INTRODUCTION

Understanding Fractions is a series of books for years 1–8. Each book's activities are designed to increase students' understanding of fractions, and have been developed with the conviction that students construct their own understanding through rich, hands- on mathematical experiences. Although the activities in each book are for a specific level, they all connect to the core of mathematics learning that is important to every primary school and middle-school student.

This book in the series is designed for grade 2. However, depending upon the experiences and background of your students, these activities may be used in grade 1 or 3. They should not be considered remedial in the upper grade levels. Many students who study fractions gain little or no understanding. The activities allow students to explore and to play with fractional relationships and to come to their own understanding. Students can actually begin to analyse shapes, relationships, and numbers, and in the process, develop their own sense of mathematical power.

In the past, the role of the maths teacher focused on presenting rules and procedures, followed by seemingly endless drills for practice. The student's role was practise, practise, practise, until mastery of algorithms was proven on the weekly test. Changing educational standards have drastically de-emphasised this computational focus. Emphasis is now on communication, reasoning, and problem solving. Teachers now foster student thinking for the problem solving and for alternative, original algorithms. Much of today's maths instruction is rooted in the work of Piaget, Suppes, Gattegno, Wirtz and Botel, Dienes, and Burns.

ORGANISATION

Understanding Fractions is not a recipe series for teaching fractional concepts. Rather, it is a series to be used by you and your students to study and explore the dynamic field of fractions. The experiences presented are intended to allow students to gradually develop a systematic awareness of fractional parts and their relationships. This level of the series depends primarily upon visual input for students to construct a personal framework for understanding fractions. The use of manipulatives is required in most lessons. Allow students to use whatever tools they find helpful to complete the work. Except for the final quiz, all work may be done in pairs or in small groups of three or four students. The student exchange of ideas increases the power of learning as it uncovers a variety of strategies for problem solving. Working together also allows students to answer each other's questions about the directions and concepts.

There is a glossary at the back of the student book. As they work through the book, students will find several words highlighted in bold type. These words appear in the glossary with an example. You may want to discuss the definitions with the class at the end of the particular lesson.

You are not asked to correct each page, but rather to discuss the students' results. Encourage students to focus more on their strategies than just on getting the right answer. The specific answers to each page are not as important as the students' awareness of a developing sense of fractions and an ability to apply previous understandings.

Try to be open to a multitude of correct responses. Some may not be the responses that you had in mind, but may demonstrate a student's own sense of the question at hand. As the teacher, you may be surprised at the students' sophisticated awareness of numbers and relationships. School for most students has been an exercise in getting the right answer in order to get a good mark, even if what they have done makes no sense to them. They have not had the opportunity to move from their baseline of knowledge to more complex thinking. This past approach undermined the student's belief in his or her ability. The teaching of mathematics today requires a tremendous shift in thinking on your part, as you learn to ask questions that are powerful catalysts for student exploration. Relax and have fun with this book. Mathematics can turn out to be the most exciting study of the day.

MATERIALS

Provide students with an ample supply of paper, pencils, rulers, crayons, and an assortment of manipulatives. Manipulatives can be as simple as buttons, plastic counters, egg cartons, marbles, and toothpicks. Commercially available manipulatives—such as colour tiles, pattern blocks, geoboards, colour squares, and Cuisenaire Rods®—are powerful tools for recreating a three-dimensional representation. Various manipulatives are referred to and recommended in the materials section. You may replace these specific manipulatives with comparable objects found in the classroom environment or made from construction paper and laminated for durability. Some activities don't list any specific materials, but you should encourage students to use any available manipulatives that will help with the learning procedure.

Using already-constructed objects whenever possible will allow you more time for planning maths content and instructional strategies. You may want to look through the book ahead of time and instruct students to create their own manipulatives packet. Students may cut out the maths tiles and pattern blocks from pages 35 and 37 in the student book and colour them appropriately.

Or you may choose to make copies of Reproducible A (page 12) and Reproducible B (page 13) and distribute them to students. If possible, have the materials laminated. Provide additional common classroom objects for students to add to their packet.

Because students need physical models to construct meaningful representations of their solutions, a variety of readily available hands-on materials will increase the opportunity each student has to bring his or her perspective to the problem at hand. Whereas some materials may seem more appropriate to you for the solution to a problem, allowing students to select their own materials increases individual, diverse thinking.

Encourage your students to use manipulatives to interpret data. Develop a maths centre that contains varieties of maths manipulatives. Recreating printed data with manipulatives provides a kinaesthetic opportunity for understanding and more options for solutions. As with any manipulative materials, the thinking activity generated from the physical manipulations is the focus.

GROUPING

The grouping recommended for this series is individual, partners, or a small collaborative group of three or four members. Although there are many approaches to learning, and some students need to work without outside distractions, most students need and benefit from access to one another's thinking. Students who are allowed to help one another will minimise misunderstandings or confusion from directions or adult interpretations. Encourage your students to discuss the directions together and make sure that their partner understands what to do before they begin. Partners should help one another and should not be afraid to say that they do not understand. Students need to know that this is how learning occurs. Suggest that students try to do the work on their own, checking with a partner as they go along to make sure that they are moving in the right direction. Students need to learn how to work as a team, being responsible for their own work, yet not doing the work for one another. When students finish their work, the group provides a powerful opportunity for immediate feedback and self-correction. Encourage students to use this checking-in period to adjust their responses. Explain that the goal is on learning, not on determining how many problems the students got right or wrong. In a group, students learn to accept the diversity of responses, which fosters creative mathematical thinking.

Sometimes, you will work together with all students. Often, this entire-class work follows the paired-group or collaborative-group sessions in which the students write and check their work together. Bringing all groups together for a focused lesson and classwide sharing permits you to correct any misunderstandings and to expand on concepts that students might bring up.

THE LESSONS AND INSTRUCTION

The lessons in ***Understanding Fractions*** involve problem solving, communication, reasoning, and mathematical connections. Each activity focuses on at least one of the curriculum strands of number, geometry, measurement, and patterns/functions. Working each lesson along with your students provides a powerful opportunity for you to develop a sense of the concepts and understandings that are emerging for them. Students are often eager to explain ideas to their teachers, especially if the teacher acknowledges his or her own difficulties. This experience makes the classroom a community of learners. Any preconceived notions of fractions that you might have will be no more important than those of the emerging mathematicians in your classroom. The lessons are meant to be exploratory and are sequentially connected. If individual lessons are presented in isolation, they will be more dependent upon adult interpretation and formal teaching.

Mathematics does not lend itself to solitary pursuit, so your role as teacher in the development of a deep and powerful understanding is crucial. The majority of students are dependent upon their instructors to lead the exploratory journey through the different strands. The social interaction of the students, the artful and timely questioning by you, and the latitude to probe, question, and discover are all critical elements in developing the mathematical minds of the students.

The lessons in ***Understanding Fractions*** are designed to enable students to increase their own mathematical power. The intent is to set the stage but not steal the show. Some activities may seem challenging, and you may be tempted to teach by modelling the solutions. However, this would not provide students with the opportunity to try the activity in whatever way they can. As long as the students have a way of beginning an activity, give them the opportunity to work it through. As members of a pair or a collaborative group, most students will meet with success. Through trial and error and discussion, students will learn.

ASSESSMENT

Because assessment is multifaceted, this series encourages a range of strategies for assessing student progress with the purpose of modifying instruction, not judging ability. Each mathematics lesson, as well as the quiz, is a part of the assessment process. Also, you can gather much information by listening to students' explanations, observing their thinking as manifested by the manipulation of physical objects, and examining their writing and reflections. These assessment tools are some ways of probing student understanding for the purpose of determining how to modify instruction. Encourage students to keep a portfolio, which will allow both you and them to see growth over time and identify successful problem-solving strategies.

LESSON 1 pages 1 and 2

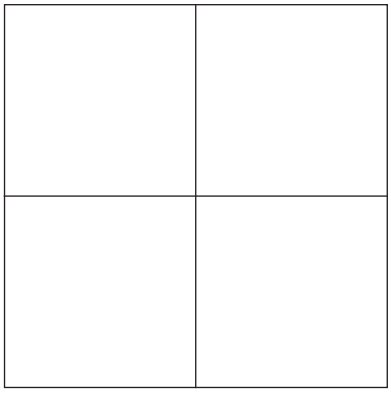
FOCUS: Explore equal parts

MATERIALS: Crayons

DIRECTIONS: Understanding the concept of equal parts is essential in work with fractions. This lesson presents equal parts in geometric figures. You may want to expand upon this idea to include equal groups. Use counters and other common objects to demonstrate equal groups. Make sure that you use items that are the same size and shape. Show groups that are equal and unequal. Emphasise the characteristics that make the parts equal—same size, shape, and number of items.

RESPONSES:

1. Rectangle A–3; Rectangle B–3
2. yes
3. no
4. Students should circle *b, c, e,* and *f.*
5. a. 2
   b. 8
   c. 4
   d. 10
   e. 3
   f. 3
6. Answers may vary. Check students' drawings. Sample answer:



I drew a square made of 4 smaller squares. I know the smaller squares are all equal because I made each one with 1-inch sides.

LESSON 2 pages 3 and 4

FOCUS: Identify one half of a figure

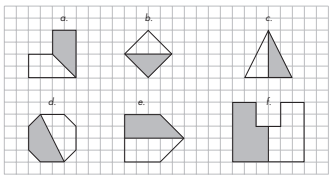
MATERIALS: Crayons, ruler, pattern blocks (optional)

DIRECTIONS: This book presents fractional concepts in just a few ways, one of which is in the context of geometric figures. The approach in this book is visual. To tap into other modalities, you may want to use pattern blocks. Students can construct figures with equal parts. They can touch the parts and place them on top of one another to prove that they are in fact the same size and shape. Use two identical pattern blocks and point out that each is one half of the larger figure that they form together.

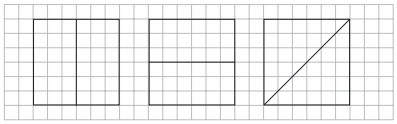
The written form of one half ($\frac{1}{2}$) is introduced in this lesson. Although it's more important at this point that students understand halves at the concrete level, the written form will be used throughout the book. Students will need to recognise this form to work through other lessons. As students colour halves of the figures, some may colour the left half, whereas others colour the right half; some may colour the top half, others, the bottom half, and so on. Some students might also think that the correct answer involves colouring the correct half of the figure. Work with students to explore the idea that either side of the figure constitutes one half.

RESPONSES:

1. a. yes b. 2
2. Check to see that students colour one half of the figure; 1 part.
3. 2 parts
4. Check students' work.
5. Answers will vary. Sample answer: All 3 rectangles are the same size and shape. They all have 2 equal parts. They are all divided in different ways. The parts in each rectangle are different shapes.
6. Answers may vary. Sample answer:



7. Answers may vary. Sample answer:



8. Answers will vary. Sample answer: I counted the small squares in each part. Since there are the same number of small squares, the parts are equal.

LESSON 3 pages 5 and 6

FOCUS: Work with one half of a number

MATERIALS: Tiles, crayons

DIRECTIONS: Another way that fractional concepts are presented in this book is with tiles. This approach combines both visual and numerical ideas. Students can see what is one half of the rectangle. Then they can count the number of tiles in the half. This is the start to understanding a fraction of a number. As with all lessons in the book, students should realise the relationship between part and whole.

To extend the ideas presented thus far, you might have students practise making halves in a variety of ways. For example, students can use string or yarn to divide their desktops in half. They can also fold pieces of paper in half along the short and long sides.

RESPONSES:

1. Check students' work.
2. 12
3. 6
4. 6
5. no
6. no
7. Check students' work.
8. Check students' work.
9. 6; The 6 blue tiles and the 6 green tiles are two equal parts of the 12 tiles, so each is one half of the 12 tiles. This means that 6 is one half of 12.
10. Check students' work.
11. 12
12. 6
13. 6
14. Check students' work.
15. Check students' work.
16. 6
17. Answers will vary. Sample answer: There are 12 squares in each rectangle. One rectangle has 2 rows of 6 squares. The other has 3 rows of 4 squares. Both show that one half of 12 is 6.

LESSON 4 pages 7 and 8

FOCUS: Work with one half of a group

MATERIALS: Tiles, crayons

DIRECTIONS: This lesson is based on a third common application of fractions—to describe parts of a group. The work here uses what Lesson 3 presented about one half of a number. Students divide groups of 24 objects into two equal parts. Students might use different strategies to break down a group into equal parts. Some students might guess and check until they discover how many items belong in each equal group. Others may use a more systematic method, such as putting all the tiles into one group and then making equal groups by counting out and placing the tiles one by one into two groups. The particular strategy doesn't matter; what may even be beneficial for students is that they understand how others approach this kind of problem.

While working on this lesson, students might begin to understand the real-world implications of fractions. To guide students to this realisation, you might discuss common items that appear in groups. For example, mention the fact that eggs are sold by the dozen. Ask students what one half of a dozen eggs is. Continue with other examples, such as products sold in multipacks. Encourage students to look around the school or their home for other kinds of groups.

RESPONSES:

1. Check students' work.
2. 24
3. 12
4. 12
5. no
6. no
7. Check students' work.
8. 12; The 12 football cards and the 12 baseball cards are two equal parts of the whole group. This means that each is one half of the group. So 12 is one half of 24.
9. Check students' work.
10. 24
11. 12
12. 12
13. Check students' work.
14. 12
15. Answers will vary. Sample answer: Both problems show groups of 24 items. One half of each group is 12. The items in each group are different. The cards are in 3 rows of 8. The coins are in 4 rows of 6.