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PRACTICE 1
Parts one and two

EXPONENTS

OBJECTIVES

In part one, students will:

- Evaluate exponential expressions.
- Solve problems by using rules of exponents.
- Show and explain solutions to problems with exponents.

In part two, students will:

- Factorise expressions by using the Distributive Property.
- Find common factors in expressions.
- Solve problems by factorising.
- Find equivalent expressions.
- Compute with exponential expressions to solve word problems.

VOCABULARY

Part one

- **exponent:** a number that tells how many times a base is used as a factor
- **power:** an expression formed by a base with its exponent, or the value of such an expression
- **base:** in a power, the factor that is multiplied by itself
- **expand:** to perform all possible multiplications in an expression to write it as a sum
- **factor (noun):** a number or expression that is multiplied by another number or expression

Part two

- **Distributive Property:** For any numbers a, b and c,
  $a(b + c) = ab + ac$ and
  $a(b - c) = ab - ac$.
- **common factor:** a number or expression that is a factor of two or more other numbers or expressions
- **factorise (verb):** to write an expression as a product of its factors

AUSTRALIAN CURRICULUM CONTENT DESCRIPTION

See page 13 to cross-reference this lesson with aligned Australian Curriculum content descriptions.

RELATED *STAMS*® *PLUS* INSTRUCTION

For instruction that supports this practice, go to:

***STAMS*® *Plus*, Book H, Lesson 1, Exponents, pp. 4–13**

***STAMS*® IWB lessons, Level H, Visualise expressions with exponents**

Use features such as sliding screens and additional examples to deepen students' understanding of expressions with exponents.

Download
http://iwb.camsandstams.com.au

Part one

EXPONENTS

PRACTICE 1

Part one

Use the rules of exponents to solve the problem.

1. Why do you subtract exponents to divide c^9 by c^5?

Expand the expression $\frac{c^9}{c^5}$ to see why exponents are subtracted in division.

The base, __c__, is the factor that is multiplied by itself. The __exponent__ is the number of times the base is used as a factor.

Rules for working with exponents:
- *Add* the **exponents** when *multiplying* **powers** with the same **base**: $a^3 \times a^4 = a^{3+4} = a^7$
- *Subtract* the exponents when *dividing* powers with the same base: $\frac{a^5}{a^3} = a^2$
- *Multiply* exponents when a power is *raised* to an exponent: $(a^2)^3 = a^6$.

$\frac{c^9}{c^5} = \frac{c \times c \times c \times c \times c \times c \times c \times c \times c}{c \times c \times c \times c \times c}$ Expand the numerator and denominator.

$= \frac{c \times c \times c \times c \times c}{c \times c \times c \times c \times c} \times c \times c \times c \times c$ Make pairs of like terms.

$= 1 \times 1 \times 1 \times 1 \times 1 \times c \times c \times c \times c$ Simplify each pair.

$= c \times c \times c \times c$ Multiply.

$= c^4$ Use an exponent to write the multiplication.

Solution: Because you are removing pairs of like terms that simplify to __1__, you subtract exponents when dividing powers that have the same __base__.

Solve each problem. Fill in the blanks.

2. Find the value of $\frac{3^9}{3^7}$.

$\frac{3^9}{3^7} = 3^{9-7} = 3^2 =$ __9__

Solution: __9__

3. Simplify the expression $a^3 \times a^2$.

$a^3 \times a^2 = a^{3+2} = a^5$

Solution: __a^5__

4. Use what you know to find the value of $4 \times 10^3 \times 3 \times 10^2$.

$4 \times 10^3 \times 3 \times 10^2 = 4 \times 3 \times 10^3 \times 10^2$

$=$ __12__ $\times 10^{3+2}$

$=$ __12__ $\times$ __100 000__

$=$ __1 200 000__

Solution: __1 200 000__

5. Simplify the expression $(b^3)^4$.

Solution: __b^{12}__

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Exponents

Solve each problem. Choose the best answer.

6. Which shows a pair of expressions that are equivalent?

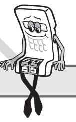
Ⓐ $(a^4)^5$ and a^9
Ⓑ $b^3 \times b^4 - b^3$ and b^4
● $(c^4 \times c^2)^2$ and c^{12}
Ⓓ $d^3 \times d \times d$ and $d^3 + 2d$

7. Which shows a pair of expressions that are equivalent?

● $\frac{(a^3)^2}{a}$ and a^5
Ⓑ $b^3 \times b^5 \div b^3$ and b^{12}
Ⓒ $\frac{c^4}{c^6} \times c^2$ and c^6
Ⓓ $\frac{d^6}{d^3} \times d^2$ and d

8. Simplify $(2a)^2$.

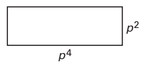
Ⓐ $2a^2$
Ⓑ $2a^3$
Ⓒ $4a$
● $4a^2$



REASONING

Solve each problem.

9. What is the area of the rectangle? Show your work.



$p^4 \times p^2 = p^{4+2} = p^6$

Solution: The area of the rectangle is __p^6__ units.

10. Can you simplify $\frac{a^6}{b^2}$? If so, simplify. If not, explain why not.

Even though the numerator and denominator each includes an exponent, the bases are not the same. An expression that does not have common bases cannot be simplified.

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At a Glance

Students apply rules for exponents to solve problems. If students have difficulty, check for these common pitfalls and use the related tips to provide help.

Solve Problems 2–5

If If students' solution to problem 4 is 12 000 000, they may have multiplied the exponents because they were multiplying everything else.

Then Ask students to multiply 100 by 1000 and then write the same problem with exponents: $10^2 \times 10^3$. Brainstorm how that helps them to solve problem 4.

Solve Problems 6–8

If If students do not find equivalent expressions for problem 6, they may not understand the rules for exponents.

Then Together, simplify each expression in the answer choices. Have students give you the rule used for each expression. Then ask them to explain how the answer choices are alike and different.

Reasoning, Problems 9–10

If If students' expression in problem 9 is a sum, they may have found the perimeter instead of the area because the *p* confused them.

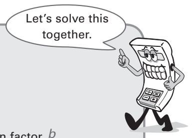
Then Redraw the diagram and assign a new variable to the expression for side length.

Part two

PRACTICE 1

Part two

EXPONENTS



Use what you know about factors to solve the problem.

1. To factorise the expression, first factorise each term.

   $b^2 + 6b$ $\quad b^2 = b \times b \quad 6b = 6 \times b$

   Each term has the factor b.

   Use the Distributive Property to factorise out the common factor, b.

   Solution: $b^2 + 6b =$ $b(b + 6)$

Solve the problem. Fill in the blanks.

2. Factorise the expression $3m^4 + 6m^2$.

   Factorise each term. $\quad 3m^4 = 3 \times m \times m \times m \times m$

   $6m^2 = 2 \times 3 \times m \times m$

   Find the highest common factor by multiplying the factors found in both terms: $\quad 3 \times m \times m = 3m^2$

   Use the Distributive Property to factorise: $\quad 3m^4 + 6m^2 = 3m^2 \times m^2 + 3m^2 \times 2$

   $= 3m^2\,(m^2 + 2)$

   Solution: $3m^4 + 6m^2 =$ $3m^2(m^2 + 2)$

Solve each problem. Choose the best answer.

3. Which is a common factor in all three terms of the expression $g^4 + 3g^2 + 6g$?

   Ⓐ g

   ● $3g$

   Ⓒ g^2

   Ⓓ $3g^2$

4. Which is a common factor in all three terms of the expression $2a^3 + 8a^2 + 6a$?

   ● $2a$

   Ⓑ $4a$

   Ⓒ $2a^3$

   Ⓓ $3a^2$

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Exponents

Solve each problem. Show your work.

5. Factorise $3p^3 + 12p^2$.

   $$3p^3 + 12p^2 = (3p^2 \times p) + (3p^2 \times 4) = 3p^2(p + 4)$$

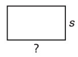
   Solution: $= 3p^3 + 12p^2 =$ $3p^2(p + 4)$

6. Factorise $10b^5 + 15b^3 + 5b$.

   $$10b^5 + 15b^3 + 5b = (5b \times 2b^4) + (5b \times 3b^2) + (5b \times 1) = 5b\,(2b^4 + 3b^2 + 1)$$

   Solution: $= 10b^5 + 15b^3 + 5b =$ $5b(2b^4 + 3b^2 + 1)$

7. The area of the rectangle is $A = s^2 + 7s$. What is the width of the rectangle?

   $$s^2 + 7s = s(s + 7)$$

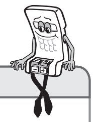
   Solution: The width of the rectangle is $s + 7$ units.

Solve the problem. Choose the best answer.

8. Which expression is equivalent to $2(w + 1)$?

   Ⓐ $2w + 1$ $\quad$ Ⓒ $w + 2$

   ● $2w + 2$ $\quad$ Ⓓ $(w + 1)(w + 1)$



REASONING

Use this information for numbers 9 and 10. Solve each problem.

The surface of Earth is 5×10^8 square kilometres. The area of Australia is 7.6×10^6 square kilometres.

9. What is the highest common factor between the surface of Earth and the area of Australia? 10^6

10. About how many times larger is the surface of Earth compared to the area of Australia?

   about 65.5 times larger; $\frac{5 \times 10^8}{7.6 \times 10^6} \approx \frac{5}{8} \times 10^{8-6} = 0.625 \times 10^2 = 62.5$

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At a Glance

Students use common factors and the Distributive Property to factorise expressions. If students have difficulty, check for these common pitfalls and use the related tips to provide help.

Solve Problems 2–4

If If students cannot find the common factor in problem 4, they may need to expand the terms of the expression.

Then Have students expand the expression: $(2 \times a \times a \times a) + (2 \times 2 \times 2 \times a \times a) + (2 \times 3 \times a)$. Ask them to circle all common factors and then rethink their answer choice.

Solve Problems 5–7

If If students' solution to problem 5 is incorrect, they may have forgotten to use the Distributive Property.

Then Ask students to describe their solution. Help them to pick out the common factor and the leftover bits that will make up the second factor in their solution.

Solve Problem 8

If If students choose A, they have forgotten to multiply both terms of the second factor by the first factor.

Then Have students start a new habit of drawing an arrow from the first factor to each term of the second factor. This should help them to remember to multiply both parts of the second factor.

Reasoning, Problems 9–10

If If students' answer to problem 10 is an expression, they may have subtracted to compare.

Then Have students read the problem aloud and brainstorm what they are being asked to do before rethinking their solution. Make sure they notice the word *times*.

REVIEWS
1–2

REVIEWS 1–2: PRACTICES 1–4

OBJECTIVES

In review 1, students will:

- Find equivalent expressions by using factorisation and rules of exponents.
- Compare and contrast exponential expressions.
- Estimate square roots and factorise exponential expressions.
- Explain solutions to word problems.

In review 2, students will:

- Write two-step equations to represent word problems.
- Solve two-step equations by using inverse operations.

VOCABULARY

Review 1

- **exponent:** a number that tells how many times a base is used as a factor
- **Distributive Property:** For any numbers a, b and c, $a(b + c) = ab + ac$ and $a(b - c) = ab - ac$.
- **common factor:** a number or expression that is a factor of two or more other numbers or expressions
- **factorise (verb):** to write an expression as a product of its factors

Review 2

- **equation:** a number sentence that contains an equals sign and shows two quantities with the same value

AUSTRALIAN CURRICULUM CONTENT DESCRIPTION

See page 13 to cross-reference this lesson with aligned Australian Curriculum content descriptions.

Review 1

REVIEW 1

REVIEW 1: PRACTICES 1 AND 2

Solve each problem. Choose the best answer.

1. Which shows an expression that is equivalent to $\sqrt{9} \times \sqrt{64}$?
   - Ⓐ $\sqrt{16} \times \sqrt{81}$
   - Ⓑ 6^2
   - Ⓒ $\sqrt{36}$
   - ● 3×2^3

2. Which is a common factor to all three terms in the expression $4h^3 + 6h^2 + 2h$?
   - Ⓐ 2
   - ● $2h$
   - Ⓒ $2h^2$
   - Ⓓ $3h$

3. Which shows a pair of expressions that are equivalent?
   - ● $\frac{m^7}{m^6}$ and m
   - Ⓑ $\frac{m^{10} \times m^6}{m^{15}}$ and m^4
   - Ⓒ $m^3 \times m^7$ and m^{21}
   - Ⓓ $(m^4)^2$ and $m^2 \times m^4$

Solve each problem. Explain your answer.

4. How are the expressions $(x^3)^3$ and $x^3 \times x^3$ the same? How are they different?

   They both are products of $x^3 \times (x^3)^3$, or x^9, has three factors of x^3, while $x^3 \times x^3$, or x^6, has two factors of $x^3 \times x^3$.

5. Without simplifying, can you tell if the given expressions are equivalent?
   $(a \times a \times a) \times (a \times a \times a)$ and $(a \times a \times a) + (a \times a \times a)$

   The expressions are not equivalent. To multiply expressions that have the same base, add the exponents. To add expressions that have the same base and exponent, multiply one term by the number of terms.

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Solve each problem. Write the solution.

6. Jason said the value of $\sqrt{60}$ is about 8. Carla said the value of $\sqrt{60}$ is closer to 7.5. Are both answers reasonable? Explain your answer.

   The closest perfect squares to 60 are 49 and 64. Since 60 is close to 64, an estimate of 8 is reasonable. Check 7.5 by multiplying: $7.5 \times 7.5 = 56.25$. 64 and 56.25 are almost the same distance from 60. Both values are reasonable.

7. Factorise $2a^4 + 4a^3 + 8a^2 + 6a^3$.

   $2a^4 + 4a^3 + 8a^2 + 6a^3 = 2a^4 + 10a^3 + 8a^2$
   $= 2a^2(a^2) + 2a^2(5a) + 2a^2(4)$

   Solution: $2a^4 + 4a^3 + 8a^2 + 6a^3 = 2a^2(a^2 + 5a + 4)$.

   Before factorising, see if you can combine like terms.

8. Factorise $\frac{c^4 + 2c^3}{c^2}$. First factorise the numerator using the Distributive Property. Then simplify the expression using the rules of exponents.

   $$\frac{c^4 + 2c^3}{c^2} = \frac{c^2(c^2 + 2c)}{c^2}$$

   Solution: $\frac{c^4 + 2c^3}{c^2} = c^2 + 2c$ or $c(c + 2)$

REASONING

Use this information for numbers 9 and 10. Solve each problem. Explain your thinking.

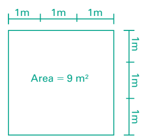
Olivia has an outdoor platform that has an area of 6 square metres.

9. She wants a tarpaulin that will cover the platform and hang down about 50 centimetres on all sides. Estimate the length of each side of the tarpaulin. Show your work.

   Each side of the platform has a length of about $\sqrt{6} \approx 2.5$ metres. Adding 0.5 of a metre to each side of the platform adds 1 metre to each dimension, so the tarpaulin would measure 2.5 + 1, or about 3.5 metres on each side.

10. She has a square tarpaulin that has an area of 9 square metres. Will it cover the table? Will it hang down 50 centimetres on each side? Draw a sketch to help you see how many square metres are on each side.

    My sketch shows me that the tarpaulin is equal to 9 square metres. Each side of the tablecloth is 3 metres. The tarpaulin will cover the platform but will only hang down 25 centimetres on each side.



At a Glance

Students solve problems involving factors, exponents and square roots. If students have difficulty, check for these common pitfalls and use the related tips to provide help.

Solve Problems 1–3

If If students choose C for problem 3, they may have multiplied exponents instead of adding them.

Then Ask students to first fully expand the expression in each choice and then explain to you how they miscalculated.

Solve Problems 4–5

If If students see no difference between the products in problem 4, they may not have distinguished a power from a product.

Then Ask students to expand the first expression. $((x^3)^3 = x^3 \times x^3 \times x^3)$ Then have them compare the two expressions.

Solve Problems 6–8

If If students solve problem 6 incorrectly, they may need help with perfect squares close to 60.

Then Have students make a list of the perfect squares through 12×12. Then have them compare the squares to 60.

Reasoning, Problems 9–10

If If students give the length of each side of the tarpaulin as about 30 metres, they may have computed the side of the table without adding the tarpaulin's overhang.

Then Have the students explain their reasoning.