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## CHAPTER 1

# PEEL

## The Search For Effective Learning in the Mathematics Classroom

In 1995, a group of committed mathematics teachers at Frankston High School, some 40km south-east of the city of Melbourne, took part in a teaching and learning project. They met weekly to discuss and share their ideas and concerns, and committed themselves to trial suggested strategies in classrooms during the following week. This simple model of ongoing reflective practice produced amazing results, both for the students and the teachers involved in the project. Their year long commitment was inspirational, particularly when it is remembered that it took place in 1995 in the midst of some of the greatest political upheaval that Victorian education has seen. I hope their deliberations will serve to inform and inspire the practice of colleagues in classrooms everywhere as we all seek to obtain the optimum learning outcomes for our students.

In this introductory chapter, I would like to share with teachers some of the research and educational policies which have influenced world-wide changes to the teaching of mathematics in recent times. Many of the initiatives and reports will be familiar to many teachers, but it is useful background in setting the scene for the discussion of this project. I will also outline some of the history of the PEEL (Project for Enhancing Effective Learning) project which was the inspiration for the 1995 venture at Frankston High School. Both current research on educational trends together with some appreciation of the PEEL project are necessary in understanding the contribution made by this project.

### **THE WORLDWIDE CALL FOR CHANGE - WHAT THE POLICY MAKERS HAVE SAID**

We all acknowledge that we are in an era of unprecedented technological advancement. The advent of computers has heralded significant and lasting change to the way we live, communicate, travel and conduct business. It has also had a profound effect on education. Mathematics education in particular has had to re-invent itself in order to maintain a relevant and productive role in the preparation of students to meet the challenges of this ever-changing world. Teachers must rise to this challenge, as the world which greets graduating students is much different to the one we encountered only a few years ago. The call for change in mathematics education has been loud and strong for some time in many parts of the world.

It is in this climate of change that the 1995 project took root. Not that the

teachers had read all of the relevant literature or were up to date with all contemporary policy - far from it. However it is true that they were ready to try different approaches which might lead to improved learning in their classes. The discussion of their motivation and expectations will follow in the next chapter. Suffice to say that this project grew out of a response to teacher dissatisfaction with poor mathematics learning on the part of many students.

It might surprise teachers to learn that despite all of the calls for change on behalf of educational authorities, the content of mathematics curricula has remained largely unchanged for 500 years. There are two outdated and misinformed public assumptions which have contributed to this state of affairs:

Mathematics is a fixed and unchanging body of facts and procedures; and to do mathematics is to calculate answers to set problems using a specific catalogue of rehearsed techniques.

*(Reshaping School Mathematics, MSEB, 1990, p. 4)*

This view of mathematics has clouded the need for change. Despite the many recent changes to education, the teaching of mathematics has continued to follow its traditional, well-worn path, seemingly oblivious to the changing needs of its clients. The result has been the alienation of generations of students from a mathematics curriculum which seems unrelated to their daily lives and ill prepares them for the application of mathematics relevant to the technological society of today.

I would like to briefly outline the main messages from the United States, Britain and Australia to this issue over recent years. The Mathematical Sciences Education Board in the U. S. made these strong observations regarding the current curriculum:

Problems are solved not by observing and responding to the natural landscape through which the mathematics curriculum passes, but by mastering time-tested routines conveniently placed along the path near every anticipated problem. Students who progress through this curriculum develop a kind of mathematical myopia in which the goal is to solve artificial word problems rather than realistic world problems. Few have the stamina to survive the curriculum of mathematics - at least not the way it is now delivered. Of 4 million who begin, only 500,000 are still studying mathematics 12 years later. Most students receive little of lasting value from the final mathematics course they study... Many of those who drop out harbour life-long feelings of guilt or distaste for school mathematics. Some of those who become disenchanted with mathematics become teachers; others help decide educational and research policy for the nation. Very few adults in the United States have had the benefit of successful completion of a mathematics curriculum.

*(Reshaping School Mathematics, MSEB, 1990, pp. 4-5)*

**This is a sad commentary indeed.**

The findings of the Cockroft Committee raised public awareness to the shortcomings of mathematics education in Great Britain in 1982. The Committee's terms of reference were to examine the teaching of mathematics in primary and secondary schools in England and to make recommendations on its relevance to the mathematics required in further and higher education, employment and adult life generally. Nearly 3000 people, all over the age of 15, were interviewed. The findings were alarming. It appeared that "even the simplest piece of mathematics induced a surprising degree of anxiety, helplessness, fear and even guilt in a number of those interviewed" (*Blueprint For Numeracy*, Department of Education and Science, 1983, p. 4). The report concluded that little use of school mathematics was made by people in their daily lives. In fact, many had invented strategies which avoided the use of mathematics altogether. It was clear that the mathematics curriculum and its methods of implementation had frustrated, confused and finally alienated many of the students it had attempted to reach. It had long ceased to address the needs of contemporary learners - its relevance was under serious question.

The U. S. Mathematical Sciences Education Board, in its publication *Everybody Counts*, drew on current knowledge about how students learn to make a strong case for the immediate change to traditional teaching practices:

Research on learning shows that most students cannot learn mathematics effectively by only listening and imitating; yet most teachers teach mathematics just this way. **Most teachers teach as they were taught, not as they were taught to teach...** Much of the failure in school mathematics is due to a tradition of teaching that is inappropriate to the way most students learn... Present educational practice in the United States offers students only one path to understanding - a long, dimly lit journey through a mountain of meaningless manipulations, with the reward of power and understanding available only to those who complete the journey.

(National Research Council, 1989, p. 6)

The path to sustaining real change to teaching practice threatens to be just as arduous as this, due in part to the conservative nature of most teachers and their natural suspicion of innovative methods. However there is little to dispute the fact that teachers need to involve students more in their own learning, encourage them to respond to and challenge prior knowledge, and create a classroom environment which stimulates all students to learn mathematics. In the 'Curriculum and Evaluation Standards for School Mathematics', the National Council of Teachers of Mathematics provided a vision for change in its discussion on 'New Goals for Students':

Towards this end, we see classrooms as places where interesting prob-