# INTRODUCTION

## What is the *FOCUS* series?

***FOCUS*** is a mathematics-strategy practice series. Each student book in the series provides brief instruction and concentrated practice for students in one targeted Mathematics Strategy. ***FOCUS*** also allows students the opportunity for self-assessment of their performance. It allows teachers the opportunity to identify and assess a student's level of mastery.

**Six Mathematics Strategies featured in the *FOCUS* series:**

- Building Number Sense
- Using Estimation
- Using Algebra
- Using Geometry
- Determining Probability and Averages
- Interpreting Graphs and Charts

The ***FOCUS*** series spans eight year levels, from year one to year eight. The introductory passages in each lesson are written at or below year level, allowing students to focus on the mathematics without struggling with the reading.

| Book | Reading Level |
|---|---|
| Book A | at or below year one readability |
| Book B | at or below year two readability |
| Book C | at or below year three readability |
| Book D | at or below year four readability |
| Book E | at or below year five readability |
| Book F | at or below year six readability |
| Book G | at or below year seven readability |
| Book H | at or below year eight readability |

## What is Determining Probability and Averages, the Mathematics Strategy featured in this *FOCUS* book?

Determining probability involves finding the likelihood that an event will occur. Probability is determined by comparing a specific outcome with all of the possible outcomes in a given situation. In the early years, students study probability in terms of *more likely, less likely* and *equally likely* outcomes. As they progress, students learn to express probability in fraction form. The numerator represents a specific outcome, and the denominator represents the total number of possible outcomes. Students in the upper year levels also learn to express probability in percentage and decimal form.

Finding the average of a group of numbers provides information about how each number relates to the group as a whole. Students in years two to eight learn how to calculate averages. The addends that they work with increase in size and number as students progress through the year levels. In years seven and eight, students are introduced to several measures of central tendency. They learn to identify and calculate the mean, median, mode and range of a data set.

Students in year one learn important readiness concepts. They sort objects into groups. They practise sorting items by size, shape and colour. Students also combine sets into one group and then make equal groups.

In the middle years students learn to calculate the total number of possible combinations in a given situation. The number of possible combinations is determined by calculating the product of the numbers of items in the given categories.

## How should I use the *FOCUS* series in the classroom?

The *FOCUS* series can be used effectively in the classroom in several ways. Here is a suggestion for using the program in **whole class, large group, small group, paired** and **individual formats.**

### To the Student

*(inside front cover of the student book)*

Read and discuss this with the whole class or large group to make sure students understand what they are to do in the book.

### Learn About

*(pages 2–3 of the student book)*

Read the two pages of instruction in the Mathematics Strategy to the whole class or large group. Model using the Mathematics Strategy. Use information from the Mathematics Strategy Tips for the Teacher on pages 12–13 of this teacher guide to prompt additional in-depth discussion of the Mathematics Strategy, as appropriate. Make sure all students understand the features of the Mathematics Strategy and how to apply the Mathematics Strategy before they go on. The Learn About requires approximately 45 minutes.

### Lesson Preview

*(pages 4–5 of the student book)*

Read the boxed directions to the whole class or large group. Emphasise what students should watch for as they read the problem. Have students read the problem individually. Guide the whole class or large group in answering the two selected-response questions. Then discuss why each answer choice is correct or not correct. Make sure all students understand how to answer the Mathematics Strategy questions before they go on. The Lesson Preview requires approximately 45 minutes.

### Lessons

*(pages 6–45 of the student book)*

For each lesson, have students read the directions and the passage individually, in pairs or in small groups. Have students answer the selected-response questions and the constructed-response question individually, in pairs or in small groups.

Have students use the Tracking Chart on page 47 of the student book to note the date that they have finished each lesson. When the questions in all five lessons in a group have been corrected, have students note the number of correct responses for each lesson and then the number of correct responses for the whole group of lessons.

Each lesson, plus tracking, requires approximately 45 minutes. Allow students 30 minutes to read the passage and answer the questions, and allow 15 minutes to discuss the responses. Discuss the answers to the questions with the whole class or large group, or with pairs, small groups or individuals. (See **What is the correction procedure?** on page 4 of this teacher guide.)

**Self-Assessment:** When students have finished each group of five lessons, have them complete the appropriate Self-Assessment. When students have finished all twenty lessons, have them complete Self-Assessment 5. Each Self-Assessment requires approximately 20 minutes.

**Discussion:** When students have finished each group of five lessons, discuss their performance individually or in small groups. When students have finished all twenty lessons, discuss their performance individually or in small groups. Each discussion requires approximately 25 minutes.

# MATHEMATICS STRATEGY TIPS FOR THE TEACHER

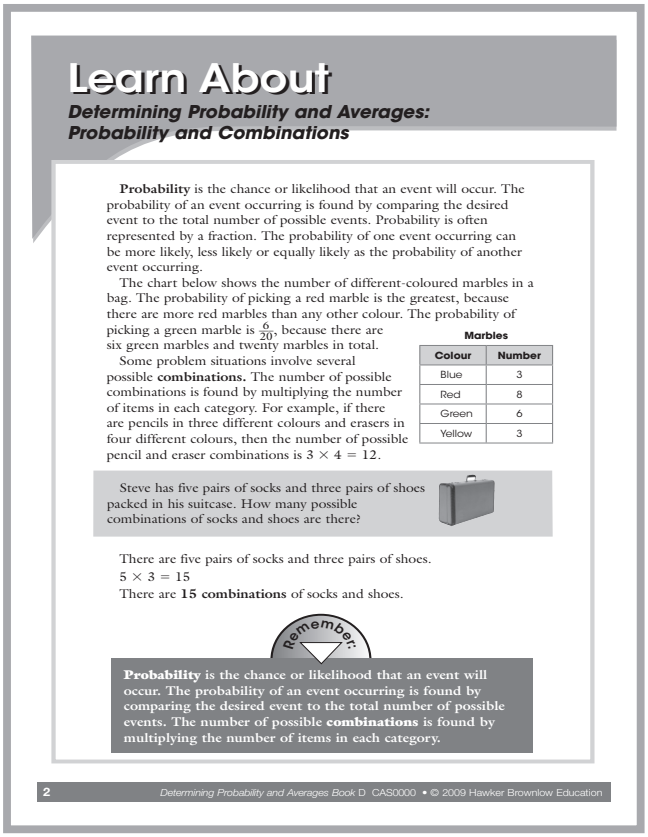
Using probability can help you determine the likelihood of a certain event's occurring.

Students explore the probability of an event as more likely than, less likely than or equally likely as another event. For example, if there are more football cards than other types of cards in a sports card collection, the likelihood of randomly picking a football card is *more likely than* that of picking any other type of sports card.

Students also use fractions to represent probability. The numerator represents the favourable outcome, and the denominator represents the total number of possible outcomes. Look at the chart on the Learn About page. There are 3 blue marbles and a total of 20 marbles. The probability of picking a blue marble is represented by the fraction $\frac{3}{20}$.

Organised lists, such as the chart on the Learn About page, can be used to determine or demonstrate the probability of events. One column lists the types of objects in a data set. The other column lists the number, measurement or value of each type of object. If one of the objects in the data set is chosen randomly, the greatest number in the chart indicates the most likely event. Conversely, the least number in the chart indicates the least likely event. Equal numbers in the chart indicate events that are equally likely to occur when items are selected randomly.

At this level, students explore the number of *possible combinations* in a data set by multiplying the number of items in each category. For example, if a shop has two types of cones and three types of ice-cream, there are $2 \times 3$, or six, possible combinations of cones and ice-cream flavours.



## Learn About

***Determining Probability and Averages: Probability and Combinations***

**Probability** is the chance or likelihood that an event will occur. The probability of an event occurring is found by comparing the desired event to the total number of possible events. Probability is often represented by a fraction. The probability of one event occurring can be more likely, less likely or equally likely as the probability of another event occurring.

The chart below shows the number of different-coloured marbles in a bag. The probability of picking a red marble is the greatest, because there are more red marbles than any other colour. The probability of picking a green marble is $\frac{6}{20}$, because there are six green marbles and twenty marbles in total.

**Marbles**

| Colour | Number |
|---|---|
| Blue | 3 |
| Red | 8 |
| Green | 6 |
| Yellow | 3 |

Some problem situations involve several possible **combinations.** The number of possible combinations is found by multiplying the number of items in each category. For example, if there are pencils in three different colours and erasers in four different colours, then the number of possible pencil and eraser combinations is $3 \times 4 = 12$.

Steve has five pairs of socks and three pairs of shoes packed in his suitcase. How many possible combinations of socks and shoes are there?

There are five pairs of socks and three pairs of shoes.
$5 \times 3 = 15$
There are **15 combinations** of socks and shoes.

Remember:

**Probability** is the chance or likelihood that an event will occur. The probability of an event occurring is found by comparing the desired event to the total number of possible events. The number of possible **combinations** is found by multiplying the number of items in each category.

2 *Determining Probability and Averages Book* D CAS0000 • © 2009 Hawker Brownlow Education

Use items from the classroom to explore *possible combinations*. Making an organised list of all possible combinations and counting the combinations may help students understand why multiplication is used to find possible combinations. For example, suppose that a pencil case contains pencils and erasers in various colours. There are yellow, blue and red pencils. There are green, blue, orange and purple erasers. Students can make a list to find the total number of possible pencil–eraser combinations. They can start by listing all of the combinations that include a yellow pencil: *yellow–green, yellow–blue, yellow–orange* and *yellow–purple*. After completing the list with the other colours of pencils and erasers, students will have three groups of four combinations. Explain to students that three groups of four is represented by the equation $3 \times 4 = 12$.

## How Is *FOCUS on Mathematics* Supported by Research?

***FOCUS on Mathematics*** is supported by research from mathematical researchers and organisations, including the National Math Advisory Panel and National Council of Teachers of Mathematics, both from the US. Much of the research on effective instruction for mathematical students parallels the recommendations of the NMAP (2008). Many of these recommendations are integrated into the ***FOCUS on Mathematics*** series, including: word-problem focus, explicit instruction with modelling and focused practice.

### Word-Problem Focus

Word problems are the proving ground for students to demonstrate their mastery of mathematical fluency and conceptual understanding. Having the ability to transfer what they have learned to new problem-solving situations is one of the major goals for mathematical education (NCTM, 2006; NMAP, 2008). "The issue of transfer, that is, the ability to use skills learned to solve one class of problems, such as similar triangles, to solve another class of problems, such as linear algebra, is a vital part of mathematics learning" (NMAP, 2008, p. 30). And yet, students, on average, have the most difficulty solving word problems.

The ***FOCUS on Mathematics*** series provides repeated and focused practice of key maths strategies in the context of word problems. With more than 800 word problems in the series, students gain multiple opportunities to practise core maths concepts and strategies.

### Explicit Instruction with Modelling

Explicit instruction is a hallmark of effective instruction for struggling and on-level students. Explicit instruction is one of the instructional methods that research has proved to be effective. "By the term *explicit instruction*, it is meant that teachers provide clear models for solving a problem type using an array of examples, that students receive extensive practice in use of newly learned strategies and skills, that students are provided with opportunities to think aloud (i.e. talk through the decisions they make and the steps they take), and that students are provided with extensive feedback" NMAP, 2008, p. 23). Each of these features, can be found in the ***FOCUS on Mathematics*** series. With explicit instruction and teacher modelling, skill efficiency is nearly guaranteed by students (Hiebert & Grouws, 2008).

***FOCUS on Mathematics*** uses explicit instruction in the teaching of the mathematical strategies. The explicit instruction occurs in the Learn About section and the Lesson Preview section. Through the Learn About section, students receive explicit instruction consisting of a definition, semi-concrete and visual representations of the maths concepts, and a usage rule for the maths strategy. Additionally, ***FOCUS on Mathematics*** is a perfect vehicle for struggling students because it does not overwhelm students with the presentation of information. In the Learn About lesson, students initially experience the maths concepts in short presentations, usually three to seven sentences long. A Remember box text feature is a point of reference for students to use while attending to lessons. The Remember box is consistently placed in each book of the series. Struggling or novice maths students usually skip or gloss over text features, which are valuable tools. With repeated exposure and external prompting by the teacher, students learn to pay attention to the text feature.

### Focused Practice

One of the major callings from from expert panels and organisations is for deeper learning and practice of mathematical skills and strategies. Focusing on specific key mathematical topics allows "teachers to commit more time each year to topics receiving special emphasis. At the same time, students would have opportunities to explore these topics in depth, in the context of related content and connected applications, thus developing more robust mathematical understandings" (NCTM, 2006, p. 4).

# ANSWER KEY

## *FOCUS on Determining Probability and Averages, Book D*

**Lesson 1** *(page 6)*

1. D 2. C 3. A 4. A

5. Solution: Connor is least likely to pick an orange fruit juice.

Sample Explanation: *I compared the number of each type of fruit juice.*

$$5 < 6 < 8$$

*There are fewer orange fruit juices than any of the other flavours. So, Connor is least likely to choose an orange fruit juice.*

**Lesson 2** *(page 8)*

1. A 2. C 3. B 4. B

5. Solution: During the third week in May, 41 students were absent.

Sample Explanation: *First, I multiplied the average of the 3 weeks (42) by 3 to find the total number of students absent during all three weeks.*

$$42 \times 3 = 126$$

*Then I added to find the total number of students absent during the first week and the second week.*

$$12 + 5 + 9 + 13 + 11 = 50$$

$$14 + 6 + 6 + 7 + 2 = 35$$

$$50 + 35 = 85$$

*Finally, I subtracted the 2-week total (85) from the 3-week total (126). The difference is the number of students absent during the third week in May.*

$$126 - 85 = 41$$

**Lesson 3** *(page 10)*

1. C 2. A 3. B 4. D

5. Solution: The probability that the coach picked a shirt whose number is divisible by 5 is $\frac{3}{12}$.

Sample Explanation: *First, I listed the numbers from 20 to 31.*

*20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31*

*Then I went through the list to find numbers that are divisible by 5. Numbers that are divisible by 5 all end in a 0 or a 5.*

*20, 25, 30*

*There are 3 numbers that are divisible by 5, so I wrote 3 over the total number of shirts (12) to express the probability.*

$$\frac{3}{12}$$

**Lesson 4** *(page 12)*

1. C 2. A 3. D 4. C

5. Solution: The probability that she will select a vowel is $\frac{3}{10}$.

Sample Explanation: *First, I counted all the letters, and then I counted all the vowels. There are 10 letters, and 3 of the letters are vowels. I wrote the number of vowels (3) over the number of letters (10) to express the probability of picking a vowel.*

**Lesson 5** *(page 14)*

1. C 2. B 3. C 4. A

5. Solution: They swam for 2 hours on the last day.

Sample Explanation: *First, I changed the times in hours and minutes to minutes.*

*1 hr 30 min = 90 min*

*1 hr 45 min = 105 min*

*Then I multiplied the average (90) by 3 days to find the total number of minutes they swam in three days.*

$$90 \times 3 = 270$$

*Next, I added the number of minutes they swam on the first and second days.*

$$45 + 105 = 150$$

*Finally, I subtracted this total (150 minutes) from 270 minutes to find the amount of time they spent swimming on the last day. I changed this time back into hours and minutes.*

$$270 - 150 = 120 \text{ min}$$

$$120 \text{ min} = 2 \text{ hr}$$

**Lesson 6** *(page 16)*

1. D 2. C 3. B 4. A

5. Solution: An average of 153 tickets were sold each day.

Sample Explanation: *There are 7 days in a week, so I divided the total number of tickets sold (1071) by 7. The quotient is the average number of tickets sold each day.*

$$1071 \div 7 = 153$$

**Lesson 7** *(page 18)*

1. D 2. B 3. D 4. B

5. Solution: Les has 32 combinations of pants, shirts and ties to choose from.

Sample Explanation: *First, I counted all the pants colours, all the shirt colours and all the tie colours.*

*pants: 4*

*shirts: 4*

*ties: 2*

*Then I multiplied the numbers to find the total number of possible combinations.*

$$4 \times 4 \times 2 = 32$$