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Introduction

Problem-solving is the process of applying acquired knowledge to different situations. It is the basic skill of mathematics and an integral part of the mathematics curriculum at all levels of instruction.

Figure it out is a series of booklets designed to teach strategies for solving mathematical problems. As students work through a booklet, they learn to read problems carefully, to think about the content of problems, and to use what they know about numbers and mathematics to decide how to find solutions.

The problems included in each booklet are open-ended, non-routine problems. Their scope extends beyond that of routine problems, or those which students can solve by merely reading and identifying the necessary mathematical operation. Each problem in ***Figure it out*** has some unique quality that requires students to think carefully about how to solve it. Many are problems that students can relate to real life.

The most exciting aspect of teaching mathematics is the discoveries students make as they work through problems. Guide them with questions, encourage the use of manipulatives, and be sure to give students time and space to discover.

The Student Book

The student book consists of lessons that teach eight different strategies that can be used to solve non-routine problems. Each lesson opens with a problem followed by **Questions** designed to help students think about the problem and how to solve it. After the **Questions** students are given guidance on how to **Apply the Strategy** to solve the problem.
Problem 1, the **Questions** and the **Apply the Strategy** section are intended to be teacher- directed. The second problem in each lesson is followed by **Think about** questions. This problem and its questions may be teacher-directed or may be completed by students on their own. Finally, each strategy lesson ends with two practice problems for students to complete independently. For more applications of the strategies, two pages of **Mixed Practice** are presented after every four strategy lessons. Students can use any strategy they find helpful to solve the problems on these pages. The last section of the booklet contains **Reviews** for each of the eight strategies. The booklet ends with a **Final Review** containing non-routine problems that can be solved using the strategies presented in the lessons.

Using the Student Book

Students should write on the answer lines provided. They should also be encouraged to write in any blank spaces in their booklets so they can keep their computation and other work close to the problems they are solving.

The Teacher Guide

The teacher guide consists of procedures for teaching the strategy lessons and guidance for presenting the **Mixed Practice** and **Review** pages. The teacher guide also contains three blackline masters that can be duplicated and distributed to students for use in solving the problems.

Using the Teacher Guide

Suggestions for instruction are provided throughout the teacher guide. These include questions to ask students, teaching tips, and diagrams and tables for student use. The teacher guide also provides answers to questions and solutions to problems posed in the student book.
It is recommended that the teacher read through the teaching notes for each lesson before presenting the lesson to students.

Teaching Strategy Lessons

Though the first problem in each strategy lesson is intended to be primarily teacher-directed, and the second is meant to be more self-guided, the teacher can approach the two problems in a similar manner. The teacher should read the problems aloud or have a student volunteer read the problems. He or she should also read each of the **Questions** and **Think about** questions aloud and lead the class in a discussion about the students' answers. During questioning, the teacher should encourage students to explain how they arrived at their answers. Explanations should be requested for correct and incorrect student responses. From students' answers, the teacher will see the wide variety of ways in which students approach the same problem. The teacher may also gain awareness of students' understanding or lack of understanding of mathematics concepts. After the questions have been answered, the teacher should help students use the strategy to solve the problem. The teaching notes provide guidance in this area. The final problems in each strategy lesson can either be teacher-directed or completed by students independently. The teacher guide provides information on how to help students think through these problems.

Following many of the problems in the teacher guide are **Challenge** problems that the teacher can present to the students. Some of these problems reinforce reasoning skills or strategy use at the level they were presented in the problems they follow. Other problems involve more advanced applications of the strategy that was taught.

Have students work through the problem. If students list pairs of numbers that have a sum of 10, they should be able to see a pattern in the digits on each side of the plus sign. By extending the pattern to 9 on the left and 1 on the right, students can be sure they have found all the possible combinations of numbers that have a sum of 10.

numbers increase by 1 ↓		numbers decrease by 1 ↓
	1 + 9	
	2 + 8	
	3 + 7	
	4 + 6	
	5 + 5	
	6 + 4	
	7 + 3	
	8 + 2	
	9 + 1	

Solution: 868-3219, 868-3228, 868-3237, 868-3246, 868-3255, 868-3264, 868-3273, 868-3282, 868-3291. Explanations may vary. Sample answer: I know I found all the possible phone numbers because I wrote all the pairs of numbers that add to 10.

On your Own
Problem 3

A chart such as the following can help students find patterns that are useful for solving the problem.

Week number	Number of cars sold
1	4
2	8
3	14
4	22
(5)	(32)
(6)	(44)
(7)	(58)

+ 4
+ 6
+ 8
+ 10
+ 12
+ 14

Have students copy the chart from the blackboard, or distribute a copy for students to write on. (Note that answers in parentheses and notes to the side indicating the pattern are for teacher use.)

Solution: 7 weeks; Explanations may vary. Sample answer: The difference between the two numbers was always an even number and it kept increasing by 2.

Problem 4

One way to solve the problem is to list the palindromes between every 2 hours in a period of 12 hours. If they begin with 12.00, students will find that there is 1 palindrome between 12.00 and 1.00 (12.21). Between 1.00 and 2.00 there are 6 palindromes (1.01,1.11,1.21, 1.31, 1.41,1.51). If they continue to list in this manner, students will discover that there are 6 palindromes between 2.00 and 3.00, 3.00 and 4.00, etc., for times up to 10.00. From 10.00 to 11.00 there is 1 (10.01), and from 11.00 to 12.00 there is 1 (11.11). The total number of palindromes in 12 hours is 57. Students should double this number to find the solution.

Alternative: Students can use a digital clock or watch to work through the problem. Have them change the time on the clock or watch by the minute and record each time that is a palindrome. By using this method, students can actually see that the numbers read the same backwards and forwards.

Solution: 114 times

12.21	1.01	2.02	3.03	4.04	5.05	6.06	7.07	8.08	9.09	10.01	11.11
	1.11	2.12	3.13	4.14	5.15	6.16	7.17	8.18	9.19		
	1.21	2.22	3.23	4.24	5.25	6.26	7.27	8.28	9.29		
	1.31	2.32	3.33	4.34	5.35	6.36	7.37	8.38	9.39		
	1.41	2.42	3.43	4.44	5.45	6.46	7.47	8.48	9.49		
	1.51	2.52	3.53	4.54	5.55	6.56	7.57	8.58	9.59		

Each of these times occurs twice in one day.

Challenge: What is the longest period of time between palindromes? (70 minutes, from 10.01 to 11.11 or from 11.11 to 12.21). What is the shortest period of time between palindromes? (2 minutes, from 9.59 to 10.01)

In one day, how many times will the numbers on a digital, 24-hour clock form a palindrome? (16 times) What are the palindromes? (01.10, 02.20, 03.30, 04.40, 05.50, 10.01,11.11, 12.21,13.31,14.41,15.51, 20.02, 21.12, 22.22, 23.32, 24.42)

Experiment

Experiential learning is sometimes the best way to find solutions or to discover strategies that will lead to solutions. Problems can often be solved by experimenting with physical objects. Models, such as counters or other concrete materials, can be used to represent objects described in problems. The use of models can enable students to visualise problems or to discover relationships that will lead to solutions. At times, students themselves can be the models used to experiment with information in a problem. By acting out the roles of people mentioned in a problem, students experience the concrete solution to the problem.

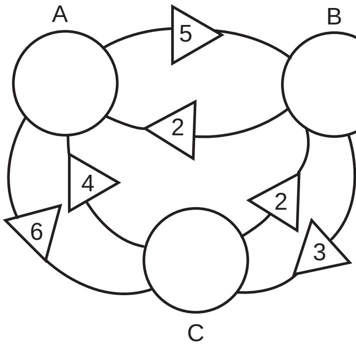
Materials:
- Student Book, pages 3–4
- 50 counters for each student
- Blackline Master 1, Teacher Guide page 27 (optional)
- 4 large pieces of newsprint or wrapping paper (optional)
- Blackline Master 2, Teacher Guide page 28 (optional)

Introduce the Strategy: Tell students that each problem in this lesson can be solved by experimenting with objects or by acting out the problem.

Moves

A → B	A has 11	B has 5
B → A	A has 14	B has 2
A → B	A has 9	B has 7
B → A	A has 12	B has 4
A → B	A has 7	B has 9
B → A	A has 10	B has 6
A → B	A has 5	B has 11
A → B	A has 0	B has 16

Challenge: Suppose you want to move 20 beans from Circle A to Circle C. The number of beans you can move from circle to circle is shown by the arrows. What moves must you make to move the beans?



(One solution is given below. Other solutions are possible.)

Moves

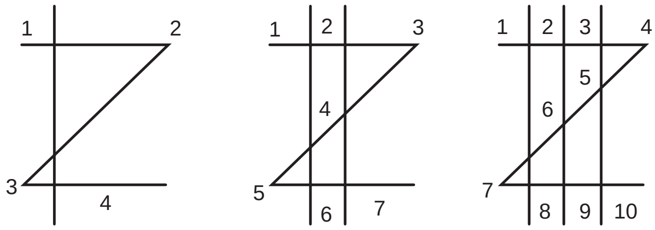
A → B	A has 15	B has 5	C has 0
A → B	A has 10	B has 10	C has 0
A → C	A has 4	B has 10	C has 6
B → C	A has 4	B has 7	C has 9
B → C	A has 4	B has 4	C has 12
B → A	A has 6	B has 2	C has 12
B → A	A has 8	B has 0	C has 12
A → C	A has 2	B has 0	C has 18
C → A	A has 6	B has 0	C has 14
A → C	A has 0	B has 0	C has 20

On your Own

Guide students if they choose to work together to solve the problems. Help them decide as a group how to experiment with the problems.

Problem 3

As students draw lines on the Z, they should notice that 3 additional pieces are made with each line drawn.



Students can continue to draw lines to solve the problem, or they can use a table such as the following. Have students copy the table from the blackboard or write on copies of the table. (Note that answers in parentheses are for teacher use.)

Number of cuts	1	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Number of pieces	4	(7)	(10)	(13)	(16)	(19)	(22)	(25)

Solution: 8 parallel vertical lines

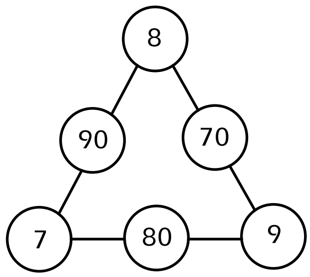
Problem 4

To help them solve the problem, have students write each of the numbers 7, 8, 9, 70, 80 and 90 on small pieces of paper, or paper attached to buttons or counters. Students can then manipulate the numbers around the diagram to avoid excessive writing and erasing.

Sample experimental method: Students multiply 70 × 80, 70 × 90 and 80 × 90. They find that all the products are greater than 5040. From here they reason that the 1-digit numbers must go in the 3 corners, since 2 corner numbers will always be multiplied when finding the product of the numbers in one line. Students then find the products of the 1-digit numbers in each line (56, 63, 72) and divide 5040 by each product to find where to place 70, 80 and 90.

Alternative: Students can use Blackline Master 2 (page 28) to solve the problem. If they do, have them cut out the circles at the bottom of the master and label each with a number from the problem.

Solution: The numbers in each line will be the same as the given solution, though the position of the triangle may vary. (It may appear rotated or flipped.)



Challenge: Have students write problems like Problem 4 and trade them with their classmates. Students can use copies of Blackline Master 2 (page 28) to help them create their problems. The circles on the bottom of the master should be used for the numbers that will go in each line of the diagram.

On your Own

Problem 3

To solve the problem, students can first divide 640 km by 80 km/h to find the number of hours the Lee family travelled (8 hours). They can then work backwards from 5.15 p.m. to find the time that is 8 hours earlier.

Encourage students to check their solutions by working forward.

Solution: 9.15 a.m.

Problem 4

To work backwards through the problem, students can use the following chain of reasoning: Jonathan had $6 left after spending $\frac{1}{2}$ of his money. Therefore, he entered the third shop with twice that amount, or $12. This $12 is $\frac{2}{3}$ of what he entered the second shop with, since he spent $\frac{1}{3}$ of his money in the second shop. If $12 is $\frac{2}{3}$, then $\frac{1}{3}$ is $6, and Jonathan entered the second shop with $12 + $6, or $18. This $18 is $\frac{3}{4}$ of what he entered the first shop with, since he spent $\frac{1}{4}$ of his money in the first shop. If $18 is $\frac{3}{4}$, then $6 is $\frac{1}{4}$, and Jonathan entered the first shop with $18 + $6, or $24.

Encourage students to check their solutions by working forward.

Alternatives: Students can use real or play money to work through the problem. They can also use the guess and check strategy by guessing a starting amount and working through the problem to see if the amount is correct.

Solution: $24; Sample response: I can start with my solution and work forward.

For example,
$24 x $\frac{1}{4}$ = $6
(first shop)

$24 – $6 = $18 $18 x $\frac{1}{3}$ = $6 $18 – $6 = $12
(second shop)
$\frac{1}{2}$ of $12 = $16
(third shop)

Challenge: Suppose Jonathan left the third shop with $18 instead of $6. How much money would he have brought on his shopping trip? ($72)

Mixed Practice

(Student Book, pages 9–10)
These pages serve as a review of the strategies *Look for Patterns*, *Experiment*, *Make a List* and *Work Backwards.* Students may apply any strategies they find helpful in solving these problems. Sample strategies are presented as possible ways to find solutions.

Problem 1

Students can look for patterns to help them solve this problem. Students may notice that because the ones digits repeat in the sequence 3, 9, 7, 1, they can extend the sequence to the twelfth term to find the solution.

Solution: 1

Problem 2

To solve the problem, students can make a list of all the numbers between 0 and 180 that contain a 6. (6, 16, 26, 36, 46, 56, 60–69, 76, 86, 96, 106, 116, 126, 136, 146, 156, 160–169, 176) They can then count the numbers. Since there are 179 numbers between 0 and 180 (1–179), and 36 of these numbers contain a 6, students can subtract 36 from 179 to find how many numbers do not contain a 6.

Solution: 143 numbers

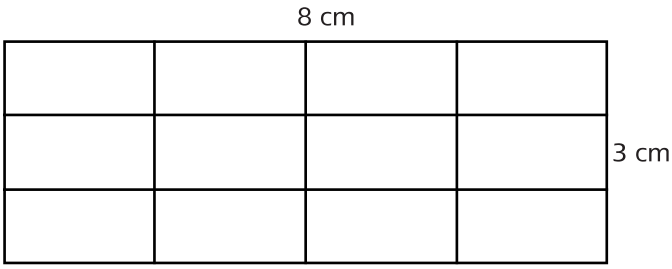
Problem 3

Students can work backwards to solve the problem.
For example:
6.10 a.m. – 10 hours 45 minutes = 7.25 p.m.

Solution: 7.25 p.m.; Sample response: I worked backwards.

Problem 4

To find the solution, students can experiment with cut-out tracings of the rectangles. They can also draw small rectangles in the large rectangle. For example:

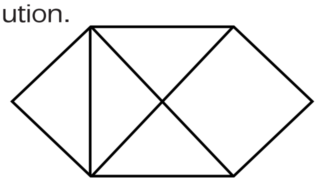


Solution: 12 small rectangles; Sample responses: I cut out a small rectangle to see how many times it fit on the large rectangle. The area of the small rectangle is 2 cm^2. The area of the large rectangle is 24 cm^2. So I divided 24 by 2 to get my answer.

Problem 5

To solve the problem, students can experiment with cut-out tracings of the pieces. They can also experiment with drawings.

Solution: Note that students' hexagons may be rotated, but positions of pieces should be the same as the given solution.



Solution:

	I	II	III
A	4	5	1
B	2	6	8
C	3	7	9

Challenge: Have students prepare their own puzzle with clues like those in Problem 2. Suggest that they start with diagrams that have 2 columns and 2 rows before making larger diagrams.

On your Own
Problem 3

Possible method of reasoning: Since N × D = N, and N = 3, then 3 × D = 3, and D = 1. Because N = 3, N × A = 3 × A. In the problem, the product of 3 × A has an A in the ones place. Therefore, A = 5, because 3 × 5 = 15. The 1 in the number 15 must be added to the product of P × N, or P × 3. Because the product of (P × 3) + 1 is C, which must be a 1-digit number, and because P cannot equal 1 or 3, P must equal 2. Since (2 × 3) + 1 = 7, you know that C = 7. Substituting the known values of N, D, A, P and C into the problem shows that E = 0.

```
   251
x   53
   753
+1255
13,303
```

Solution: P = 2, A = 5, D = 1, N = 3, C = 7, E = 0

Problem 4

To help students use the logic box, first be sure they understand that the clues tell which numbers can and cannot belong to certain people. Point out that knowing which number does not belong to a person helps to narrow the possibilities for that person's number. Then tell students that they can record information from the clues in the logic box by marking an X in a square to show that a number and a person do not match. They should mark a tick to show a match. You may need to hint that if a match is found, students can mark an X in all the other squares in that column and row.

Sample use of logic box:

	29	48	53	72	101
Ann	Clue C √	×	×	Clue A ×	Clue A ×
Bob	Clue C ×	Clue C ×	Clue C ×	Clue C ×	Clue C √
Cal	×	Clue B ×	√	Clue B ×	Clue C ×
Don	×	√	×	Clue D ×	Clue C ×
Eve	×	×	×	√	×

Solution: Ann = 29, Bob = 101, Cal = 53, Don = 48, Eve = 72

Solve a Simpler Problem

Solving a simpler problem involves putting the original problem aside and working on a problem that is related but easier to work out. Sometimes it is easier to work out a similar problem with simpler conditions. The approach to the solution of the simpler problem is then applied to the original problem. At other times, a pattern seen when solving one or more simpler problems can lead to the solution of the original problem.

Materials:

- Student Book, pages 15–16
- 6 index cards for each student (optional)
- 1 real or model clock or a drawing of a clock for each student (optional)

Introduce the Strategy: Tell students that to solve each of the problems in this lesson they can first solve simpler problems.

Problem 1
Questions

Read each question aloud. Then work through the question with students.

a. (9, 8, 7; 987, 978, 897, 879, 798, 789)
Encourage students to experiment with arranging sets of 3 digits to help them answer the question. If they need help finding the answer, have them first find the largest 3-digit number that can be made. (987) Then have them find the 2 largest 3-digit numbers that can be made. (987, 978) Student can continue to make 1 number at a time until they have made 6 numbers. Help them discover that all of the numbers use the same digits (9, 8 and 7), and that the 6 numbers are all the possible arrangements of those digits.

b. (9, 8, 7, 6; 9876, 9867, 9768, 9786, 9678, 9687)
Students should experiment with arranging sets of 4 digits to answer the question. From question a, they may be able to deduce that the digits 9, 8, 7 and 6 should be used to make the largest 4-digit number. If they experiment with the arrangements of these digits, they will find that 9 must be in the thousands place of the 6 largest 4-digit numbers.

c. (Answers will vary. Sample answer: 9, 8, 7, 6, 5, 4)
Student may guess at the answer to this question, but encourage them to make an educated guess by using what they know from answering the previous questions.

d. (Answers will vary. Sample answer: yes, because there are a lot of other numbers to arrange and rearrange without moving the 9)
This question hints to students that the solution can be found by solving the simpler problem of finding all the arrangements of the digits 8, 7, 6, 5 and 4.

e. (Answers will vary. Sample answers: Try to solve the problem again; start in the hundred thousands place and look at every number to be sure the largest digits are in the largest places and that the other numbers are as large as possible.)

f. (Answers will vary.) Sample answer: I knew that the number in the hundred thousands place should be the largest, the number in the ten thousands place should be the next largest, and so on.

Apply the Strategy

From answering the questions, students should know which digits they need to use, and that they can arrange and rearrange those digits to form the 6 numbers. If students do not know that there must be a 9 in the hundred thousands place of all 6 numbers, they have missed an important step in simplifying the problem. However, they will still be able to solve the problem, if only by writing as many 6-digit numbers as they can. Some students will immediately see that the first three digits of the number must be 987, and that they can find the arrangements of the digits 6, 5 and 4 to find the solution.

Alternative: Students can write each of the digits from 4 to 9 on an index card or slip of paper to help them arrange and rearrange the numbers in an organised manner.

Solution: 987,654; 987,645; 987,564; 987,546; 987,465; 987,456

Problem 2
Think about:

Read the questions aloud and discuss them if necessary. Students can use a real clock, a model clock, or a drawing of a clock to help them solve the problem and answer the questions.

- (24 hours)
- (8.00 a.m.; Explanations will vary. Sample answers: Since it will be 7.00 a.m. in 24 hours, and 25 hours is 24 + 1 hour, add 1 hour to 7.00 a.m.; add 25 hours to 7.00 a.m.)
If students have added 25 hours to 7.00 a.m. to answer the question, help them see that they can also find the answer by adding 1 hour to 7.00 a.m.
- (11.00 a.m.; Explanations will vary.
Sample answers:
Since it will be 7.00 a.m. in 48 hours, and 52 hours is 48 + 4 hours, add 4 hours to 7.00 a.m.; add 52 hours to 7.00 a.m.)
If students added 52 hours to 7.00 a.m. to answer the question, help them see that they can also find the answer by adding 4 hours to 7.00 a.m.
- (24)

Have students work through the problem. If they have not found a simple way to solve the problem after answering the questions, have students list the first three multiples of 24. (24, 48, 72) Then ask what the time will be in 24 hours after 7.00 a.m. (7.00 a.m.), 48 hours after 7.00 a.m. (7.00 a.m.), and 72 hours after 7.00 a.m. (7.00 a.m.). Have students say how knowing the multiples of 24 can help them solve the problem. Students should indicate that they can find how many groups of 24 hours are in 800 hours, and, if any hours are left over, they can count forward from 7.00 a.m. by the hours left over to find the solution.

To solve the problem, some students will divide 800 by 24 (33 R8) and add the remainder to 7.00 a.m. (7.00 a.m. + 8 hours = 3.00 p.m.). Other students will count by 24 to get as close to 800 as possible. They will then find how many hours are left over and add these to 7.00 a.m.

Alternative: Students can use a calculator to add 24 + 24 + 24 . . . until the sum is as close to 800 as possible. (That sum is 792.) If they find the difference between that sum and 800 hours (8 hours), students can add the difference to 7.00 a.m. to find the solution.

Solution: 3.00 p.m.