INTRODUCTION

Understanding Fractions is a series of books for years 1–8. Each book's activities are designed to increase students' understanding of fractions, and have been developed with the conviction that students construct their own understanding through rich, hands- on mathematical experiences. Although the activities in each book are for a specific level, they all connect to the core of mathematics learning that is important to every primary school and middle-school student.

This book in the series is designed for grade 4. However, depending upon the experiences and background of your students, these activities may be used in grade 3 or 5. They should not be considered remedial in the upper grade levels. Many students who study fractions gain little or no understanding. The activities allow students to explore and to play with fractional relationships and to come to their own understanding. Students can actually begin to analyse shapes, relationships, and numbers, and in the process, develop their own sense of mathematical power.

In the past, the role of the maths teacher focused on presenting rules and procedures, followed by seemingly endless drills for practice. The student's role was practise, practise, practise, until mastery of algorithms was proven on the weekly test. Changing educational standards have drastically de-emphasised this computational focus. Emphasis is now on communication, reasoning, and problem solving. Teachers now foster student thinking for the problem solving and for alternative, original algorithms. Much of today's maths instruction is rooted in the work of Piaget, Suppes, Gattegno, Wirtz and Botel, Dienes, and Burns.

ORGANISATION

Understanding Fractions is not a recipe series for teaching fractional concepts. Rather, it is a series to be used by you and your students to study and explore the dynamic field of fractions. The experiences presented are intended to allow students to gradually develop a systematic awareness of fractional parts and their relationships. Whereas the earlier levels in this series depend primarily upon visual input for students to construct a personal framework for understanding fractions, Book 4 directs students to self-discovery of the operational algorithms and the move from simpler to more complex fractions. Their response to diagrams in this book requires students to develop their mathematical writing ability. Except for the two quizzes, all work may be done in pairs or in small groups of three or four students. The student exchange of ideas increases the power of learning as it uncovers a variety of strategies for problem solving. Working together also allows students to answer one another's questions about the directions and concepts.

You are not asked to correct each page, but rather to discuss the students' results. Encourage students to focus more on their strategies than just on getting the right answer. The specific answers to each page are not as important as the students' awareness of a developing sense of fractions and an ability to apply previous understandings.

Try to be open to a multitude of correct strategies. Some may not be the strategies that you had in mind, but may demonstrate a student's own sense of the question at hand. As the teacher, you may be surprised at the students' sophisticated awareness of numbers and relationships. School for most students has been an exercise in getting the right answer in order to get a good mark, even if what they have done makes no sense to them. They have not had the opportunity to move from their baseline of knowledge to more complex thinking. This past approach undermined the student's belief in his or her ability. The teaching of mathematics today requires a tremendous shift in thinking on your part, as you learn to ask questions that are powerful catalysts for student exploration. Relax and have fun with this book. Mathematics can turn out to be the most exciting study of the day.

MATERIALS

Provide students with an ample supply of paper, pencils, rulers, coloured textas, and an assortment of manipulatives. Manipulatives can be as simple as buttons, plastic counters, egg cartons, marbles, and toothpicks. Commercially available manipulatives—such as colour tiles, pattern blocks, geoboards, colour squares, and Cuisenaire Rods®—are powerful tools for recreating a three-dimensional representation. Various manipulatives are referred to and recommended in the materials section. You may replace these specific manipulatives with comparable objects from the classroom environment or made from construction paper and laminated for durability. Some activities don't list any specific materials, but you should encourage students to use any available manipulatives that will help with the learning procedure. Calculators are optional, but may prove to be useful with many activities.

Using already-constructed objects whenever possible will allow you more time for planning maths content and instructional strategies. You may want to look through the book ahead of time and instruct students to create their own manipulatives packet. Students may cut out the maths tiles and pattern blocks from pages 35 and 37 in the student book and colour them appropriately.

Or you may choose to make copies of Reproducible C (page 18) and Reproducible D (page 19) and distribute them to students. If possible, have the materials laminated. There are other reproducible pages in the back of this teacher guide that can be copied and used as manipulatives. Provide additional common classroom objects for students to add to their packet.

Because students need physical models to construct meaningful representations of their solutions, a variety of readily available hands-on materials will increase the opportunity each student has to bring his or her perspective to the problem at hand. Whereas some materials may seem more appropriate to you for the solution to a problem, allowing students to select their own materials increases individual, diverse thinking.

Encourage your students to use manipulatives to interpret data. Develop a maths centre that contains varieties of maths manipulatives. Recreating printed data with manipulatives provides a kinaesthetic opportunity for understanding and more options for solutions. As with any manipulative materials, the thinking activity generated from the physical manipulations is the focus.

GROUPING

The grouping recommended for this series is individual, partners, or a small collaborative groups of three or four members. Although there are many approaches to learning, and some students need to work without outside distractions, most students need and benefit from access to one another's thinking. Students who are allowed to help one another will minimise misunderstandings or confusion from directions or adult interpretations. Encourage your students to discuss the directions together and make sure that their partner understands what to do before they begin. Partners should help each other and should not be afraid to say that they do not understand. Students need to know that this is how learning occurs. Suggest that students try to do the work on their own, checking with a partner as they go along to make sure that they are moving in the right direction. Students need to learn how to work as a team, being responsible for their own work, yet not doing the work for one another. When students finish their work, the group provides a powerful opportunity for immediate feedback and self-correction. Encourage students to use this checking-in period to adjust their responses. Explain that the goal is on learning, not on determining how many problems the students got right or wrong. In a group, students learn to accept the diversity of responses, which fosters creative mathematical thinking.

Sometimes, you will work together with all students. Often, this entire-class work follows the paired-group or collaborative-group sessions in which the students write and check their work together. Bringing all groups together for a focused lesson and classwide sharing permits you to correct any misunderstandings and to expand on concepts that students might bring up. See the teacher guide for reproducibles to use during some full-class-focused lessons.

THE LESSONS AND INSTRUCTION

The lessons in ***Understanding Fractions*** involve problem solving, communication, reasoning, and mathematical connections. Each activity focuses on at least one of the curriculum strands of number, geometry, measurement, and patterns/functions. Working each lesson along with your students provides a powerful opportunity for you to develop a sense of the concepts and understandings that are emerging for them. Students are often eager to explain ideas to their teachers, especially if the teacher acknowledges his or her own difficulties. This experience makes the classroom a community of learners. Any preconceived notions of fractions that you might have will be no more important than those of the emerging mathematicians in your classroom. The lessons are meant to be exploratory and are sequentially connected. When individual lessons are presented in isolation, they become more dependent upon adult interpretation and formal teaching.

Mathematics does not lend itself to solitary pursuit, so your role as teacher in the development of a deep and powerful understanding is crucial. The majority of students are dependent upon their instructors to lead the exploratory journey through the different strands. The social interaction of the students; the artful and timely questioning by you; and the latitude to probe, question, and discover are all critical elements in developing the mathematical minds of the students.

The lessons in ***Understanding Fractions*** are designed to enable students to increase their own mathematical power. The intent is for you to set the stage but not steal the show. Some activities may seem challenging, and you may be tempted to teach by modelling the solutions. However, this would not provide students with the opportunity to try the activity in whatever way they can. As long as the students have a way of beginning an activity, give them the opportunity to work it through. As members of a pair or a collaborative group, most students will meet with success. Through trial and error and discussion, students will learn.

ASSESSMENT

Because assessment is multifaceted, this series encourages a range of strategies for assessing student progress with the purpose of modifying instruction, not judging ability. Each mathematics lesson, as well as the quizzes, is a part of the assessment process. Also, you can gather much information by listening to students' explanations, observing their thinking as manifested by the manipulation of physical objects, and examining their writing and reflections. These assessment tools are some ways of probing student understanding for the purpose of determining how to modify instruction. Encourage students to keep a portfolio, which will allow both you and them to see growth over time and identify successful problem-solving strategies.

LESSON 1 page 1

FOCUS: Review halves, thirds, and sixths

MATERIALS: Ruler; red, blue, green, and yellow crayons or textas

DIRECTIONS: As an introduction, invite students to demonstrate different ways of dividing figures into equal parts. Ask students to name the fractions for these equal parts and to explain how they know the parts are equal. Students should inductively know that the equal parts are the same shape and size. They should also understand that there are two halves, three thirds, four quarters, and so on, in different figures. As students work through the lesson, they will discover that there is rarely only one correct way to divide the figures.

If students are having trouble with some of the figures, provide some helpful suggestions. For example, use the centre point to divide the triangle into thirds. Find the centre point of each side of the triangle to divide it into quarters. When dividing the hexagon and octagon, make sure each part has the same number of sides on the outside of the figure. You may wish to have volunteers demonstrate the different ways they divided the figures in the lesson.

RESPONSES:

1.–4. Check students' work. Accept all reasonable responses.

LESSON 2 page 2

FOCUS: Review and compare halves, thirds, and sixths

MATERIALS: Ruler; red, blue, and green crayons or textas; Optional: Reproducible A (page 16 of teacher guide)

DIRECTIONS: You may want to use Reproducible A to create an overhead-projector overlay of the hexagon to introduce the lesson or to demonstrate the solutions. You might also want to suggest that students use the pattern blocks to help complete the lesson. The colours used in the lesson match the colours of the pattern blocks. Using the blocks allows students to physically break down and put back together the pieces of the hexagon, providing concrete examples of both fractional parts and geometric shapes. The blocks can also help in comparing sizes of the fractional parts.

For many of the questions in this lesson and in many other lessons, students may answer with a whole number or a fraction. You may want to state a preferred method. To emphasise the concept further, you may want students to express answers in fractional form.

RESPONSES:

1. Check students' work.
2. a
3. a. triangle
   b. rhombus (or diamond)
   c. $\frac{2}{6}$, $\frac{2}{6}$
   d. $\frac{6}{6}$
   e. $\frac{3}{3}$
   f. $\frac{2}{2}$
   g. $\frac{3}{6}$, $\frac{3}{6}$
   h. a half
   i. a third
   j. a half

LESSON 3 pages 3 and 4

FOCUS: Explore equivalent fractions

MATERIALS: Ruler, pattern blocks

DIRECTIONS: In this lesson, students explore equivalent fractions. To reinforce the concepts, provide students with several different manipulatives that show fraction equivalencies; for example, rulers, fraction strips, pattern blocks, and so on. Students will see that $\frac{1}{3}$ of a hexagon is the same as $\frac{2}{6}$ of the hexagon and also that $\frac{1}{3}$ of a fraction strip is the same as $\frac{2}{6}$ of the strip.

As students work through the lesson, allow them to use the manipulatives in any way they find helpful. Students might use the pattern blocks to check their answers for problems 3–6. For problems 7 and 8, the 'trading' further emphasises the concept of equivalency. As students trade pieces of the pattern blocks, they can place pieces on top of one another to prove that pieces are equivalent. To extend the lesson, challenge students to write number sentences that express the various pairs of equivalent fractions they discovered.

RESPONSES:

1. Check students' work.
2. a. 2, $\frac{1}{2}$
   b. 3, $\frac{1}{3}$
   c. 6, $\frac{1}{6}$
3. a. $\frac{1}{3}$
   b. $\frac{1}{2}$
   c. $\frac{1}{2}$
4. a. $\frac{2}{2}$
   b. $\frac{3}{3}$
   c. $\frac{6}{6}$
5. Check students' work.
6. a. They are the same size.
   b. They are the same size.
   c. They are the same size.
7. a. $\frac{3}{6}$
   b. $\frac{6}{6}$
   c. $\frac{4}{6}$
   d. $\frac{6}{6}$
   e. $\frac{6}{6}$
   f. $\frac{3}{3}$
   g. $\frac{2}{2}$
   h. $\frac{3}{3}$
   i. $\frac{6}{6}$
   j. $\frac{3}{6}$
8. a. $\frac{2}{6}$
   b. $\frac{2}{6}$
   c. $\frac{2}{6}$
9. a. $\frac{3}{6}$
   b. $\frac{3}{6}$
   c. $\frac{3}{6}$

LESSON 4 pages 5 and 6

FOCUS: Solve problems by using equivalent fractions

MATERIALS: Ruler, crayons or textas

DIRECTIONS: The activities in this lesson are deliberately open-ended. There are no restrictions as to the number of pieces of pizza that each guest gets. If students ask about this, allow them to decide the number of pieces. In so doing, students will expand their thinking, as they are compelled to consider a variety of possible combinations. Working with pattern blocks will give students the opportunity to study the range of possibilities.

The ideas in this lesson can be used as a starting point for discussions about divisibility, multiples, and factors. You may wish to pursue these concepts with students who demonstrate understanding of the possible combinations in problem 5.

RESPONSES:

Explanations will vary but should express an understanding of divisibility and factors.

1. Check students' work.
2. Sample drawing: 4 of hexagon *A*
3. Sample drawing: 5 of hexagon *B*
4. Sample drawing: 2 of hexagon *B* or 1 of hexagon *C*
5. Sample drawings: 6 of hexagon *A*, 4 of hexagon *B*, or 2 of hexagon *C*