

## INTRODUCTION

*Understanding Fractions* is a series of books for years 1–8. Each book's activities are designed to increase students' understanding of fractions, and have been developed with the conviction that students construct their own understanding through rich, hands-on mathematical experiences. Although the activities in each book are for a specific level, they all connect to the core of mathematics learning that is important to every primary school and middle-school student.

This book in the series is designed for grade 1. However, depending upon the experiences and background of your students, these activities may be used in grade K or 2. They should not be considered remedial in the upper grade levels. Many students who study fractions gain little or no understanding. The activities allow students to explore and to play with fractional relationships and to come to their own understanding. Students can actually begin to analyse shapes, relationships, and numbers, and in the process, develop their own sense of mathematical power.

In the past, the role of the maths teacher focused on presenting rules and procedures, followed by seemingly endless drills for practice. The student's role was practise, practise, practise, until mastery of algorithms was proven on the weekly test. Changing educational standards have drastically de-emphasised this computational focus. Emphasis is now on communication, reasoning, and problem solving. Teachers now foster student thinking for the problem solving and for alternative, original algorithms. Much of today's maths instruction is rooted in the work of Piaget, Suppes, Gattegno, Wirtz and Botel, Dienes, and Burns.

## ORGANISATION

*Understanding Fractions* is not a recipe series for teaching fractional concepts. Rather, it is a series to be used by you and your students to study and explore the dynamic field of fractions. The experiences presented are intended to allow students to gradually develop a systematic awareness of fractional parts and their relationships. This level of the series depends primarily upon visual input for students to construct a personal framework for understanding fractions. The use of manipulatives is required in most lessons. Allow students to use whatever tools they find helpful to complete the work. Except for the quiz, all work may be done in pairs or in small groups of three or four students. The student exchange of ideas increases the power of learning as it uncovers a variety of strategies for problem solving. Working together also allows students to answer each other's questions about the directions and concepts.

There is a glossary at the back of the student book. As they work through the book, students will find several words highlighted in bold type. These words appear in a glossary, with an example. You may want to discuss the definitions with the class at the end of the particular lesson.

You are not asked to correct each page, but rather to discuss the students' results. Encourage students to focus more on their strategies than just on getting the right answer. The specific answers to each page are not as important as the students' awareness of a developing sense of fractions and an ability to apply previous understandings.

Try to be open to a multitude of correct responses. Some may not be the responses that you had in mind, but may demonstrate a student's own sense of the question at hand. As the teacher, you may be surprised at the students' sophisticated awareness of numbers and relationships. School for most students has been an exercise in getting the right answer in order to get a good mark, even if what they have done makes no sense to them. They have not had the opportunity to move from their baseline of knowledge to more complex thinking. This past approach undermined the student's belief in his or her ability. The teaching of mathematics today requires a tremendous shift in thinking on your part, as you learn to ask questions that are powerful catalysts for student exploration. Relax and have fun with this book. Mathematics can turn out to be the most exciting study of the day.

## **MATERIALS**

Provide students with an ample supply of paper, pencils, rulers, crayons, and an assortment of manipulatives. Manipulatives can be as simple as buttons, plastic counters, egg cartons, marbles, and toothpicks. Commercially available manipulatives—such as colour tiles, pattern blocks, geoboards, colour squares, and Cuisenaire Rods®—are powerful tools for recreating a three-dimensional representation. Various manipulatives are referred to and recommended in the materials section. You may replace these specific manipulatives with comparable objects found in the classroom environment or made from construction paper and laminated for durability. Some activities don't list any specific materials, but you should encourage students to use any available manipulatives that will help with the learning procedure.

Using already-constructed objects whenever possible will allow you more time for planning maths content and instructional strategies. You may want to look through the book ahead of time and instruct students to create their own manipulatives packet. Students may cut out the maths tiles and pattern blocks from pages 35 and 37 in the back of the student book and colour them appropriately.

Or you may choose to make copies of Reproducible A (page 12) and Reproducible B (page 13) and distribute them to students. If possible, have the materials laminated. Provide additional common classroom objects for students to add to their packet.

Because students need physical models to construct meaningful representations of their solutions, a variety of readily available hands-on materials will increase the opportunity each student has to bring his or her perspective to the problem at hand. Whereas some materials may seem more appropriate to you for the solution to a problem, allowing students to select their own materials increases individual, diverse thinking.

Encourage your students to use manipulatives to interpret data. Develop a maths centre that contains varieties of maths manipulatives. Recreating printed data with manipulatives provides a kinaesthetic opportunity for understanding and more options for solutions. As with any manipulative materials, the thinking activity generated from the physical manipulations is the focus.

## **GROUPING**

The grouping recommended for this series is individual, partners, or a small collaborative group of three or four members. Although there are many approaches to learning, and some students need to work without outside distractions, most students need and benefit from access to one another's thinking. Students who are allowed to help one another will minimise misunderstandings or confusion from directions or adult interpretations. Encourage your students to discuss the directions together and make sure that their partner understands what to do before they begin. Partners should help one another and should not be afraid to say that they do not understand. Students need to know that this is how learning occurs. Suggest that students try to do the work on their own, checking with a partner as they go along to make sure that they are moving in the right direction. Students need to learn how to work as a team, being responsible for their own work, yet not doing the work for one another. When students finish their work, the group provides a powerful opportunity for immediate feedback and self-correction. Encourage students to use this checking-in period to adjust their responses. Explain that the goal is on learning, not on determining how many problems the students got right or wrong. In a group, students learn to accept the diversity of responses, which fosters creative mathematical thinking.

Sometimes, you will work together with all students. Often, this entire-class work follows the paired-group or collaborative-group sessions in which the students write and check their work together. Bringing all groups together for a focused lesson and classwide sharing permits you to correct any misunderstandings and to expand on concepts that students might bring up.

## THE LESSONS AND INSTRUCTION

The lessons in *Understanding Fractions* involve problem solving, communication, reasoning, and mathematical connections. Each activity focuses on at least one of the curriculum strands of number, geometry, measurement, and patterns/functions. Working each lesson along with your students provides a powerful opportunity for you to develop a sense of the concepts and understandings that are emerging for them. Students are often eager to explain ideas to their teachers, especially if the teacher acknowledges his or her own difficulties. This experience makes the classroom a community of learners. Any preconceived notions of fractions that you might have will be no more important than those of the emerging mathematicians in your classroom. The lessons are meant to be exploratory and are sequentially connected. If individual lessons are presented in isolation, they will be more dependent upon adult interpretation and formal teaching.

Mathematics does not lend itself to solitary pursuit, so your role as teacher in the development of a deep and powerful understanding is crucial. The majority of students are dependent upon their instructors to lead the exploratory journey through the different strands. The social interaction of the students, the artful and timely questioning by you, and the latitude to probe, question, and discover are all critical elements in developing the mathematical minds of the students.

The lessons in *Understanding Fractions* are designed to enable students to increase their own mathematical power. The intent is to set the stage but not steal the show. Some activities may seem challenging, and you may be tempted to teach by modelling the solutions. However, this would not provide students with the opportunity to try the activity in whatever way they can. As long as the students have a way of beginning an activity, give them the opportunity to work it through. As members of a pair or a collaborative group, most students will meet with success. Through trial and error and discussion, students will learn.

## ASSESSMENT

Because assessment is multifaceted, this series encourages a range of strategies for assessing student progress with the purpose of modifying instruction, not judging ability. Each mathematics lesson, as well as the quiz, is a part of the assessment process. Also, you can gather much information by listening to students' explanations, observing their thinking as manifested by the manipulation of physical objects, and examining their writing and reflections. These assessment tools are some ways of probing student understanding for the purpose of determining how to modify instruction. Encourage students to keep a portfolio, which will allow both you and them to see growth over time and identify successful problem-solving strategies.

**FOCUS:** Work with same size and shape

**MATERIALS:** Pattern blocks, crayons

**DIRECTIONS:** Before students can begin working with equal parts, they need to understand the concept of same size and shape. Have students use pattern blocks to discover figures that are the same shape and size. They can place the blocks on top of one another to prove that all sides and corners of the figures are equal. For additional practice, encourage students to look around the classroom for examples of objects that are the same size and shape. Also ask students to think of other examples of things that are the same size and shape.

Work through the lesson with students where necessary. At first, students physically match figures to prove that they are the same size and shape. Next, they visually determine which figures are the same size and shape. Some students may want to trace the first figure so that they can check which figures match. Finally, students draw a figure that is the same size and shape as the one given. To do this, they simply need to count the number of squares inside the original figure and then duplicate it on a grid.

**RESPONSES:**

1. b. Students use the trapezoid pattern block.
2. Students place the block on top of the trapezoid in the book.
3. yes
4. yes
5. Check students' work.
6. Check students' work.
7. Check students' work.

**FOCUS:** Introduce symmetry

**MATERIALS:** Paper, scissors, pattern blocks

**DIRECTIONS:** This lesson provides another look at same size and shape, in the geometric concept of symmetry. Students discover how to make two figures the same size and shape by folding and cutting. The term *line of symmetry* is not introduced at this point. You may choose to have students identify the lines of symmetry in the figure if you feel it's appropriate.

Work through the lesson with students where necessary. Provide students with paper, scissors, and pattern blocks. You may want to demonstrate the procedure described in items 1 to 6 before having students do it themselves. Once students understand the physical process, they can apply what they have learned to visually identify figures shown with a line of symmetry, and then to draw lines of symmetry in other figures.

**RESPONSES:**

- 1.–6. Check students' work.
7. yes
8. yes
9. Check students' work.
10. Students should circle figures *b* and *c*.
11. Check students' work.

**FOCUS:** Explore equal parts

**MATERIALS:** Pattern blocks

**DIRECTIONS:** Understanding the concept of equal parts is essential in work with fractions. This lesson shows equal parts in geometric figures. Point out the characteristics that make the parts equal (same size and shape). Encourage students to use the pattern blocks to create figures that have equal parts.

Work through the lesson with students where necessary. First, students work with two equal parts, which should be a familiar idea by now. As students complete the lesson, help them to understand that figures can be divided into more than just two equal parts. They see figures divided into three and four equal parts. You may want to challenge students to divide various figures into more than four parts.

**RESPONSES:**

1. Students study the rectangle.
  - a. 2
  - b. yes
2. Students study the rectangle.
  - a. 3
  - b. yes
  - c. 3
3. Students should circle figures *b* and *c*.
4. a. 3  
b. 4  
c. 2
5. Check students' work.

**FOCUS:** Explore equal parts of groups

**MATERIALS:** Counters, crayons

**DIRECTIONS:** Another important concept in the understanding of fractions is equal parts of groups. Now that students have been exposed to same size and shape and equal parts of figures, they can move on to apply this concept to groups. Emphasise that the key to making equal parts of groups is the number of items. To make equal groups, there must be the same number of items in each group. Before completing the lesson, students can practise making equal groups with counters or other small objects. To stress equality, make sure the objects are the same.

Work through the lesson with students where necessary. Help students understand that a group of objects can be divided into different numbers of equal groups. In this lesson, students make two equal groups and three equal groups. You may want to point out that there are many other possibilities for making equal groups; for example, four and five equal groups. The essential element is that the groups must have the same (equal) number of items.

**RESPONSES:**

1. Check students' work.
2. 3
3. 2 groups of 3 = 6
4. a. Check students' work. 2 groups of 5 = 10  
b. Check students' work. 2 groups of 4 = 8
5. Check students' work.
6. 3
7. 3 groups of 3 = 9
8. a. Check students' work. 3 groups of 4 = 12  
b. Check students' work. 3 groups of 5 = 15