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Introduction

The story of Al and Gebra (pronounced JEH-bruh) provides a mental model for many basic algebraic concepts. A mental model is a way of holding abstract thoughts in our head.

Multiple readings of *The Journey of Al & Gebra to the Land of Algebra* will enable the reader to easily access the appropriate mental model for solving an algebra problem. For example, a student who needs to remember how to add positive and negative numbers can recall the chapter in *Al & Gebra* where the princess eats positive berries that make her grow and negative berries that make her shrink. Whichever she eats the most of determines the results. The student then would know how to solve the problem.

Al & Gebra's storyline is understandable by students as young as year three or four, yet sophisticated enough for individuals in secondary school or beyond.

The Journey of Al & Gebra to the Land of Algebra is a starting point for learning algebra. It provides a foundation for concrete, logical understanding of abstract thought. It is not intended to take the place of instructional programs that provide more detailed explanations and practice material.

Enjoy the journey!

The Departure

In a land of dragons and dungeons, queens and kings, lords and ladies, and moats and castles, there lived a young man and a young woman. Their names were Al and Gebra. When this story begins, Al and Gebra are kneeling before the king. His royal court is watching silently from behind the throne.

"Al and Gebra," the king said as he held the tip of his sword over their heads. "You have reached the age of adulthood, and you have been recommended for the ranks of Lord and Lady. But before I can bestow this honour upon you, you must prove yourselves worthy."

"How must we do this, Your Majesty?" Gebra asked without looking up.

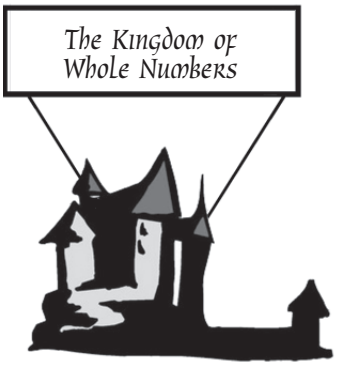
"Your task will not be an easy one," the king answered. "You must travel to each of the 14 kingdoms in our great forest. You will discover that every kingdom is facing a serious problem. You must find an answer to each problem and bring the solutions back to me."

"We will do as you ask," Al promised as he and Gebra rose, bowed slightly and silently exited the chamber.

"I hope so," the king whispered under his breath as he watched them depart.

The First Kingdom

Positive and Negative Numbers



The dark shadows of the First Kingdom loomed in the distance as Al and Gebra emerged from the forest. They approached the huge stone walls cautiously, pausing before the drawbridge.

"The Kingdom of Whole Numbers," Al read from the sign above the castle. "I wonder what that means."

"The people in this kingdom call themselves Whole Numbers," a squeaky voice whispered from the shadows.

"Who's there?" Gebra asked, her horse dancing nervously.

"Here I am!" Al and Gebra saw the bony arm of a small, thin woman waving at them from behind a huge rock.

"Why do the people in your kingdom call themselves Whole Numbers?" Al asked.

"Because we're all named after whole numbers: Zero, One, Two, Three, Fifty-nine, Seventy-two, Eighty-one and so on. That's why." The tiny woman bobbed into full view as she talked.

Whole number: any of the numbers 0, 1, 2, 3 and so on

"But whole numbers make dumb names," she continued. "Who wants to be named 'Thirty-seven?' Or 'One hundred and six?' Or 'Two thousand and eighty?' I don't."

"You certainly are negative!" Gebra responded.

"That's me! Negative Number, at your service." The bony woman took a bow.

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"Negative Number!" Al said with a laugh. "That does have a nicer ring to it than 'One hundred and six.' Does the Kingdom of Whole Numbers have a problem that needs to be solved?"

"Why don't you find out for yourself?" Negative Number snapped.

Al thought the tiny lady might be helpful in some way, so he said, "If you would like to come inside with us, you can ride with me on my horse."

Before anyone knew what had happened, the tiny lady was in the saddle, holding Al tightly around his waist. "Giddy-up!" she yelled, tapping the horse's sides with the heels of her narrow, pointed shoes.

Al's horse responded, and within seconds the party of three was entering the courtyard. Once inside Al and Gebra saw people running everywhere, many of them arguing with one another. "Why does everyone look so confused?" Gebra asked Negative Number, who was barely visible behind Al.

"Because we can't find our rooms," she said. Negative Number's voice sounded desperate.

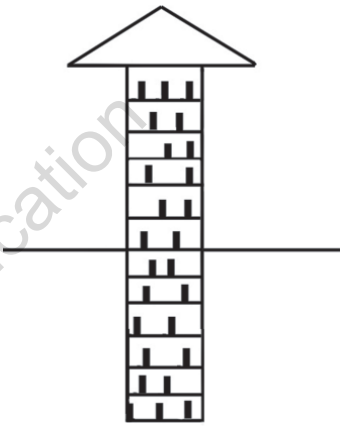
"Some rooms are in the dungeon, and some are in the tower.

"Each of us knows the number of the floor that our room is on, but we don't know whether to go up into the tower or down into the dungeon to find it.

"For example, my room is on the fifth floor, but I don't know whether that's the fifth floor up, in the tower, or the fifth floor down, in the dungeon," Negative Number said, then fell silent.

"This is a serious problem," said Gebra. "We need to talk to someone in charge." So Gebra urged her horse toward a line of armoured soldiers.

"Halt!" the soldiers' voices shouted when Al and Gebra were about three metres in front of them. Al's horse reared and Negative Number tumbled to the ground.



"Some way to show your gratitude!" she snarled at Al.

"Take us to your leader," Al demanded of the soldiers. In unison they lifted their right feet, then their lefts. Chanting and marching, the soldiers resembled a human door as the group parted down the middle. Finally the human door opened enough to reveal a queen sitting on a platinum throne.

"Her Majesty, Mean Queen Jean," a small boy announced through a megaphone.

Al and Gebra dismounted their horses, walked to the foot of the queen's throne, and bowed. "Your Majesty," they said before straightening up.

"If you don't mind my asking, why do the people of this kingdom call you *mean?*" Gebra asked. Al cleared his throat and glanced sideways at Gebra, questioning the propriety of posing such a question.

"It's simple," the queen replied. "They call me *mean* because I am average. I am the average height, average weight, and average age – and I have the average IQ of all the adults in the kingdom. *Mean* is another word for average, so they call me *mean.*"

Mean: The arithmetic mean, often called the average or simply the mean, is the sum of the values divided by the total number of values.

Examples: The mean of 5, 7 and 9 is 7 (5 + 7 + 9 = 21; 21 ÷ 3 = 7).
The mean of 1, 57, 100 and 2 is 40 (1 + 57 + 100+ 2 = 160; 160 ÷ 4 = 40).

"Your Majesty, Mean Jean," Al said, bowing slightly. "We are in search of a problem to solve. Does your kingdom have any problems?"

A deafening roar of clanging armour filled the chamber as knights clasped their hands over their mouths, trying to stifle their roars of laughter.

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"Silence!" the queen shouted, glaring at the knights, then looked back at Al and Gebra. "Yes," she said, "we have a problem, but why don't you just get a good night's sleep before we talk about it. Room numbers, please!"

The small boy disappeared into a nearby chamber, then reappeared with a large stone bowl, which he lifted up to Al and Gebra. "Your room numbers!" he said. Gebra looked inside the bowl and saw many small pieces of paper, with a numeral written on each one. She reached inside and took one piece of paper. "Number 12," she held the paper up for all to see.

"So," said the queen, "your room is on the twelfth floor. Now go to your room."

"But should I go to the twelfth floor up in the tower, or to the twelfth floor down in the dungeon? They're opposites!" Gebra's voice sounded tense.

"That's for you to figure out!" The queen sounded equally annoyed.

"That's the problem," the child whispered from somewhere near Gebra's elbow. "We don't know how to show whether the numbers mean that we should go up into the tower or down into the dungeon. All we know is that they're, well, numbers!"

"We need to think of something," Gebra whispered to Al.

"To begin with, the ground that we're standing on is the line that separates the tower from the dungeon. We can call this line 'zero,'" Al suggested.

"So I'm standing on zero. That's just what I thought," Negative Number mocked.

"Now we need a name for the numbers that are up in the tower," Gebra continued.

"If your room is in the tower, that would be positive," Al said. "So the numbers that are above zero can be called 'positive numbers.'"

Positive numbers: Numbers that are greater than zero are positive.

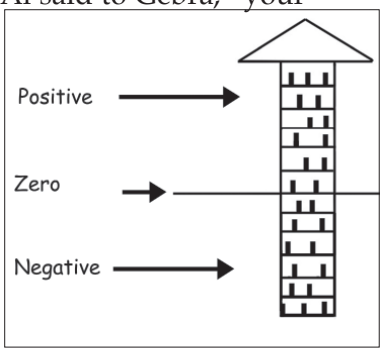
"Good idea. And numbers in the dungeon are less than zero. What should we call numbers that are less than zero?" Gebra wondered aloud.

"This is a waste of time," Negative Number grumbled.

"Negative Number, would you please stop being so negative?" Gebra said with a scowl, then smiled as an idea lit up her face. "That's it, Negative Number! We can call numbers that are less than zero 'negative numbers.'"

"Negative numbers are the opposite of positive numbers," Negative Number boasted.

"So, if your room is in the dungeon," Al said to Gebra, "your room number is negative – negative twelve (-12). If it is in the tower, it is positive – positive twelve (+12)."



"I want to make a proclamation!" the queen said, rising from her throne. "We will no longer be called The Kingdom of Whole Numbers! From this

Negative numbers: Numbers that are less than zero are negative.

day forward we will be called the Kingdom of Integers – whole numbers and their opposites."

Within minutes the sign above the drawbridge that had read "The Kingdom of Whole Numbers" was removed, and a new sign was put in its place:

All the integers in the kingdom lined up to have a positive (+) or a negative (-) sign drawn in front of the numbers on

The Kingdom of Integers: whole numbers and their opposites

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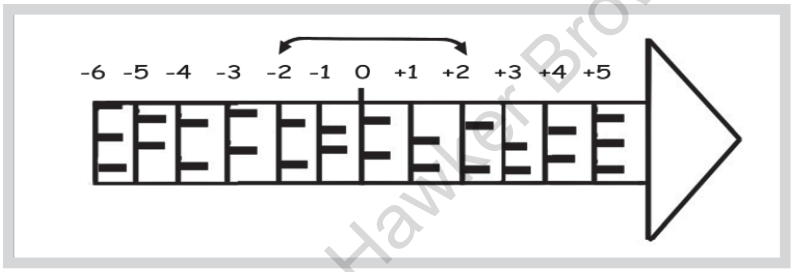
their room-assignment slips. By midnight everyone was tucked in bed and sound asleep. "You're a hero," Gebra whispered to Negative Number before heading up the stairs to her room on the +12th (positive twelfth) floor.

"That's right. From now on half of the numbers in the world will be named after you," Al added before going to his room on the -23rd (negative twenty-third) floor.

"Don't want to be a hero!" Negative Number shouted after them. "But I'll try," she added quietly, when they were out of hearing range.

The next morning Negative Number and the boy were waiting for Al and Gebra in the courtyard. "Look," Negative Number said excitedly, as she pointed to a drawing she had made in the sand. "If the tower were to fall down, this is what it would look like!"

"And another thing," the lad said, pointing to the chart. "Positive 2 and negative 2 are the same distance from zero.



So even though negative 2 is negative, it still means that someone must walk two floors from zero."

"You're right," Al said. "Negative numbers have value."

"Absolutely," the boy said.

Al smiled. "Good thinking," he said. "We'll say that the value of a negative number is its opposite, and we'll call that the 'absolute value.'"

"Absolutely," the boy repeated.