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Lesson 1 EXPONENTS

LESSON OBJECTIVES

Students will:

- Understand the rules for working with exponents.
- Apply rules of exponents and the Distributive Property to factorise and expand expressions involving exponents.

RELATED AUSTRALIAN CURRICULUM CONTENT DESCRIPTIONS

See page 26 to cross-reference this lesson with aligned Australian Curriculum content descriptions.

PREREQUISITES

Students should be able to:

- Apply the formulas for the area of a rectangle and the volume of a rectangular prism.
- Add, subtract and multiply mentally.

VOCABULARY

PAGE 4

- **exponent:** a number that tells how many times a base is used as a factor
- **power:** an expression formed by a base with its exponent, or the value of such an expression
- **base:** in a power, the factor that is multiplied by itself
- **factor (noun):** a number or expression that is multiplied by another number or expression

PAGE 6

- **Distributive Property:** For any numbers a , b and c , $a(b + c) = ab + ac$ and $a(b - c) = ab - ac$.
- **common factor:** a number or expression that is a factor of two or more other numbers or expressions
- **factorise (verb):** to write an expression as a product of its factors
- **expand:** to perform all possible multiplications in an expression to write it as a sum

MATHS BACKGROUND

Students have used exponents in their work with formulas such as $A = \pi r^2$ (area of a circle) and $V = s^3$ (volume of a cube). They have learned that an exponent represents multiplying with a repeated factor. ($A = \pi r^2 = \pi \times r \times r$ and $V = s^3 = s \times s \times s$)

In this lesson, students learn how to apply rules of exponents and the Distributive Property to work with expressions. The example below shows how the rule for multiplying powers is used in applying the Distributive Property to expand an expression.

$$\begin{aligned}a^3(a^2 + 2) &= a^3 \times a^2 + a^3 \times 2 \\ &= a^{3+2} + 2a^3 \\ &= a^5 + 2a^3\end{aligned}$$

Students will build on the skills and concepts they learn in this lesson when they work with square roots, the Pythagorean theorem and the distance formula in subsequent lessons. In later years, they will use exponent rules when they study scientific notation and exponential functions, with applications such as population growth, compound interest, depreciation and the half-life of a radioactive substance.



Interactive Whiteboard

Visualise expressions with exponents

Go to the *IWB lessons* to bring parts one and two to life. Use features such as sliding screens with additional examples to deepen students' understanding of expressions with exponents.



Download


<http://iwb.camsandstams.com.au>

Modelled Instruction

Lesson 1 EXPONENTS
PART ONE: Learn about expressions with exponents


How can you simplify expressions with exponents?

The area of this square is $b \times b$, or b^2 .



$A = b^2$

The volume of this cube is $a \times a \times a$, or a^3 .



$V = a^3$

The 2 and 3 in the expressions above are called **exponents**.
Exponents can be used as a shortcut for representing repeated multiplication.
What are the rules for working with expressions involving exponents?

An expression like 2^4 is called a **power**.
The **base**, 2, is a **factor** that is multiplied by itself.
The exponent, 4, is the number of times the base is used as a factor.

Use an exponent to rewrite $a \times a \times a$ as a power. a^3

Use exponents to rewrite $(a \times a \times a) \times (a \times a \times a)$. $(a^3) \times (a^3)$ or a^6

Use exponents to rewrite $\frac{a \times a \times a}{a \times a}$. $\frac{a^3}{a^2} = a$ or a^1

Use exponents to rewrite $(a^2)^3$. $(a^2)^3 \times (a^2)^3$ or a^6

The rules for working with exponents can help you simplify expressions with exponents:

- Add the exponents when **multiplying** powers with the same base. $a^2 \times a^3 = a^5$
- Subtract the exponents when **dividing** powers with the same base. $\frac{a^5}{a^2} = a^3$
- Multiply exponents when a power is **raised** to an exponent. $(a^2)^3 = a^6$

What is any number divided by itself? Use the subtraction rule to find $\frac{a^2}{a^2}$.
Then use the result to make a general statement about n^n for any number n .

Guided Instruction

Exponents

Think It Through

Fill in the blanks as you solve the problem.

Simplify each expression.

$b^2 \times b^5$ $\frac{c^6}{c^2}$ $(d^3)^5$

■ The expression $b^2 \times b^5$ shows multiplying two powers with the same base.
What should you do with the exponents? add
Add the exponents. $2 + 5 = 7$

Solution: $b^2 \times b^5 = b^7$

■ The expression $\frac{c^6}{c^2}$ shows dividing two powers with the same base.
What should you do with the exponents? subtract
Subtract the exponents. $6 - 2 = 4$

Solution: $\frac{c^6}{c^2} = c^4$

■ The expression $(d^3)^5$ shows raising a power to an exponent .
What should you do with the exponents? multiply
Multiply the exponents. $2 \times 5 = 10$

Solution: $(d^3)^5 = d^{15}$

Your Turn Now, use what you know to solve this problem.

1. Simplify $\frac{m^2 \times m^4}{m^3}$. Show your work.

$$\frac{m^2 \times m^4}{m^3} = \frac{m^{2+4}}{m^3}$$

$$= \frac{m^6}{m^3}$$

$$= m^{6-3}$$

$$\frac{m^2 \times m^4}{m^3} = m^3$$

You can work with the expanded form of an expression containing exponents to check your answer.

$$\frac{c^6}{c^2} = \frac{c \times c \times c \times c \times c \times c}{c \times c} = c^4$$

AT A GLANCE

Students activate their background knowledge about multiplication and then learn rules for working with exponents, which represent repeated multiplication.

STEP BY STEP

PAGE 4

- Introduce the **Question** at the top of the page.
- Have students read the information in **Explore**. Explain that b^2 is read *b to the second power* or *b squared*, and a^3 is read *a to the third power* or *a cubed*.
- Read **Think** with students. Point out that a is a variable that can represent a number; for example, if $a = 2$, then $a^3 = 2^3 = 2 \times 2 \times 2 = 8$. Pause so students can read aloud the expressions.

EAL/D Support: One everyday meaning of *base* is *bottom*. Help students use this meaning to remember that the base in a power is the *bottom* number or variable.

- Discuss **Connect** with students. For each rule, help them see the connection between the exponents on the left side and the exponent on the right side.

- Organise students in pairs or groups for **Let's Talk** and monitor their discussions.
- Be sure students understand that the result when any nonzero number is divided by itself is 1, so $\frac{a^2}{a^2} = 1$. According to the exponent rules, $\frac{a^2}{a^2} = a^{2-2} = a^0$, so $a^0 = 1$. Therefore, $n^0 = 1$ for any nonzero number n .

PAGE 5

- Read the **Think It Through** problem with students.
- Guide students as they solve the problem by simplifying each expression.

Tip: If students get confused as to which operation to perform on the exponents, refer them to the simple examples presented in **Connect**.

- Monitor students as they complete **Your Turn**. Then discuss the correct answer.

Error Alert: Students who wrote m^2 may have found the correct numerator, m^8 , but then divided exponents instead of subtracting.



ADDITIONAL ACTIVITY

See **Hands-on Activity** (page 38).

Modelled Instruction

PART TWO: Learn more about expressions with exponents

How can you factorise expressions with exponents?

Explore

You can use the **Distributive Property** and the rules for exponents to work with expressions with exponents. The Distributive Property deals with expressions involving multiplication and addition or subtraction.

Distributive Property
 $a(b + c) = ab + ac$
 $a(b - c) = ab - ac$

The area of this rectangle is $n^2 + 5n$. What expression represents the length of the rectangle?

Area = $n^2 + 5n$
 n
 ?

Think

The area of a rectangle is equal to the length times the width.

Think:
 $\underline{\text{length}} \times \text{width} = \underline{\text{area}}$
 $\underline{\text{length}} \times n = n^2 + 5n$

Connect

When each term of an expression has a **common factor**, you can use the Distributive Property to **factorise** the expression.

The terms of the expression on the right are n^2 and $5n$.

$A = n^2 + 5n$

Factorise each term. Use the rules for exponents for n^n : $n^2 = n^{1+1} = n^1 \times n^1$

$A = n \times n + n \times 5$

Use the Distributive Property to factorise the common factor, n .

$A = n(n + 5)$

So, $A = n(n + 5)$, and also $A = \text{length} \times \text{width}$.

If the width of the rectangle is n , then the length must be $(n + 5)$.

The factor n is common to both terms.

Let's Talk

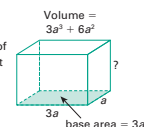
How can you use the rules for exponents and the Distributive Property to **expand** $x(x^2 + x + 1)$?

Guided Instruction

Think It Through

Fill in the blanks as you solve the problem.

The volume of this prism is $3a^3 + 6a^2$, and the area of its base is $3a^2$. What expression represents the height of the prism?



The volume of a prism is equal to the area of its base times its height. Think:

base area \times height = volume
 $3a^2 \times \underline{\hspace{2cm}} = 3a^3 + 6a^2$

The terms of the expression $3a^3 + 6a^2$ are $3a^3$ and $6a^2$.

$3a^3 = 3 \times a \times a \times a$ $6a^2 = 2 \times 3 \times a \times a$

Circle all the matching factors to find the common factor of the terms.

$3a^3 = (3 \times a \times a) \times a$ $6a^2 = 2 \times (3 \times a \times a)$

The common factor is $3 \times a \times a$, or $3a^2$.

Use the Distributive Property to factorise the expression for volume.

$3a^3 + 6a^2 = 3a^2 \times \underline{\hspace{1cm}} + 3a^2 \times \underline{\hspace{1cm}}$
 $= 3a^2 \left(\underline{\hspace{1cm}} + \underline{\hspace{1cm}} \right)$
 base area height

Solution: The expression $\underline{a + 2}$ represents the height of the prism.

Your Turn Now, use what you know to solve this problem.

2. Use the Distributive Property to expand $2b(b^2 + 4)$.

$2b(b^2 + 4) = 2b \times b^2 + 2b \times 4$
 $= 2b^{1+2} + (2 \times 4)b$

$2b(b^2 + 4) = \underline{2b^3 + 8b}$

Factorising and expanding are opposite processes. Both use the Distributive Property.

Expand $n(n + 8)$:
 $n(n + 8) = n^2 + 8n$

Factorise $n^2 + 8n$:
 $n^2 + 8n = n(n + 8)$



AT A GLANCE

Students learn how to factorise and expand expressions involving exponents, using the Distributive Property.

STEP BY STEP

PAGE 6

- Introduce the **Question** at the top of the page.
- Read **Explore** with students. Use numbers to illustrate the Distributive Property; for example:
 $2(3 + 5) = 2 \times 3 + 2 \times 5$ $3(7 - 2) = 3 \times 7 - 3 \times 2$
 $2(8) = 6 + 10$ $3(5) = 21 - 6$
 $16 = 16$ $15 = 15$

EAL/D Support: The word *property* has many meanings. Make sure students understand that in mathematics, a property is an idea about relationships that often involves numbers.

- Read **Think** with students. Pause so they can read aloud the answers.
- Read **Connect** with students. Test their understanding by asking why n is a common factor. (*Each term in the expression can be divided by n .*)

- Organise students in pairs or groups for **Let's Talk** and monitor their discussions.
- Be sure students understand that each term inside the brackets must be multiplied by x , and that $x = x^1$. For example, the first multiplication is $x^1 \times x^2 = x^{1+2} = x^3$. The full expanded result is $x^3 + x^2 + x$.

PAGE 7

- Read the **Think It Through** problem with students.
- Guide students as they solve the problem.

Tip: Point out that the base is a rectangle. Have students verify that the area of the base is the area of the rectangle with dimensions $3a$ and a :

$3a \times a = 3 \times a^1 \times a^1 = 3 \times a^{1+1} = 3a^2$

- Monitor students as they complete **Your Turn**. Then discuss the correct answer.

Error Alert: Students who wrote $2b^3 + 4$ may have neglected to multiply 4 by $2b$.



ADDITIONAL ACTIVITY

See **Reteaching Activity** (page 38).

Modelled Practice

PART THREE: Choose the right answer

Solve the problem. Then read why each answer choice is correct or not correct.

Solve

Which expression is equivalent to $a^2 \times a^4 \times a^7$?

- Ⓐ a^6
- Ⓑ a^7
- Ⓒ a^8
- Ⓓ a^9

Check

Check whether you chose the correct answer.

$a = a^1$, so $a^2 \times a^4 \times a = a^2 \times a^4 \times a^1$.

Each of the powers has the same base.

To multiply powers with the same base, add the exponents.

$$a^2 \times a^4 \times a^1 = a^{2+4+1} = a^7$$

$$2 + 4 + 1 = 7$$

So, the correct answer is Ⓑ.

Why are the other answer choices not correct?

Ⓐ a^6	The exponent of the last factor, a , was neglected. Because $a = a^1$, the exponents are 2, 4 and 1.
Ⓒ a^8	The exponents should be added, not multiplied.
Ⓓ a^9	The first and second exponents were multiplied and then the third exponent was added. All three exponents should be added.

Guided Practice

Your Turn Solve each problem. Use the hints to avoid mistakes.



- If a variable has no written exponent, its exponent is 1. For example, $a = a^1$.
- To multiply powers with the same base, add exponents; do not multiply exponents.
- To divide powers with the same base, subtract exponents; do not divide exponents.

3. Which is a common factor of all three terms in the expression below?

$$4b^3 + 2b^2 + 6b^4$$

- Ⓐ b^3
- Ⓑ b^4
- Ⓒ $2b^2$
- Ⓓ $2b^3$

4. Which operation should you perform on the exponents to simplify the expression?

$$\frac{x^6}{x^2}$$

- Ⓐ addition
- Ⓑ division
- Ⓒ multiplication
- Ⓓ subtraction

5. A rectangle has an area of $m^3 + 2m^2$ and a width of $m + 2$.

$$A = m^3 + 2m^2 \quad m + 2$$

What is the length of the rectangle?

- Ⓐ m^2
- Ⓑ m^3
- Ⓒ $(m^3 + 2m^2) - (m + 2)$
- Ⓓ $(m^3 + 2m^2) + (m + 2)$

6. Which shows a pair of expressions that are equivalent?

- Ⓐ $(p^2)^3$ and p^5
- Ⓑ t^8 and t^2
- Ⓒ $q^4 + q^2$ and q^6
- Ⓓ $y^2 \times y^2 \times y$ and y^5

AT A GLANCE

Students reinforce their understanding of exponents through solving a multiple-choice problem and analysing correct and incorrect answer choices.

STEP BY STEP

PAGE 8

- Tell students that this page models finding the correct answer to a multiple-choice problem.
- Have students read the problem in **Solve** and choose the best answer. Remind them to check their maths.
- Examine **Check** with students. Discuss the correct and incorrect choices.

PAGE 9

- Monitor students as they complete **Your Turn**.
- Organise students in pairs or small groups and have them discuss why each answer choice is correct or not and what errors may have been made.
- Review the answers with the class.



ADDITIONAL ACTIVITY

See **Vocabulary Activity** (page 38).

Answer Analysis

3. Ⓐ Chose variable power of first term.
 Ⓑ Chose greatest variable power.
 ● $4b^3 + 2b^2 + 6b^4 = 2b^2 \times 2b + 2b^2 \times 1 + 2b^2 \times 3b^2$, so $2b^2$ is a common factor.
 Ⓓ Chose variable power of first term and correct numerical factor.
4. Ⓐ Confused with multiplication of exponents.
 Ⓑ Identified operation shown in expression.
 Ⓒ Confused with raising a power to an exponent.
 ● To divide powers with the same base, subtract the exponents.
5. ● The length is m^2 because area = length \times width, and $m^3 + 2m^2 = m^2(m + 2)$.
 Ⓑ Thought m^3 was a common factor of m^3 and $2m^2$.
 Ⓒ Subtracted width from area.
 Ⓓ Added width and area.
6. Ⓐ Added exponents instead of multiplying.
 Ⓑ Divided exponents instead of subtracting.
 Ⓒ These terms are added, not multiplied.
 ● $y^2 \times y^2 \times y = y^2 \times y^2 \times y^1 = y^{2+2+1} = y^5$.

Modelled Practice

PART FOUR: Write the best answer

Study the model. It is a good example of a written answer.

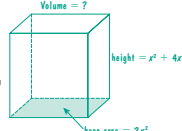
Student model

The base of a rectangular prism has an area of $2x^2$. The height of the prism is $x^2 + 4x$. Write an expression, without brackets, that gives the volume of the prism.

Show each step. Then explain how you found the solution.

Volume = base area \times height

$$\begin{aligned} V &= 2x^2(x^2 + 4x) \\ &= 2x^2(x^2) + 2x^2(4x) \\ &= 2x^2(x^2) + 2x^2(4x) \\ &= 2x^{2+2} + (2 \times 4)x^{2+1} \\ &= 2x^4 + 8x^3 \end{aligned}$$



The student shows each step.

The student correctly answers the question asked.

The student gives important details about how to find the answer.

The student uses the maths words *Distributive Property*, *expand powers*, *base and exponents*.

Solution: The volume of the prism is $2x^4 + 8x^3$ cubic units.

Explanation:

I drew a rectangular prism and labelled it with the given height and base area to help me see the problem. I knew that the volume of a prism is the base area times the height, so I wrote an expression for the volume, using the given expressions for the base area and the height. Then I used the *Distributive Property* to expand the volume expression, using the rules for multiplying powers with the same base (add the exponents).

10

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Guided Practice

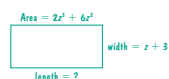
Exponents

Your Turn Solve the problem. Use what you learned from the model.

7. A rectangular playing field has an area of $2z^2 + 6z^2$. The width of the field is $z + 3$. What is the length of the field?

Show each step. Then explain how you found the solution.

Area = length \times width



Factorize the expression for area.

$$\begin{aligned} A &= 2z^2 + 6z^2 \\ &= 2 \times z^{2+1} + 3 \times 2 \times z^2 \\ &= \cancel{2} \times z^2 \times z + 3 \times \cancel{2} \times z^2 \leftarrow 2z^2 \text{ is a common factor} \\ &= 2z^2(z + 3) \end{aligned}$$

length width

Solution: The length of the field is $2z^2$ units.

Explanation:

I drew a rectangle and labelled it to help me see the problem. I wrote a formula to show that the area of a rectangle is length times width. Because I was given the expression for the area, I needed to factorize to find the length. I factorised each term and then used the rules for multiplying powers to write z^2 as $z^2 \times z^1$. Because $2z^2$ is a common factor in each term, I used the *Distributive Property* to factorise the expression as $2z^2(z + 3)$. Because $(z + 3)$ is the width, the common factor $2z^2$ is the length.

- CHECKLIST**
- Did you ...
- show each step?
 - answer the question asked?
 - give important details?
 - use maths words?

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AT A GLANCE

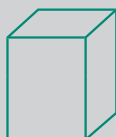
Students study a model answer to an extended-response problem.

STEP BY STEP

PAGE 10

- Tell students that this page models building the solution to a problem one step at a time and writing to explain the solution.
- Have students read the problem in **Show**. Discuss how each mathematical step leads to the solution.

Tip: Emphasise the importance of making a sketch of the geometric figure described in the problem. If students have trouble, a simpler sketch of the prism, not showing hidden edges, is sufficient.



- Read **Explain** with students. Have them circle the maths words in the explanation.
- Direct students' attention to the notes in the right margin. Tell them that this model would receive a high score for the reasons described in these notes.

PAGE 11

- Monitor students as they complete **Your Turn**.
- Encourage students to follow the **Checklist** to write the best answer.
- Have students discuss their work with a partner. Then discuss the correct answer as a class.

Answer and Explanation

7. See the sample answer. This answer shows all of the steps taken to solve the problem, including writing the formula for area and substituting the given expressions for area and width. The solution answers the question. The explanation provides important details about how the problem was solved and uses the maths words *expression*, *factorise*, *common factor* and *Distributive Property*.



ADDITIONAL ACTIVITY

See **Real-World Connection** (page 38).



ADDITIONAL ACTIVITY

See **School-Home Connection** (page 38).

Independent Practice

PART FIVE: Prepare for a test



As you solve problems involving exponents, remember to:

- add exponents to multiply powers with the same base.
- subtract exponents to divide powers with the same base.
- multiply exponents to raise a power to an exponent.
- use the Distributive Property to factorise or expand an expression.

Solve each problem.

8. Which of the following expressions is equivalent to the expression below?

$$x(x^2 + 4)$$

- Ⓐ $x^2 + 4x$
- Ⓑ $x^3 + 4x$
- Ⓒ $x^3 + 4$
- Ⓓ $x^2 + x + 4$

9. A prism has a volume of $b^3 + 2b^2 + b$. The height of the prism is b . What is the area of the base?

- Ⓐ $b^2 + 2b + 1$
- Ⓑ $b^2 + 2b + b$
- Ⓒ $b^2 + 2b^2 + b$
- Ⓓ $b(b^2 + 2b + 1)$

10. Which expression can be simplified by multiplying the exponents?

- Ⓐ $\frac{t^6}{t^3}$
- Ⓑ $(c^3)^2$
- Ⓒ $q^2 \times q^2$
- Ⓓ $r^3 + r^5$

11. Which expression has terms with a common factor of a^2 ?

- Ⓐ $2a + 2a^2$
- Ⓑ $4a^3 + 5a$
- Ⓒ $5a^3 + 3a^2$
- Ⓓ $6a^3 + 2a^2 + 3$

Independent Practice

12. A rectangle has length $5z$ and width $z^2 + 3z$. What is the area of the rectangle?

- Ⓐ $z^2 + 8z$
- Ⓑ $z^2 + 3z^2$
- Ⓒ $5z^2 + 3z$
- Ⓓ $5z^3 + 15z^2$

13. Which expression is equivalent to $y^8 \times y^4$?

- Ⓐ y^2
- Ⓑ y^4
- Ⓒ y^{12}
- Ⓓ y^{22}

14. Look at the expression below.

$$\frac{p^{12}}{p^7}$$

What operation should you perform on the exponents to divide the powers?

subtraction

Divide the powers.

$$\frac{p^{12}}{p^7} = p^{12-7} = p^5$$

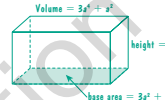
15. A rectangular prism has a volume of $3a^4 + a^2$. The area of the base of the prism is $3a^2 + 1$. What is the height of the prism?

Show each step. Then explain how you found the solution.

Volume = base area \times height

Factorise the expression for volume.

$$\begin{aligned} V &= 3a^4 + a^2 \\ &= 3 \times a^2 \times a^2 + a^2 \\ &= 3a^2 \times (a^2 + 1) \times a^2 \leftarrow a^2 \text{ is a common factor} \\ &= a^2(3a^2 + 1) \end{aligned}$$



Solution: The height of the prism is a^2 units.

Explanation:

I drew a rectangular prism and labelled it to help me see the problem. I wrote a formula to show that the volume of the prism is the area of the base times the height. Because I was given the expression for the volume, I needed to factorise to find the height. I factorised each term and then used the rules for multiplying powers to write a^4 as $a^2 \times a^2$. Because a^2 is a common factor in each term, I used the Distributive Property to factorise the expression as $a^2(3a^2 + 1)$. Because $(3a^2 + 1)$ is the area of the base, the common factor a^2 is the height.

AT A GLANCE

Students practise using exponents to solve problems that might appear on a mathematics test.

STEP BY STEP

PAGES 12–13

- Tell students that they will practise solving problems that involve expressions with exponents.
- Point out the tips at the top of page 12. Explain to students that these tips will help them answer the problems correctly.
- You may wish to have students review the hints for avoiding mistakes on page 9 as well.
- Tell students to complete problems 8–15 on pages 12 and 13. Encourage them to check their answers.
- Discuss the correct responses as a class.

Answers and Explanations

- Ⓑ Use the Distributive Property to expand the expression: $x(x^2 + 4) = x \times x^2 + x \times 4 = x^3 + 4x$.
- Ⓐ Factorise: $b^3 + 2b^2 + b = b(b^2 + 2b + 1)$; because volume of a prism = height \times base area, and the height is b , the base area must be $b^2 + 2b + 1$.
- Ⓑ To raise a power to an exponent, multiply the exponents: $(c^4)^3 = c^{12}$.
- Ⓒ $5a^3 + 3a^2 = a^2 \times 5a + a^2 \times 3$, so a^2 is a common factor of each term in the expression.
- Ⓓ The area of a rectangle is length \times width, so multiply $5z$ by $z^2 + 3z$. Use the Distributive Property to expand the expression: $5z(z^2 + 3z) = 5z \times z^2 + 5z \times 3z = 5z^3 + 15z^2$. So, the area of the rectangle is $5z^3 + 15z^2$.
- Ⓒ To multiply two powers with the same base, add the exponents: $y^8 \times y^4 = y^{8+4} = y^{12}$.

(continued on page 37)

(continued from page 36)

14. The expression $\frac{n^{12}}{n^3}$ shows dividing two powers with the same base. To divide powers, subtract the exponents: $\frac{n^{12}}{n^3} = n^{12-3} = n^9$.

15. See the sample answer. This answer shows all of the steps the student took to solve the problem. The solution answers the question. The explanation provides important details about how the student solved the problem and uses the maths words *expression*, *factorise*, *common factor* and *Distributive Property*.



ASSESSMENT AND REMEDIATION

- Ask students to expand the expression $p^2(2p^3 + p)$. ($2p^5 + p^3$)
- For students who are still struggling, use the chart below to guide remediation.
- After providing remediation, check students' understanding. Ask students to explain their thinking while expanding the expression $a^3(4a + 3a^2)$. ($4a^4 + 3a^5$)

If the error is . . .	Students may . . .	To remediate . . .
$2p^6 + p^2$	have multiplied the exponents.	Have students first use the Distributive Property, then write the powers as repeated multiplication, and then count the number of p factors to find the correct exponents. $p^2(2p^3 + p) = (p^2 \times 2p^3) + (p^2 \times p)$ $= (p \times p \times 2 \times p \times p \times p) + (p \times p \times p)$ $= 2p^5 + p^3$
$2p^5 + p$	have neglected to multiply p by p^2 .	Remind students that there are two terms inside the brackets and that <i>each</i> must be multiplied by p^2 . Have them write the following. $p^2(\underline{\quad} + \underline{\quad}) = (p^2 \times \underline{\quad}) + (p^2 \times \underline{\quad})$ Tell them to fill in the actual terms inside the brackets, and then multiply those terms by p . $p^2(\underline{2p^3} + \underline{p}) = (p^2 \cdot \underline{2p^3}) + (p^2 \cdot \underline{p})$ $= 2p^5 + p^3$
$2p^3 + p^2 + p$	have added the terms instead of multiplying.	Explain that brackets immediately following a number or expression indicate multiplication. Write the following expressions on the board and have students read them aloud, emphasising the word <i>times</i> for multiplication. $3(8) \quad (\text{three } \mathbf{times} \text{ eight})$ $x^2(x + 4) \quad (x \text{ squared } \mathbf{times} \text{ the quantity } x \text{ plus four})$ $p^2(2p^3 + p) \quad (p \text{ squared } \mathbf{times} \text{ the quantity two } p \text{ cubed plus } p)$



ADDITIONAL ACTIVITY

For students who have mastered the skills in this lesson, see **Challenge Activity** (page 38).

ADDITIONAL ACTIVITIES



Hands-on Activity

Fold paper to demonstrate powers of 2 and 3.

Materials: 2 pieces of paper per student

Have students fold a piece of paper in half and then open it. Ask how many sections are formed. (2) Ask them to express that number as a power. (2¹) Have them refold and then fold one more time. Ask the same questions. (4, 2²) Then add one more new fold. (8, 2³)

Now use another sheet of paper. Modify the activity, using two folds each time instead of one fold. (3, 3¹; 9, 3²; 27, 3³)



Reteaching Activity

Use numbers to show exponent rules and the Distributive Property.

Have students evaluate $2^2(2^3 + 3)$ in the ways shown below to illustrate the properties explored in the lesson.

$$\begin{aligned} 2^2(2^3 + 3) &= (2 \times 2)(2 \times 2 \times 2 + 3) && \text{Definition} \\ &= 4(8 + 3) && \text{of exponent} \\ &= 4(11) \\ &= 44 \end{aligned}$$

$$\begin{aligned} 2^2(2^3 + 3) &= 2^2 \times 2^3 + 2^2 \times 3 && \text{Distributive} \\ &= 2^{2+3} + 2^2 \times 3 && \text{Property} \\ &= 2^5 + 2^2 \times 3 && \text{Multiply} \\ &= 32 + 4 \times 3 && \text{powers} \\ &= 44 \end{aligned}$$

Use a similar process to illustrate the rules for dividing powers with the same base or raising a power to an exponent.



Vocabulary Activity

Play "Concentration" to reinforce terms.

Materials: index cards

In pairs, have students write one vocabulary term on each index card. Then have students write a definition or example of each term on another index card. Have students shuffle the cards and place them face down in an array. Students take turns flipping over two cards. If the player chooses a matching term and

definition, he or she keeps the pair. If the cards do not match, he or she replaces them. The student with the most matched pairs wins.



Real-World Connection

Factorise expressions for surface area of cans.

Materials: a can or other cylindrical container

Hold up a can and say, "Many containers, like this can, have a cylinder shape." Name or elicit examples, such as coffee cans, water tanks and oil tanks.

On the board, write the expression for the surface area of a cylinder, $2\pi r^2 + 2\pi rh$, where r is the radius, h is the height and π is approximately 3.14. Tell students that this expression can be used to find the amount of material it takes to make a cylindrical container.

Have students find a common factor and use the Distributive Property to factorise the expression. ($2\pi r(r + h)$)



School-Home Connection

Inform families about exponents.

Give each student a copy of the School-Home Connection activity sheet for Lesson 1 (page 159) to share with the family. The activity included in the letter has the family play a game involving expressions with exponents.



Challenge Activity

Find dimensions of a square and a cube.

On the board, draw a square and label its area as a^6 . Ask students to write an expression for the length of a side of the square and explain their answer.

$$(a^3, \text{ because } a^3 \times a^3 = a^6)$$

Then draw a cube and label its volume a^6 . Ask students to write an expression for the length of an edge of the cube and explain their answer.

$$(a^2, \text{ because } a^2 \times a^2 \times a^2 = a^6)$$