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Introduction

Problem-solving is the process of applying acquired knowledge to different situations. It is the basic skill of mathematics and an integral part of the mathematics curriculum at all levels of instruction.

Figure it out is a series of booklets designed to teach strategies for solving mathematical problems. As students work through a booklet, they learn to read problems carefully, to think about the content of problems, and to use what they know about numbers and mathematics to decide how to find solutions.

The problems included in each booklet are open-ended, non-routine problems. Their scope extends beyond that of routine problems, or those which students can solve by merely reading and identifying the necessary mathematical operation. Each problem in ***Figure it out*** has some unique quality that requires students to think carefully about how to solve it. Many are problems that students can relate to real life.

The most exciting aspect of teaching mathematics is the discoveries students make as they work through problems. Guide them with questions, encourage the use of manipulatives, and be sure to give students time and space to discover.

The Student Book

The student book consists of lessons that teach eight different strategies that can be used to solve non-routine problems. Each lesson opens with a problem followed by **Questions** designed to help students think about the problem and how to solve it. After the **Questions**, students are given guidance on how to **Apply the Strategy** to solve the problem. **Problem 1**, the **Questions**, and the **Apply the Strategy** section are intended to be teacher-directed. The second problem in each lesson is followed by **Think about** questions. This problem and its questions may be teacher-directed or may be completed by students on their own. Finally, each strategy lesson ends with two practice problems for students to complete independently. For more applications of the strategies, two pages of **Mixed Practice** are presented after every four strategy lessons. Students can use any strategy they find helpful to solve the problems on these pages. The last section of the booklet contains **Reviews** for each of the eight strategies. The booklet ends with a **Final Review** containing non-routine open-ended problems that can be solved using the strategies presented in the lessons.

Using the Student Book

Students should write on the answer lines provided. They should also be encouraged to write in any blank spaces in their booklets so they can keep their computation and other work close to the problems they are solving.

The Teacher Guide

The teacher guide consists of procedures for teaching the strategy lessons and guidance for presenting the **Mixed Practice** and **Review** pages. The teacher guide also contains five blackline masters that can be duplicated and distributed to students for use in solving the problems.

Using the Teacher Guide

Suggestions for instruction are provided throughout the teacher guide. These include questions to ask students, teaching tips, and diagrams and tables for student use. The teacher guide also provides answers to questions and solutions to problems posed in the student book. It is recommended that the teacher read through the teaching notes for each lesson before presenting the lesson to students.

Teaching Strategy Lessons

Though the first problem in each strategy lesson is intended to be primarily teacher-directed, and the second is meant to be more self-guided, the teacher can approach the two problems in a similar manner. The teacher should read the problems aloud or have a student volunteer read the problems. He or she should also read each of the **Questions** and **Think about** questions aloud and lead the class in a discussion about the students' answers. During questioning, the teacher should encourage students to explain how they arrived at their answers. Explanations should be requested for correct and incorrect student responses. From students' answers, the teacher will see the wide variety of ways in which students approach the same problem. The teacher may also gain awareness of students' understanding or lack of understanding of mathematics concepts. After the questions have been answered, the teacher should help students use the strategy to solve the problem. The teaching notes provide guidance in this area. The final problems in each strategy lesson can either be teacher-directed or completed by students independently. The teacher guide provides information on how to help students think through these problems.

Following many of the problems in the teacher guide are **Challenge** problems that the teacher can present to the students. Some of these problems reinforce reasoning skills or strategy use at the level they were presented in the problems they follow. Other problems involve more advanced applications of the strategy that was taught.

A note on teaching problem-solving:
In order for students to learn the skills needed to solve problems, it is important for the teacher to create a problem-solving environment in the classroom. This involves three things. First, students must see the teacher as a problem-solver and absorb the teacher's problem-solving process. The teacher should verbalise his or her thought process. Second, problem-solving takes time. Always provide students with sufficient time to explore problems. Third, problem-solving is a

Problem 4

To work through the problem, students can list all the sets of four-digit numbers and sets of five-digit numbers that have a digit sum of 6. Remind students that they can use 0 as one of the digits. Students can then choose the numbers in their sets that can be used to write palindromes.

Sample list:

4-digit numbers	5-digit numbers
3003	30003
2112	20202
1221	12021
	11211
	10401

Solution: See sample list above; all the digits of the numbers are less than 5.

Look for Patterns

Recognising and extending patterns is a strategy frequently used for solving problems in mathematics and in life. Sometimes, patterns can be used to simplify a problem-solving process. In other instances, patterns are a critical part of a solution. In preparation for using patterns to solve problems, it is important that students gain experience in looking for patterns. Initially they may find patterns that may not be useful for solving the problems. However, they will eventually be able to decide which patterns are applicable. All patterns should be considered interesting and worthy of discussion.

Materials:

- Student Book, pages 3–4
- 50 counters for each student (optional)

Introduce the Strategy: Tell students that finding number patterns can help them solve each of the problems in this lesson.

Problem 1

Questions

Read each question aloud. Then work through the question with students.

a. (Answers will vary. Sample answers: The numbers increase; the numbers increase by 4; all the numbers are odd.)
b. (Answers may vary. Students should note that the numbers increase by 4.)
c. (Answers will vary. Sample answers: Write every number in the sequence up to 239; write several more numbers in the sequence and look for a pattern that will help find the answer without writing all the numbers.)
d. (Answers may vary. Students should note that the third term is 1 less than 12.)

Apply the Strategy

While answering question *d,* students may have noticed at least one pattern that can be used to solve the problem without writing every number in the sequence. If not, point out that each term in the sequence is 1 less than a multiple of 4. This may lead students to notice that each term can be found by subtracting 1 from each successive multiple of 4. The following table shows how that information can be used to solve the problem.

Term	Number	Multiple of 4
1	3	1 x 4 = 4
2	7	2 x 4 = 8
3	11	3 x 4 = 12
4	15	4 x 4 = 16
5	19	5 x 4 = 20
6	23	6 x 4 = 24
7	27	7 x 4 = 28
8	31	8 x 4 = 32
9	35	9 x 4 = 36
10	39	10 x 4 = 40
▪	▪	▪
▪	▪	▪
▪	▪	▪
?	239	? x 4 = 240 (60 x 4 = 240; sixtieth term)

Alternative: Another pattern that can be used to solve the problem is to notice which numbers in the sequence have a 9 in the ones place. These are all the terms that are multiples of 5. For each successive multiple of 5, the number in the sequence is 20 more than the number of the fifth term before it. To see this pattern, students should extend the sequence at least as far as the 15th term. The following list shows this use of the pattern.

Term 5 = 19	Term 35 = 139
10 = 39	40 = 159
15 = 59	45 = 179
20 = 79	50 = 199
25 = 99	55 = 219
30 = 119	60 = 239

Solution: the sixtieth term

e. (Answers will vary. Sample answers: Write all the numbers in the sequence to 239; use a different pattern.)

Challenge: What is the 501st term in the sequence? (2003) In the sequence of terms, what pattern can you find that will help you list 10 terms that will have a 7 in the ones place of their numbers? List the 10 terms and their numbers. (Any term that has a 2 or a 7 in its ones place will have a 7 in the ones place of its number; Term 2–7, Term 7–27, Term 12–47, Term 17–67, Term 22–87, Term 27–107, Term 32–127, Term 37–147, Term 42–167, Term 47–187)

Problem 2

Think about:

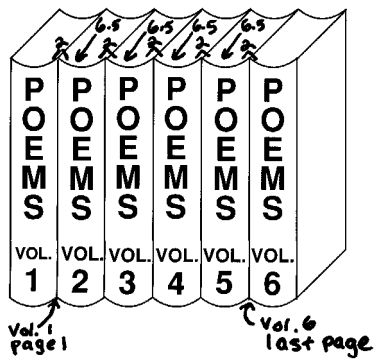
Read the questions aloud and discuss them if necessary.

- (Answers will vary. Sample answers: 10 is greater than 3; 10 is 7 more than 3; one number is odd and one number is even; the number of jars sold increased.)
- (no)
  Help students trace the increase in sales from week to week. They should note that by the end of week 2, Joey had sold 7 more jars than he had sold by the end of week 1, and that by the end of week 3, he had sold 21 more jars than he had sold by the end of week 2.

Ways to Seat _____ People at Tables of _____ and of _____ People

Tables of _____ people	Total number of people	Tables of _____ people	Total number of people	Total people seated

Problem 2



5 x 2 cm = 10 cm
4 x 6.5 cm = 26 cm

10
+26
36 cm

Solution: 36 centimetres

Problem 3

Solution:

1459	4159	5149	9145
1495	4915	5194	9154
1549	4519	5419	9415
1594	4591	5491	9451
1945	4915	5914	9514
1954	4951	5941	9541

Challenge: What fraction of the numbers is even? ($\frac{6}{24}$ or $\frac{1}{4}$) What fraction of the numbers is <u>not</u> divisible by 5? ($\frac{18}{24}$ or $\frac{3}{4}$)

Problem 4

Because 36 is the fifth number in the sequence, the fiftieth number is (45 x 7) + 36 = 351.

Solution: 351

Problem 5

20 + 21 + 22 + 23 + 24 + 25 + 26 = 161 too high
10 + 11 + 12 + 13 + 14 + 15 + 16 = 91 too low
11 + 12 + 13 + 14 + 15 + 16 + 17 = 98 closer
12 + 13 + 14 + 15 + 16 + 17 + 18 = 105

Solution: 12, 13, 14, 15, 16, 17, 18

Problem 6

0 0 1 □ 1 2 □ □ 2 3 □ □ □ 3 4 □ □ □ □ 4

4 □ □ □ □ 4
1 □ 1
3 □ □ □ 3
2 □ □ 2
0 0

Solution: Note that three solutions are given. More solutions are possible.
4131243200 2412134003 2342131400

Problem 7

	Amount spent	Amount remaining	
Fruit shop	\$16 ($\frac{1}{2}$)	\$16 ($\frac{1}{2}$)	
Butcher's shop	\$22 ($\frac{1}{3}$ + \$4)	\$32 ($\frac{2}{3}$ – \$4)	$\frac{1}{3}$ = \$18
Supermarket	\$70 ($\frac{1}{2}$ + \$8)	\$54 ($\frac{1}{2}$ – \$8)	$\frac{1}{2}$ = \$62
Start = \$124			

Solution: \$124; Explanations may vary. Sample response: I can prove my answer is correct by working forward. I start with my answer \$124; $\frac{1}{2}$ of that is \$62, plus \$8, so the Ground family spent \$70 in the supermarket and had \$54 left. $\frac{1}{3}$ of \$54 is \$18, plus \$4 is \$22; so they spent \$22 in the butcher's shop and had \$32 left. $\frac{1}{2}$ of \$32 is \$16; so they spent $\frac{1}{2}$ in the fruit shop and had \$16.

Problem 8

<u>Equation A</u>	<u>Equation B</u>	<u>Equation C</u>
F G = B T	G G G = B T	B = T T T T T T T T

<u>Step 1</u>: F G = B T

F G = T T T T T T T T T (from C)

<u>Step 2</u>: G G G = B T

G G G = T T T T T T T T T (from C)

<u>Step 3</u>: G G G = T T T T T T T T T

G = T T T

<u>Step 4</u>: F G = T T T T T T T T T (from *Step 1*)

F T T T = T T T T T T T T T (from *Step 3*)

<u>Step 5</u>: F T T T = T T T T T T T T T

F = T T T T T T

Solution: 6 tennis balls