

FOREWORD

The theory of mental models was first introduced by Kenneth Craik, a British philosopher and psychologist, in his 1943 book *The Nature of Explanation*. After his untimely death in a bicycle accident in 1945 (at age 31), the idea did not re-emerge in the literature until 1983 with the publication of Philip Johnson-Laird's book *Mental Models*.

Mental models have been defined as internalized, mental representations of things in the world—or internal symbols or representations of external phenomena. The representation that a person develops determines how he/she understands and interacts with the world. The more accurate the mental model, the more successful the interaction.

Mental models are believed to play a major part in cognition. When learners develop a more complete, accurate, and richer model of a particular domain, they become more competent in that domain. A teacher's main responsibility is to mediate students' learning so they formulate accurate, complete mental representations of abstract concepts as efficiently as possible.

The term *mental model*, as used in the field of education and in this workbook, is intended to describe strategies, visual representations, analogies, and stories that assist in the development of accurate internal symbols. Each mental model is designed to move the student closer to a deeper and richer understanding of the standards and abstract concepts necessary for success in the academic setting, as well as meaningful interaction with the world.

INTRODUCTION

Mental Models for Math

This workbook contains examples of mental models that teachers in middle school through high school can use in explaining mathematical concepts.

The following information, taken from *Understanding Learning: the How, the Why, the What* by Dr. Ruby K. Payne, explains the characteristics and purposes of mental models.

- Mental models are how the mind holds abstract information, i.e., information that has no sensory representation.
- All subject areas or disciplines have their own blueprint or mental models.
- Mental models tell us what is and is not important in the discipline. They help the mind to sort.
- Mental models often explain the “why” of things working the way they do.
- Mental models tell the structure, purpose, or pattern of the discipline.
- Mental models are held in the mind as stories, analogies, movements, or two-dimensional drawings.
- Mental models “collapse” the amount of time it takes to teach/learn something.
- Mental models of a discipline are contained within the curriculum.

Mathematics, for example is about assigning order and value to the universe.

The mental models in this workbook involve four types: Pictures, Movement, Stories, and Analogies. All can be used effectively by teachers.

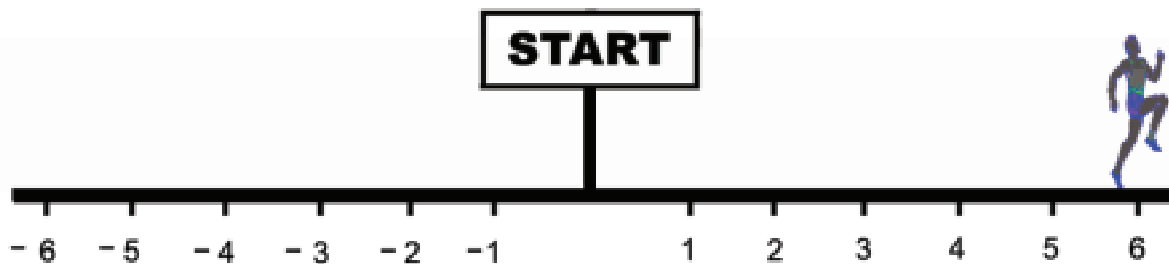


Absolute Value

A runner ran in a positive direction. He ran 6 feet.

$$|6| = 6$$

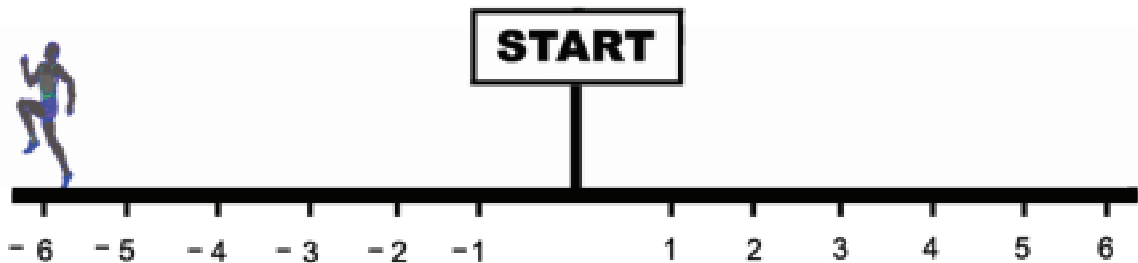
(The absolute value of 6 is 6.)



Another runner ran in a negative direction. He also ran 6 feet.

$$|-6| = 6$$

(The absolute value of negative 6 is 6.)



MENTAL MODEL—NUMBERS AND OPERATIONS

Standard: Numbers and Operations

Understand numbers, ways of representing numbers, relationships among numbers, and number systems.

Understand and use ratios and proportions to represent quantitative relationships.

Explanation of Mental Model:

This “poster style” mental model is designed for frequent viewing.

STEP SHEET

1. The ratio of your mouth to eyes is 1:2. List three additional ratios of your facial features (mouths, eyes, ears, eyebrows, noses).
2. Study the mental model and write the definition of *proportional* in your own words.
3. Study Mental Model #1 and explain in your own words how you know that Ray’s and Plutonia’s faces are proportional.
4. Study Mental Model #2 and explain in your own words how you know that Ray’s and NaPortia’s faces are not proportional.
5. Study Mental Model #3 and compute the number of eyes that June from Jupiter has.
6. Draw a space creature’s face that has features that are proportional to yours.
7. Draw a face that has proportional features but has a different number of eyes than you have.
8. Create other space creatures and let your neighbor determine which ones have features that are numerically proportional to yours.
9. Design a mental model that explains ratio and proportion.



$$y = x$$

I want to be just like you.

I am an independent adult.
You can depend on me!

Dependent Variable

Independent Variable

NOTE: Some mathematicians avoid use of this terminology because of possible confusion with other disciplines.