding the skills most needed in today's workplaces (the percentage of respondents who selected each skill is reported in parenthesis)]

What are the essential skills of the 21st Century?

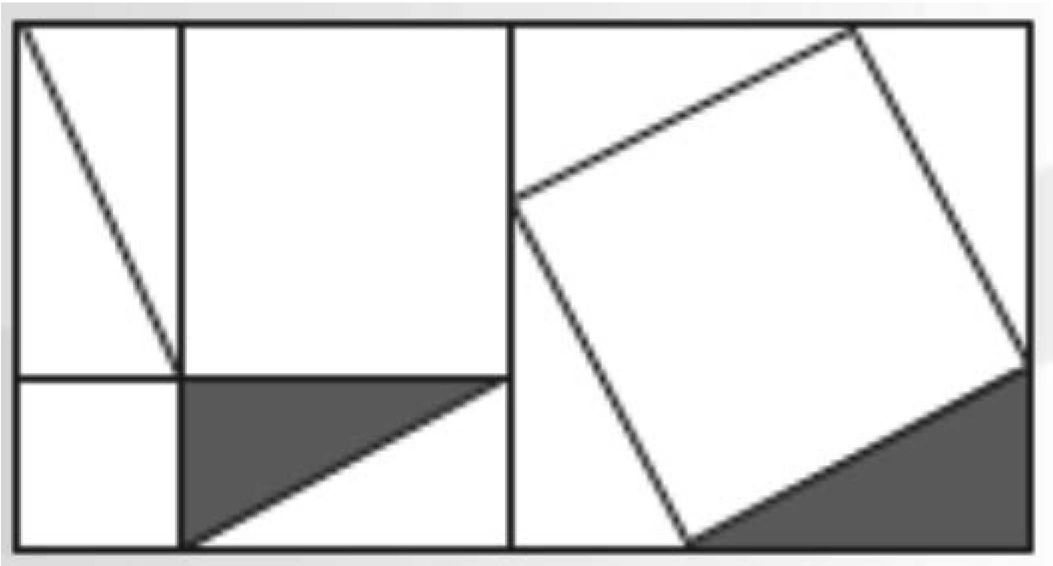
- Creativity and innovation
- Critical thinking and problem solving
- Communication and collaboration
- Information and media literacy
- Technological literacy
- Flexibility and adaptability
- Initiative and self-direction
- Social and cross-cultural skills
- Productivity and accountability
- Leadership and responsibility

(Partnership for 21st Century Skills, 2011)

Creativity and Innovation

- Compare approaches
- Find innovative solutions
- Evaluate & revise
- Look for patterns
- Make generalizations
- Discover insights

Students are shown two different approaches to proving the Pythagorean theorem, such as dissecting a square in different ways, dropping a perpendicular to the hypotenuse from the opposite vertex, or arguing using similar triangles. Students use the Internet to research other proofs of the theorem. Each student writes a brief report comparing two different proofs & presents it to the group.



Critical Thinking & Problem Solving

- Look for structure
- Make complex choices
- Construct viable arguments
- Analyze & synthesize evidence
- Interprets parts of a whole

Students examine a local building that has stairs but no ramp for wheelchairs. Working in groups, students identify the best place to install a ramp. Then they determine the appropriate slope, decide whether or not the ramp should have a switchback, & design the ramp using the Pythagorean Theorem.

Communication & Collaboration

- Articulate thoughts & ideas
- Reason abstractly & quantitatively
- Rephrase explanations
- Work efficiently & respectfully in diverse teams
- Model problems with mathematics

Working in small groups, students compare the cost of buying a hybrid versus a non-hybrid car. Students research pricing & fuel miles-per-gallon estimates for comparable models of hybrids an non-hybrids. They factor in average local gas prices over x years. Using their knowledge of linear functions, students then analyze the overall of cost of the hybrid versus the non-hybrid vehicle over x number of years, using assumptions regarding the average price of gas & how many miles the car will be driven each year. Students analyze how changing the number of years or number of miles to be driven would affect the outcome. Each group presents its results & conclusions to the class.

Information Literacy

- Identify, access, evaluate, & use data
- Explore new areas of mathematics

Students work in groups to investigate a field of modern mathematics research, such as the "four-color map theorem," fractals, buckeyballs, or the mathematics of DNA structure. Each group persuades the class of the importance of this new area of mathematics to society.

Media Literacy

- Interpret how statistics, probabilities, & media messages are constructed & employed
- Examine tools & purposes of statistical messages
- Explore the legal & ethical issues surrounding the access, use, & potential distortion of mathematics
- Present statistics strategically

Students study how the U.S. Bureau of Labor Statistics (BLS) uses the Consumer Price Index (CPI) to measure inflation. Then students examine how the European Union measures inflation using the Harmonized
Index of Consumer Prices. After comparing the two approaches, students read media articles about the CPI and inflation and discuss how the government's way of measuring the CPI affects public policy decisions.

Information Communications, Technology Literacy

- Use graphing calculators, spreadsheets, computer graphing, computer algebra systems, GPS devices, and online resources
- Construct graphical representations of functions and of data

Students use statistical software such as Tinkerplots or Fathom to compare the variables associated with different passenger vehicles. Students identify miles-per-gallon and price for each vehicle. They create a scatter plot to show the relationship between the two variables for the different vehicles. Then students look for patterns that might indicate a relationship between the two variables and create an equation to model the behavior of the relationship.

Flexibility & Adaptability

- Collaboratively work in pairs and small groups to address mathematical challenges
- Vary and share responsibilities
- Solve open-ended problems in a climate of ambiguity and changing priorities

Working in small groups or pairs, students compare the amount of fresh water needed to produce various animal and vegetable foods. Then, each group creates a one- week menu (five lunches) for the school cafeteria that aims to minimize water use while being both appealing and nutritious for students. Students post their proposed menus in a prominent location at the school.

Initiative & Self-Direction

- Monitor, define, prioritize, and complete tasks independently
- Balancing tactical and strategic goals to solve problems.
- Reflect on past experiences to inform future problem solving

Students work alone or in small groups to tackle a non-standard mathematical problem such as, "Two primes make one square" (http:// nrich.maths.org/5743). First, students identify prime numbers between 1 and 100 that are the sum of two square numbers (for example, 4=2+2; 9=2+7; and 16=5+11). Then, students try to find square numbers that are not the sum of two primes. By identifying prime numbers and listing the squares of numbers from 4 to 20, students may discover that 121 and 289 cannot be expressed as the sum of two square numbers. Students may also discover that whenever an odd square number is the sum of two primes, one of the primes must be the number 2. Students are asked why one of the primes must be 2.

Social & Cross-Cultural Skills

- Learn about the use of mathematics in other cultures
- Recognize contributions to mathematics from a variety of cultures
- Understand the practical needs that led to mathematical innovations
- Apply tools of mathematics to understanding cross-cultural problems and issues

Students investigate the geometric patterns in Medieval Islamic tiling. They explore the use of the "girih" in art and architecture and its similarity to modern mathematical tiling patterns known as Penrose tiles. Students also examine the symmetric patterns of the Alhambra, a walled city and fortress in Granada, Spain built under Muslim rule.

Productivity & Accountability

- Set goals
- Establish priorities and schedules
- Meet goals to complete a project

Students work in small groups to complete a task related a mathematical topic the class has *already* covered. Students make plans for the steps required to complete task and they accept roles and responsibilities to complete the work. Each team creates a presentation to teach the rest of the class that describes their problem-solving steps *and* their solution. Students focus not only on the content of their presentation, but also on using effective presentation techniques, such speaking in an appropriate voice, making eye contact, and incorporating visual representations of the mathematical topic. Students use technology in their presentation where appropriate.

Leadership & Responsibility

- Use interpersonal and problem-solving skills to leverage strengths of peers
- Solve mathematical problems important to the community
- Consider the ethical implications of mathematically- based decisions

Students work together to redesign a school playground or public park in their community, and create a scale model of the new design for public view. First, students discuss how the space is currently used and how they think space should be reallocated and redesigned to enhance popular activities or create additional space for activities to benefit the community. They measure each section of the current area, propose design changes, and negotiate with one another on such questions as: whether to enlarge a soccer field; whether to transform a tennis court into a paved space for young children to ride on tricycles; how to improve drainage to create enough space to add a basketball hoop; or how to create an area that would be fun for doing skateboard tricks. Students also research the costs of various proposals and try to minimize costs. Once students reach a consensus on the overall plan and compute necessary dimensions, they divide up the work of creating a scale model of the new design. One team of students creates a twodimensional scale drawing based on actual measurements of the existing space; another team uses materials such as cardboard, wood, and fabric to create items such as goal posts, playground equipment, also sized to scale. They display the scale model in a central location at the school or in a community building.

A Sample Scope and Sequence
Teach Less, Learn More
Grade 8

1. Expressions and Equations
    a. Evaluating expressions with all rational numbers
    b. Solving one-step and two-step equations
2. The Cartesian Plane
    a. Graphing on the Cartesian plane
    b. Transformations in the Cartesian plane
3. Linear Programming
    a. Slope-intercept and standard forms
    b. Systems of equations
    c. Functions
    d. Lines of best fit
4. Applications of Equations
    a. Pythagorean Theorem
    b. Volumes of three-dimensional shapes

Grade 3

1. Multiplication and Division
    a. Antecedents of Addition and Subtraction
    b. Decomposition and Composition
    c. Area of a Rectangle
2. Fractional Awareness
    a. Unit fractions
    b. Equivalent fractions
    c. On a ruler and number line
3. Identify and Classify Shapes
4. Measure Time

What are the foundations of mathematics?

Early numeracy

- Subitizing
- Magnitude
- Counting
- Correspondence
- Cardinality
- Composition
- Decomposition
- Conservation
- Hierarchical Inclusion
- Part-Whole
- Compensation
- Unitizing

Essential skills

- Problem solve
- Read & communicate
- Estimate & verify answers & solutions
- Reason logically
- Use technology
- Read, write, compare, order
- Compute fluently
- Deconstruct numbers and determine the GCF, LCM, & prime factorization
- Write & use ratios, rates, & proportions
- Gather, organize, display & interpret data
- Write, simplify & solve algebraic expressions & equations
- Represent, analyze, extend & generalize patterns
- Understand & represent functions

Pedagogies, Practices, Strategies

- Print and digital
- Whole group, small group, individual
- Inquiry-based and direct-instruction
- Metacognitively-modeled
- Growth mindset
- Conceptual, procedural, application
- Concrete, pictorial, abstract
- Habits of mind

Habits of Mind

- **Determine reasonability**
  - Check answers to problems.
  - Evaluate the reasonableness of results.
  - Make assumptions and approximations to simplify a complicated situation.
  - Attend to the meaning of quantities, not just how to compute them.
  - Reflect on whether the results make sense, improving the model if it has not served its purpose.
  - Interpret mathematical results in context.
  - Detect possible errors by strategically using estimation and other mathematical knowledge.
  - Make conjectures about the form and meaning of the solution. Justify conclusions.
- **Analogize**
  - Consider analogous problems.
  - Try special cases and simpler forms of the original problem in order to gain insight into its solution.
  - Look for entry points to a solution.
  - Understand the approaches that others use to solve complex problems and identify correspondences between different approaches.
  - Analyze situations by breaking them into cases.
- **Conceptualize**
  - Construct arguments using concrete referents such as objects, drawings, diagrams, and actions.
  - Map relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas.
  - Draw diagrams of important features and relationships.
  - Use concrete objects or pictures to help conceptualize and solve a problem.
- **Contextualize**
  - Pause as needed during the manipulation process in order to probe into the referents for the symbols involved.
  - Create a coherent representation of the problem at hand.
  - Apply mathematics to solve problems arising in everyday life, society, and the workplace.

  - Explain the meaning of a problem.
  - Interpret expressions.
- **De-contextualize**
  - Abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents.
  - Identify important quantities in a practical situation.
  - Make sense of quantities and their relationships in problem situations.
- **Pattern**
  - Discern patterns or structures.
  - Look for patterns, generalities, and shortcuts.
  - Graph data and search for regularity or trends
  - Analyze relationships mathematically to draw conclusions.
  - Explain correspondences between equations, verbal descriptions, tables, and graphs.
- **Conjecture**
  - Examine claims.
  - Analyze givens.
  - Understand and use stated assumptions, definitions, and previously established results in constructing arguments.
  - Determine domains to which an argument applies.
  - Analyze constraints.
  - Make conjectures and build a logical progression of statements to explore the truth of conjectures.
  - Reason inductively about data, making plausible arguments that take into account the context from which the data arose.
  - Compare the effectiveness of two plausible arguments.
  - Recognize and use counterexamples.
- **Be precise**
  - Express numerical answers with a degree of precision appropriate for the problem context.
  - Calculate accurately and efficiently.
  - Consider the units involved in a problem.
  - Specify units of measure.
  - Make explicit use of definitions.
- **Employ equalities**
  - State the meaning of the symbols, including using the equal sign consistently and appropriately.
  - Know and flexibly use different properties of operations and objects.
  - Transform algebraic expressions.
- **Communicate**
  - Listen to or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.
  - Give carefully formulated explanations to each other.
  - Communicate solutions precisely to others.
  - Respond to the arguments of others. Use clear definitions in discussion with

others and in reasoning.
  - Distinguish correct logic or reasoning from that which is flawed, and if there is a flaw in an argument, explain what it is.
- **Monitor**
  - Plan a solution pathway rather than simply jumping into a solution attempt.
  - Analyze goals.
  - Monitor and evaluate progress and change course if necessary.
  - Maintain oversight of the problem solving process, while attending to the details.
- **Use resources**
  - Use technological tools to explore and deepen understanding of concepts.
  - Consider available tools when solving a problem (pencil and paper, concrete models, a ruler, a protractor, a calculator, a graphing calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software).
  - Use technology to visualize the results of varying assumptions, explore consequences, and compare predictions with data.
  - Identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems.

Evidence Gathering

- Clarifying, sharing, and understanding goals for learning and criteria for success with learners
- Engineering effective classroom discussions, questions, activities, and tasks that elicit evidence of students' learning
- Providing feedback that moves learning forward
- Activating students as owners of their own learning
- Activating students as learning resources for one another

Scoring Guide

Q #	Understanding **2**-Understands the task **1**-Misinterprets parts of task **0**-No attempt/ misinterprets the entire task	Solving **2**-A procedure that could lead to the correct solution **1**-Partially correct procedure **0-**No attempt/ totally incorrect plan	Answering **2**-Correct solution **1**-Copying error; computational error, partial answer for task with multiple answers; no answer statement; answer labeled incorrectly **0**-No answer/totally illogical answer	Total
1				
2				
3				
4				
5				
6				
7				
8				
Error Analysis				

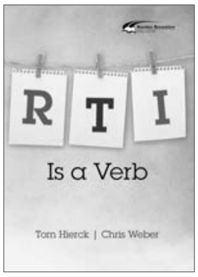
How do we Shift to Next Generation Mathematics?

- A culture of "teach less, learn more"
- Pilot in grade level
- Pilot in school
- Pilot in one unit of instruction
- Professional Development on:
  - Student engagement
  - Inquiry-based and direction instruction pedagogies
  - Environmental innovations, e.g., station rotation
  - Rich resources
  - Alternative strategies

RELATED RESOURCES

Available from Hawker Brownlow Education

Qty	Code	Title	Price
	HB7897	Collaborative System of Support	$35.95
	CO569X	Common Formative Assessment	$45.95
	BKD8054	Rigorous Curriculum Design: How to Create Curricular Units of Study that Align Standards, Instruction, and Assessment	$49.95
	CO2977	RTI Is a Verb	$36.95
	BKD8474	RTI Roadmap for School Leaders: Plan and Go	$45.95
	BKD3338	Strategies for Mathematics Instruction and Intervention, 6–8	$42.95
	BKD3313	Strategies for Mathematics Instruction and Intervention, K–5	$42.95
	BKD8023	The Data Teams Experience: A Guide for Effective Meetings	$49.95
	BKD4891	Uniting Academic and Behavior Interventions: Solving the Skill or Will Dilemma	$42.95
		Total (plus freight) $	



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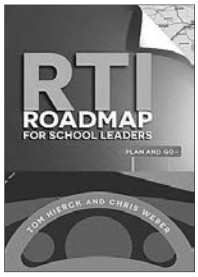

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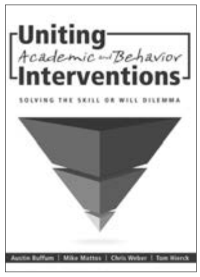

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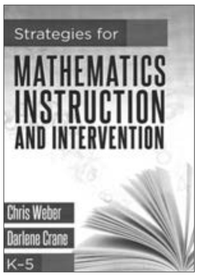

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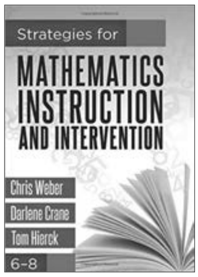

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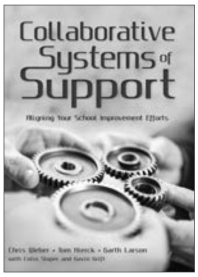

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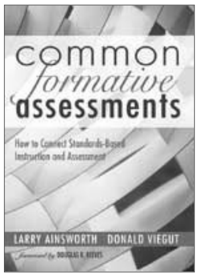

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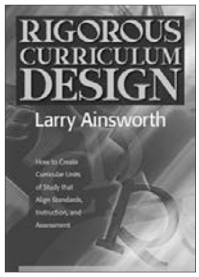

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