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Introduction

Problem-solving is the process of applying acquired knowledge to different situations. It is the basic skill of mathematics and an integral part of the mathematics curriculum at all levels of instruction.

Figure it out is a series of booklets designed to teach strategies for solving mathematical problems. As students work through a booklet, they learn to read problems carefully, to think about the content of problems, and to use what they know about numbers and mathematics to decide how to find solutions.

The problems included in each booklet are open-ended, non-routine problems. Their scope extends beyond that of routine problems, or those which students can solve by merely reading and identifying the necessary mathematical operation. Each problem in ***Figure it out*** has some unique quality that requires students to think carefully about how to solve it. Many are problems that students can relate to real life.

The most exciting aspect of teaching mathematics is the discoveries students make as they work through problems. Guide them with questions, encourage the use of manipulatives and be sure to give students time and space to discover.

The Student Book

The student book consists of lessons that teach eight different strategies that can be used to solve non-routine problems. Each lesson opens with a problem followed by **Questions** designed to help students think about the problem and how to solve it. After the **Questions**, students are given guidance on how to **Apply the Strategy** to solve the problem. **Problem 1**, the **Questions**, and the **Apply the Strategy** section are intended to be teacher- directed. The second problem in each lesson is followed by **Think about** questions. This problem and its questions may be teacher-directed or may be completed by students on their own. Finally, each strategy lesson ends with two practice problems for students to complete independently. For more applications of the strategies, two pages of **Mixed Practice** are presented after every four strategy lessons. Students can use any strategy they find helpful to solve the problems on these pages. The last section of the booklet contains **Reviews** for each of the eight strategies. The booklet ends with a **Final Review** containing non-routine, open-ended problems that can be solved using the strategies presented in the lessons.

Using the Student Book

Students should write on the answer lines provided. They should also be encouraged to write in any blank spaces in their booklets so they can keep their computation and other work close to the problems they are solving.

The Teacher Guide

The teacher guide consists of procedures for teaching the strategy lessons and guidance for presenting the **Mixed Practice** and **Review** pages. The teacher guide also contains three blackline masters that can be duplicated and distributed to students for use in solving the problems.

Using the Teacher Guide

Suggestions for instruction are provided throughout the teacher guide. These include questions to ask students, teaching tips, and diagrams and tables for student use. The teacher guide also provides answers to questions and solutions to problems posed in the student book.
It is recommended that the teacher read through the teaching notes for each lesson before presenting the lesson to students.

Teaching Strategy Lessons

Though the first problem in each strategy lesson is intended to be primarily teacher-directed, and the second is meant to be more self-guided, the teacher can approach the two problems in a similar manner. The teacher should read the problems aloud or have a student volunteer read the problems. He or she should also read each of the **Questions** and **Think about** questions aloud and lead the class in a discussion about the students' answers. During questioning, the teacher should encourage students to explain how they arrived at their answers. Explanations should be requested for correct and incorrect student responses. From students' answers, the teacher will see the wide variety of ways in which students approach the same problem. The teacher may also gain awareness of students' understanding or lack of understanding of mathematics concepts. After the questions have been answered, the teacher should help students use the strategy to solve the problem. The teaching notes provide guidance in this area. The final problems in each strategy lesson can either be teacher-directed or completed by students independently. The teacher guide provides information on how to help students think through these problems.

Following many of the problems in the teacher guide are **Challenge** problems that the teacher can present to the students. Some of these problems reinforce reasoning skills or strategy use at the level they were presented in the problems they follow. Other problems involve more advanced applications of the strategy that was taught.

Problem 1

Questions

Read each question aloud. Then work through the question with students. Encourage students to use the picture below the problem to help them answer the questions. Students can also use a piece of clay that can be formed into a log. If clay is not available, a straw can be used to represent a log.

a. (minutes)
   After students answer, ask where they found their answer in the problem. (The problem asks *how long* it will take.)

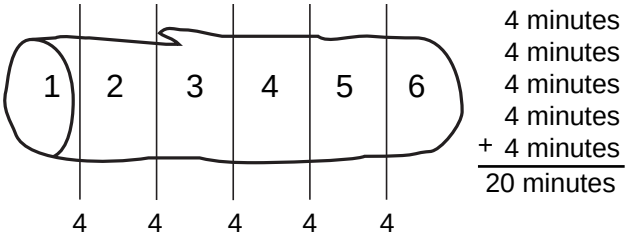
b. (1 cut)
   Ask students how they can use the drawing to answer the question. (Answer may vary. Sample answers: Draw lines to stand for each cut; colour the log in two different colours to show that it has been cut once.)

c. (4 minutes)
   Have a volunteer read from the problem the sentence that answers the question. (In 4 minutes, Jenna can saw a log into 2 pieces.)

d. (5 cuts)
   Ask students how this question can help them solve the problem. (Answers may vary. Students should note that if they know how many cuts are needed to saw 6 pieces, they can then find how much time it takes to saw that many cuts.)

Apply the Strategy

Students should show 5 cuts and 6 pieces on their drawings. For example:



4 minutes
4 minutes
4 minutes
4 minutes
+ 4 minutes
20 minutes

If they use clay or a straw, students should make the actual cuts or use a texta to draw the cuts. Students should note that each cut takes 4 minutes to make, so they can multiply 5 × 4 or add 4 + 4 + 4 + 4 + 4 to get the answer.

Alternatives: After they work through the questions, students can compare the number of cuts needed to make a number of pieces. The following table can help students compare.

Number of cuts	Number of pieces
1	2
2	3
3	4

Have students copy the table from the blackboard or distribute a copy for students to write on. Then have students enter in the table increasing numbers of cuts and the number of pieces made from the cuts. Students should find after several entries that the number of cuts is always one less than the number of pieces.

Solution: 20 minutes

e. (Answers will vary.)

Problem 2

For solving problems of this type, counters such as beans or buttons may be used in conjunction with a picture. To help them think about the questions and work through the problem, give each student 34 counters. Students can then arrange the counters around the tables illustrated on the page.

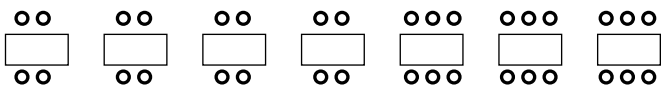
Think about:

Read the questions aloud and discuss them if necessary.

- (4 students)
  Have a volunteer read from the problem the sentence that answers the question. (There are small tables for 4 students and large tables for 6 students.)
- (28 students)
  To guide students, ask how they could use the picture of the seven tables to answer this question. (Draw 4 students around each table. Then find the number of students drawn by counting, by adding seven 4's, or by multiplying 7 x 4.)
- (42 students)
  Discuss whether students can use the same strategy to answer this question as they used for the preceding question.
- (Answers may vary. Sample answer: Draw 4 students at each table, then add 2 to one table at a time until there are 34 students in all.)

Have students work through the problem. Encourage them to use their drawings in any way that will help them solve the problem. Students may find it useful to guess the number of students at each table and then to draw them in to test their guesses. Accept any use of drawings that will lead to the correct answer.

Sample drawing:



Alternatives: Students can also use the following table to solve this problem.

Tables with 4 students	Tables with 6 students	Total number of people
7	0	28
6	1	(30)
5	2	(32)
(4)	(3)	(34)

Have students copy the table from the blackboard or distribute a copy for students to write on. (Note that answers in parentheses are for teacher use.) Explain to students that they can fill in the table to solve the problem. To help them get started, point out that the number of tables in each row always equals 7. Then explain that they can draw a picture, use counters, or multiply to find the total number of people who can sit at the tables in each row. Students can continue the table until the total number of people is 34. Once they have completed the table, have students say the number patterns they see in the columns. (They should note that the first column decreases by 1 as the second column increases by 1. They should also note that the total number of people increases by 2.)

Solution: 4 small tables, 3 large tables

Challenge: If Problem 2 did not tell you how many tables to use, what would be the solution to the problem? Would there be more than one solution? (There are 2 solutions: 1 small table and 5 large tables; 7 small tables and 1 large table.)

To help them solve this problem, have students work out additional ways to seat 34 people at tables of 4 and 6 people. Students can use any of the strategies used to solve Problem 2. For students who enjoy or need extra practice, have them show their results in a table such as the one below. Students should copy the table from the blackboard or write on copies of the table. (Note that answers in parentheses are for teacher use.)

Small tables	Large tables	Total tables	Total people
(1)	(5)	(6)	(34)
(4)	(3)	(7)	(34)
(7)	(1)	(8)	(34)

Once students have completed the table and found the solutions, have students look for patterns. Discuss any patterns students see.

You may also wish to modify Problem 2 so that the total number of tables is always 7 but the total number of people changes. Have students fill in a table like the one below to show all the different numbers of people that can be seated. Students should copy the table from the blackboard or write on copies of the table. (Note that the answers in parentheses are for teacher use.)

Small tables	Large tables	Total people
(0)	(7)	(42)
(1)	(6)	(40)
(2)	(5)	(38)
(3)	(4)	(36)
(4)	(3)	(34)
(5)	(2)	(32)
(6)	(1)	(30)
(7)	(0)	(28)

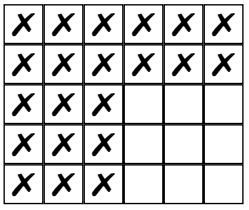
Have students look for patterns. Discuss their findings.

On your Own

Problem 3

You may need to review the meaning of *half* before students work through the problem.

Sample use of drawing:

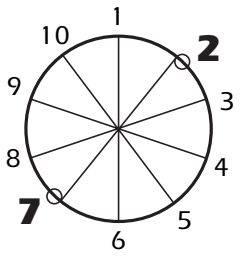


Alternatives: Students can use counters with Blackline Master 1 (page 26) to work through the problem. Have students hold the page so there are 5 rows and 6 columns to match the picture. Have students place one counter in each box to show which stamps have been pasted in the album. They can then count the boxes without counters. If counters of two colours are available, one colour can be used to represent stamps pasted in the book and the other used for stamps needed.

Solution: 9 stamps. Accept any reasonable explanation.

Problem 4

Sample uses of drawing:



Tell students that the table shows amounts made after placing consecutive amounts in a Magic Money box for 2 days. Have them complete the left column of their tables to $10 and then the right columns. Students should then look for patterns. Help them discover that the amounts in the box after 2 days are all odd numbers, that the amounts increase by 4, and that the amount in the box is 3 more than 4 times the original amount.

After they work through the table, give students the following problems.

- If there is $99 in the box after 2 days, how much money was first placed in the box? ($24)
- Suppose the box is used for 3 days. At the end of the third day, there is $31 in the box. How much money was placed in the box on the first day? ($3)

Problem 2

Before students read the problem, review telling time to the hour, quarter hour and half hour.

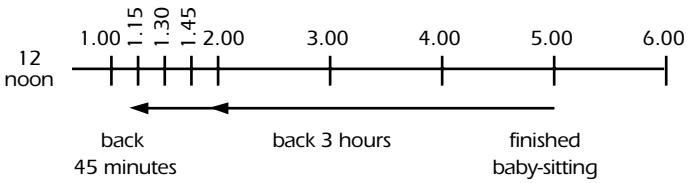
Think about:

Read the questions aloud and discuss them if necessary.

- (less than 1 hour)
- (2.00 p.m.; 3 hours before 5.00 p.m. is 2.00 p.m.)
- (Answers will vary. Sample answer: after 12 noon because he started baby-sitting after 12 noon.) Encourage students to reason that it is 5 hours from noon to 5.00, and since Jackson baby-sat for 3 hours and practised for less than 1 hour, the total time is less than 5 hours, or after noon.

Have students work backwards through the problem. They should reason that they must first subtract 3 hours from 5.00 to find when Jackson began baby-sitting. (2.00 p.m.) They can then subtract 45 minutes from the answer to find when Jackson began to practise. At this step, some students may subtract 1 hour and then add 15 minutes.

Alternatives: Students can use a non-digital clock or a picture of a clock to help them work through the problem. They may also find a timeline useful.



Solution: *Jackson began practising at 1.15 p.m.*

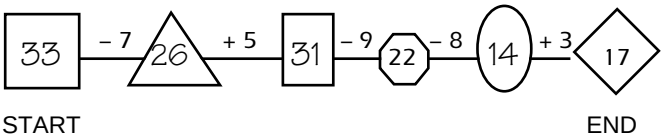
On your Own

Problem 3

If students have difficulty getting started, have them look at the last two shapes in the chain. Explain that since the last number is 17, and the operation is +3, the number in the oval +3 must equal 17. To illustrate, write ■ + 3 = 17, or N + 3 = 17, on the blackboard. If necessary, have students use counters to find the missing number. Explain that they can write the same kinds of number sentences to find the other missing numbers.

When they have completed the chain, students should work forward to check the equation.

Solution:



Problem 4

To help students work through the problem, have them begin by finding Sally's fine for the first 3 days. (30¢) Working backwards from $1, students can then find the remaining fine. (70¢) Working backward from 70¢, students can subtract 7¢ for each day the video was overdue. They can then count the number of 7s and the number of 10s to find the answer. Some students will easily see that there are 10 7s in 70, and that the video was overdue for 10 days plus the first 3 days.

Alternatives: Students can work forward and complete the following table to solve the problem. Have students copy the table from the blackboard or write on copies of the table. (Note that answers in parentheses are for teacher use.)

Number of days	1	2	3	4	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
Fine for each day	10¢	20¢	(30¢)	(37¢)	(44¢)	(51¢)	(58¢)	(65¢)	(72¢)	(79¢)	(86¢)	(93¢)	($1.00)

The fines for each day to 3 days are found by adding 10¢ to the previous day's total. The remaining fines are found by adding 7¢ to the previous day's total.

Solution: 13 days. Number sentences may vary. Samples:
10¢ + 10¢ + 10¢ + 7¢ + 7¢ + 7¢ + 7¢ + 7¢ + 7¢ + 7¢ + 7¢ + 7¢ + 7¢ = 100¢ = $1
(3 days x 10¢) + (10 days x 7¢) = 100¢ = $1

