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## Introduction

Problem-solving is the process of applying acquired knowledge to different situations. It is the basic skill of mathematics and an integral part of the mathematics curriculum at all levels of instruction.

**Figure it out** is a series of booklets designed to teach strategies for solving mathematical problems. As students work through a booklet, they learn to read problems carefully, to think about the content of problems, and to use what they know about numbers and mathematics to decide how to find solutions.

The problems included in each booklet are open-ended, non-routine problems. Their scope extends beyond that of routine problems, or those which students can solve by merely reading and identifying the necessary mathematical operation. Each problem in **Figure it out** has some unique quality that requires students to think carefully about how to solve it. Many are problems that students can relate to real life.

The most exciting aspect of teaching mathematics is the discoveries students make as they work through problems. Guide them with questions, encourage the use of manipulatives, and be sure to give students time and space to discover.

### The Student Book

The student book consists of lessons that teach eight different strategies that can be used to solve non-routine problems. Each lesson opens with a problem followed by **Questions** designed to help students think about the problem and how to solve it. After the **Questions**, students are given guidance on how to **Apply the Strategy** to solve the problem.

**Problem 1**, the **Questions**, and the **Apply the Strategy** section are intended to be teacher-directed. The second problem in each lesson is followed by **Think about** questions. This problem and its questions may be teacher-directed or may be completed by students on their own. Finally, each strategy lesson ends with two practice problems for students to complete independently. For more applications of the strategies, two pages of **Mixed Practice** are presented after every four strategy lessons. Students can use any strategy they find helpful to solve the problems on these pages. The last section of the booklet contains **Reviews** for each of the eight strategies. The booklet ends with a **Final Review** containing non-routine, open-ended problems that can be solved using the strategies presented in the lessons.

### Using the Student Book

Students should write on the answer lines provided. They should also be encouraged to write in any blank spaces in their booklets so they can keep their computation and other work close to the problems they are solving.

### The Teacher Guide

The teacher guide consists of procedures for teaching the strategy lessons and guidance for presenting the **Mixed Practice** and **Review** pages. The teacher guide also contains two blackline masters that can be duplicated and distributed to students for use in solving the problems.

### Using the Teacher Guide

Suggestions for instruction are provided throughout the teacher guide. These include questions to ask students, teaching tips, and diagrams and tables for student use. The teacher guide also provides answers to questions and solutions to problems posed in the student book. Some problems have more than one correct answer. All possible answers are listed. It is recommended that the teacher read through the teaching notes for each lesson before presenting the lesson to students.

### Teaching Strategy Lessons

Though the first problem in each strategy lesson is intended to be primarily teacher-directed, and the second is meant to be more self-guided, the teacher can approach the two problems in a similar manner. The teacher should read the problems aloud or have a student volunteer read the problems. He or she should also read each of the **Questions** and **Think about** questions aloud and lead the class in a discussion about the students' answers. During questioning, the teacher should encourage students to explain how they arrived at their answers. Explanations should be requested for correct and incorrect student responses. From students' answers, the teacher will see the wide variety of ways in which students approach the same problem. The teacher may also gain awareness of students' understanding or lack of understanding of mathematics concepts. After the questions have been answered, the teacher should help students use the strategy to solve the problem. The teaching notes provide guidance in this area. The final problems in each strategy lesson can either be teacher-directed or completed by students independently. The teacher guide provides information on how to help students think through these problems.

Following many of the problems in the teacher guide are **Challenge** problems that the teacher can present to the students. Some of these problems reinforce reasoning skills or strategy use at the level they were presented in the problems they follow. Other problems involve more advanced applications of the strategy that was taught.

### Apply the Strategy

Students should use what they discovered in answering the questions to help them guess the value of the letter M. Have students rewrite the problem for each guess they make. If students need help guessing, ask them what number times 4 will give a 2 in the ones place. (3)

**Solution:** A = 2      P = 9      M = 3

e. (Answers will vary. Sample answer: Try to make the letters stand for other numbers.)

**Challenge:** BEE      (B = 1, E = 0, Z = 9)

$$\begin{array}{r} \text{BEE} \\ - \text{B} \\ \hline \text{ZZ} \end{array}$$

### Problem 2

#### Think about:

Read the questions aloud and discuss them if necessary. You may need to review the concept of multiples to help students answer the questions and solve the problem.

- (7)
- (4)
- (7)
- (4)
- (The number must be a multiple of 4 and 7.)

Have students work through the problem. From the questions students may be aware that the number of ears, tails and legs Harrison counted is a multiple of 7. To use a picture to check their guesses, students should count the cats' legs. Some students may immediately see that they can draw 7 cats to find the solution.

**Alternative:** Students can use the following table to solve the problem. Have them copy the table from the blackboard or write on copies of the table.

Number of animals	1	2	3	4	5		
Number of dogs' ears, tails and legs	7	14					
Number of cats' legs	4	8					

Explain to students that the numbers at the top of each column stand for the number of both kinds of animals: 1 dog and 1 cat, 2 dogs and 2 cats, etc. Students should complete several columns of the table and look for numbers that appear in both rows (28; 56). To find the solution, students should check that the total number of animals is between 15 and 30.

**Solution:** 14 cats; 8 dogs

**Challenge:** Suppose Harrison was in an enormous pet shop in which the number of cats and dogs was between 400 and 500. If the number of ears, tails and legs on all the dogs is the same as the number of all the cats' legs, what are some possible totals for the number of cats and dogs in the shop? (420, 448, 476)

### On your Own

#### Problem 3

Suggest that students use counters to help them find the solution. To help them get started, point out that the number must be greater than 6 since they can subtract 6 from the number. You may also want to hint that the answer is a whole number, since students will be trying to find 1 less than half of the number.

To guide students in their guessing process, work aloud through the problem using 6 as a guess for the mystery number. Explain that if you add 8 to 6, the sum is 14. This is greater than twice 6, or 12. Point out that because 6 does not work for the first clue in the problem, 6 is not the mystery number. Work through the second clue to further prove the point: if you subtract 6 from 6, the difference is 0. Half of 6 is 3, and 0 is not 1 less than 3.

Draw the following tables on the blackboard and have students use them to work through their guesses. Have students copy the tables or work in copies of the tables. Point out how the tables enable students to make comparisons.

GUESS	Guess + 8	< ?	Guess x 2	GUESS	Guess - 6	= ?	$\frac{1}{2}$ guess - 1
6	14	no	12	6	0	no	3 - 1 = 2
7	15	no	14	7	1	no	cannot do
8	16	no	16	8	2	no	4 - 1 = 3
9	17	yes	18	9	3	no	cannot do
10	18	yes	20	10	4	yes	5 - 1 = 4

**Solution:** 10

#### Problem 4

To help students get started, have them begin with a guess for the weight of the barrel. Suggest that they begin with 10, as it is an easy number to work with. Students should reason that the flour would then weigh 30 kilograms since 40 kilograms (weight of full barrel) - 10 kilograms (weight of barrel) = 30 kilograms. They can double the weight of the flour to get the weight of the nails, 60 kg, and the barrel, 10 kg. Their combined weight is 70 kg, which is too much. At this point, some students will make new guesses by randomly choosing a weight for the barrel. Other students will reason that the flour must weigh less than 30 kg because 30 kg is half the weight of the barrel plus the nails. With this information, students can decrease their guesses.

**Solution:** 20 kg

**Challenge:** Suppose all the weights in Problem 4 were the same, but the nails weighed 5 times as much as the flour. How much would the barrel weigh? (35 kg)

## Draw a Picture

Drawing a picture of the objects, events or relationships in a problem often helps students to clarify what must be done to solve the problem. From their illustrations, students can learn what must be calculated to find a solution. Illustrations also help students to discover relationships between parts of problems and to gain understanding of spatial relationships. Because students must read a problem carefully in order to illustrate it, this strategy is very useful for beginning problem-solvers. The strategy is also helpful for students who learn best with visual aids.

### Materials:

- Student Book, pages 5–6
- 18 counters for each student (optional)
- Blackline Master 1, Teacher Guide page 27 (optional)

**Introduce the Strategy:** Explain that drawing a picture is often a good way to get a better understanding of a problem. Tell students that they can draw a picture to solve each of the problems in this lesson.

### Problem 1

Before students work through the problem, you may need to review fractions as part of a set. Use a set of 12 large counters to reinforce the concept. Have students find  $\frac{1}{2}$ ,  $\frac{1}{3}$ ,  $\frac{1}{4}$  and  $\frac{1}{6}$  of the set.

### Questions

Read each question aloud. Then work through the question with students.

- a. (1 coin)  
If necessary, have students use 6 counters to find the answer. Students should separate the coins into 6 equal groups and find how many are in 1 group. (1 coin) Discuss that another way of saying  $\frac{1}{6}$  is 1 out of 6.
- b. (Answers will vary. Sample answer: Separate 18 coins into 6 equal groups and count the number of coins in 1 group.)
- c. (more; less)  
To help students answer the questions, have them separate 6 counters into 3 equal groups and find  $\frac{1}{3}$  of 6, or the number in 1 group (2 counters). From question a, students know that  $\frac{1}{6}$  of 6 is 1. Students should note that 2 is greater than 1, so  $\frac{1}{3}$  is more than  $\frac{1}{6}$ . If they make 2 equal groups to find  $\frac{1}{2}$  of 6 (3 counters), students will note that 2 is less than 3 so  $\frac{1}{3}$  is less than  $\frac{1}{2}$ . Explain that 1 out of 3 is more than 1 out of 6, and 1 out of 3 is less than 1 out of 2.
- d. (Answers may vary. With the information from question c, students should be able to conclude that Kelsie has more five-cent coins.)

## Apply the Strategy

Though the eventual goal is for students to draw their own pictures, in learning the strategy it is helpful to provide a picture for students to use. Students can use the circles on the student book page, Blackline Master 1, or draw 18 circles if this will help them to solve the problem.

After students have labelled 1 out of every 6 circles as twenty cents, they should label 1 out of every 3 circles as a ten-cent and 1 out of every 2 as a five-cent. Students may ring groups of circles or write coin values in the circles. Sample uses of picture:

$9 \times 5\text{¢} = 45\text{¢}$      $20\text{s} = 60\text{¢}$      $5\text{s} = 45\text{¢}$   
 $6 \times 10\text{¢} = 60\text{¢}$                      $10\text{s} = 60\text{¢}$   
 $3 \times 20\text{¢} = 60\text{¢}$

Once they have labelled the circles, they can use paper and pencil, mental maths or a calculator to find the total value of the coins. Students may add the values of each type of coin by group or add individual coin values.

**Solution:** \$1.65, or 165¢

e. (Answers will vary.)

### Problem 2

#### Think about:

Read the questions aloud and discuss them if necessary.

- (48 ears)  
Students can draw a picture or multiply  $3 \times 16$  to find the answer.
- (80 ears)  
Students can draw a picture or multiply  $5 \times 16$  to find the answer.
- (16 heads)  
Watch for students who think they should draw Danny's head as well as those of the space people.
- (3 ears)  
Have a volunteer read from the problem the information that helps them answer the question. (People from the planet Mondo have 3 ears. People from the planet Verbo have 5 ears.)

Have students work through the problem. Students may find it useful to record their lists in a chart. In the following chart, the first three columns show numbers of discs landing in one ring of the number board. The last column is the total number of discs in each row. If they use the chart, students should understand that they need to keep track of the point totals in each row to be sure they are finding sums of 75.

Sample list:

Sums of 75			Total discs used
5	9	15	
15	0	0	15
12	0	1	13
9	0	2	11
6	0	3	9
3	0	4	7
0	0	5	5
6	5	0	11
3	5	1	9
0	5	2	7

**Solution:** 9 discs; 3 discs on 5, 5 discs on 9, 1 disc on 15

Sample equations:

$$5 + 5 + 5 + 9 + 9 + 9 + 9 + 9 + 15 = 75$$

$$(3 \times 5) + (5 \times 9) + (1 \times 15) = 75$$

(Order does not matter in either equation.)

**Challenge:** Suppose Denise and Latisha scored 100 points. List the ways they could have scored 100 points using exactly 12 discs. (You do not have to use all 3 numbers.)

**Solution:**

Sums of 100		
5	9	15
20	0	0
17	0	1
14	0	2
11	0	3
8	0	4
5	0	5
2	0	6
11	5	0
8	5	1
5	5	2
2	5	3
2	10	0

### On your Own

You may need to help students make charts or tables to record their work for Problems 3 and 4. Samples are given.

### Problem 3

If students need help getting started, have them use clues a and b to list the possible numbers:

159	195	519	591	915	951
168	186	618	681	816	861
267	276	627	672	726	762
357	375	537	573	735	753
456	465	546	564	645	654

Then have students use clue c. This clue eliminates all rows except the fourth because all of the other numbers have an even digit or the digit 9. Clue d will then provide the solution.

Once they have found the solution, students should check that the number fits all the clues.

**Solution:** 357

### Problem 4

Sample list:

Birds	Bird legs	Lions	Lion legs	Total legs
23	46	—	—	46
21	42	1	4	46
19	38	2	8	46
17	34	3	12	46
15	30	4	16	46
13	26	5	20	46
11	22	6	24	46
9	18	7	28	46
7	14	8	32	46
5	10	9	36	46
3	6	10	40	46
1	2	11	44	46

9 is 2 more than 7

**Solution:** 9 birds; 7 lions

**Challenge:** Suppose the problem said 'The number of birds and lions were both odd numbers' instead of 'The number of birds was 2 more than the number of lions'. What would your solutions be?

21 birds; 1 lion	13 birds; 5 lions	5 birds; 9 lions
17 birds; 3 lions	9 birds; 7 lions	1 bird; 11 lions

### Mixed Practice

These pages serve as a review of the strategies *Act it out*, *Guess and Check*, *Draw a Picture*, and *Make a List*. Though uses of particular strategies are presented, students may apply any strategies they find useful for solving these problems.

Sample use of table:

Number of correct solutions (+10 points)	Number of incorrect solutions (-1 point)	Test score
10	0	100
9	1	89
8	2	(78)
(7)	(3)	(67)
(6)	(4)	(56)
(5)	(5)	(45)

**Solution:** 5 solutions

**On your Own**  
**Problem 3**

Encourage students to make a chart such as the following and to look for patterns in the numbers of copies sold. (Note that answers in parentheses and notes indicating the pattern of increase are for teacher use.) Students should see that in each week, the number of copies increases by 500 more than the previous week's increase. Students can use this pattern to complete the chart and find the solution.

Week	Number of copies sold
1	200
2	700
3	1700
4	3200
5	(5200)
6	(7700)
7	(10,700)
8	(14,200)

↘ +500  
 ↘ +1000  
 ↘ +1500  
 ↘ +2000  
 ↘ +2500  
 ↘ +3000  
 ↘ +3500

**Solution:** 14,200 copies

**Challenge:** In which week will the number of copies be greater than 20,000? (week 10; 22,700 copies exactly)

**Problem 4**

Students can work in a chart such as the following to solve the problem. (Note that answers in parentheses and notes indicating the pattern of increase are for teacher use.)

Number of friends	Number of calls
2	1
(3)	(3)
(4)	(6)
(5)	(10)
(6)	(15)
(7)	(21)
(8)	(28)
(9)	(36)
(10)	(45)

↘ +2  
 ↘ +3  
 ↘ +4  
 ↘ +5  
 ↘ +6  
 ↘ +7  
 ↘ +8  
 ↘ +9

**Solution:** 45 calls

**Challenge:** How much would 45 long distance calls cost if each call lasted for 6 minutes and the rate for the calls was \$10 per hour? (\$45)

**Solve a Simpler Problem**

Solving a simpler problem involves putting the original problem aside and working on a problem that is related, but easier to work out. Sometimes it is easier to work out a similar problem with simpler conditions. The approach to the solution of the simpler problem is then applied to the original problem. At other times, a pattern seen when solving one or more simpler problems can lead to the solution of the original problem.

**Materials:**

- Student Book, pages 13–14

**Introduce the Strategy:** Tell students that to solve each of the problems in this lesson they can first solve simpler problems.