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ALGEBRA IDEA 1

Introducing the Product of Two Negatives

Objective: To have students understand intuitively and abstractly why the product of two negatives is positive

Materials: A video camera and a videotape player

Procedure: Typically, a discussion of the product of two negative numbers follows a discussion of the product of a negative number and a positive number (which may be shown as the multiple addition of a negative number or by some other convenient method). This product generally poses no great difficulty for the teacher to demonstrate or for the students to understand.

It is more difficult to develop an analogue for the product of two negative numbers. We can show that the product of two negatives evolves from the pattern:

$$(-1)(+3) = -3$$
$$(-1)(+2) = -2$$
$$(-1)(+1) = -1$$
$$(-1)(0) = 0$$

Continuing the pattern seen by successive entries in the second and third numbers of each line gives the following:

$$(-1)(-1) = +1$$
$$(-1)(-2) = +2$$
$$(-1)(-3) = +3$$

How can students get a more genuine intuitive feel for this concept? Perhaps a "real-life" illustration will do.

Consider making a videotape recording of a clear plastic water tank that has a transparent drain tube that we know can empty (*negative*) the tank at the rate of 3 gallons per minute. We tape this event for several minutes, allowing the tank contents to lower. If we run the tape for 2 minutes, the tank will show a decrease of 6 gallons. For 3 minutes, the tank will have emptied 9 gallons.

Suppose we run the tape in reverse (*negative*) for 1 minute. The tank will have refilled by 3 gallons. At twice the normal rate in reverse (*negative*), the tank will gain (*positive*) in content at the rate of 6 gallons per minute. Here a student can see that the product of two negatives (emptying a tank and running a film in reverse) results in a positive.

If this is not sufficiently convincing, then perhaps tell students to consider **good guys** (positive) or **bad guys** (negative) who are **entering** (positive) or **leaving** (negative) a town:

1. If the **good guys** (+) **enter** (+) the town, that is **good** (+) for the town. $(+)(+) = +$
2. If the **good guys** (+) **leave** (–) the town, that is **bad** (–) for the town. $(+)(-) = -$
3. If the **bad guys** (–) **enter** (+) the town, that is **bad** (–) for the town. $(-)(+) = -$
4. If the **bad guys** (–) **leave** (–) the town, that is **good** (+) for the town. $(-)(-) = +$

This example gives students an intuitive feel for the product of two negatives. You, the teacher, should choose the approach most likely to succeed with a particular class.

For the more curious or gifted students, a proof of this concept might be in order. They should know that $(-1) + 1 = 0$. By multiplying both sides of this equation by –1, we get

$$(-1)[(-1) + 1] = (-1)[0]$$

Using the distributive property, we get

$$(-1)(-1) + (-1)1 = (-1)0$$

Because we know that

$$(-1)1 = -1 \text{ and } (-1)0 = 0$$

we then have

$$(-1)(-1) + (-1) = 0$$

By adding 1 to both sides of this equation (or by asking, "What must be added to (–1) to get 0?"), we get

$$(-1)(-1) = 1$$

which "proves" the relationship.

We might also like to reach back to another topic to validate further that the product of two negatives is a positive. Recall that $(a^x)^y = a^{xy}$, which holds true for $a = 2$, $x = y = -1$.

Therefore,

$$(2^{-1})^{-1} = 2^{(-1)(-1)}$$

By the definition of a negative exponent, we get

$$(2^{-1})^{-1} = \left(\frac{1}{2}\right)^{-1} = 2$$

We can then conclude that

$$2^1 = 2^{(-1)(-1)} \text{ or } 1 = (-1)(-1)$$

because if the bases are equal, so must the exponents be equal.

ALGEBRA IDEA 2

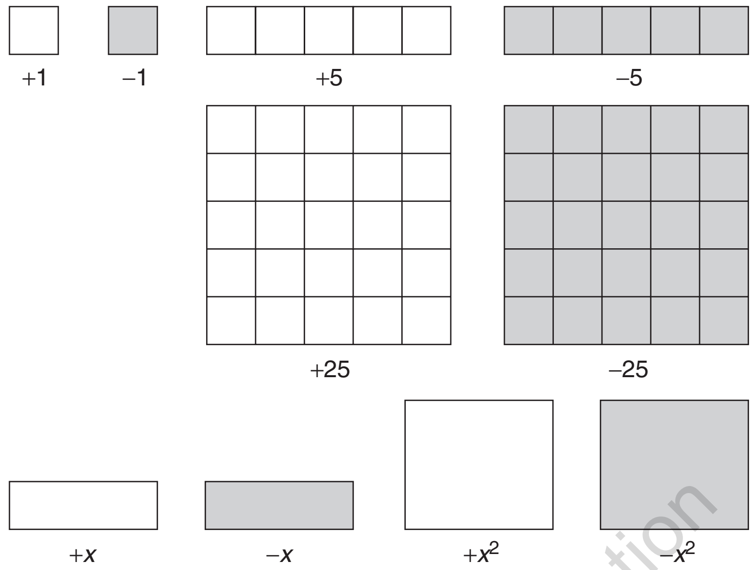
Multiplying Polynomials by Monomials (Introducing Algebra Tiles)

Objective: To examine the multiplication of polynomials by monomials geometrically

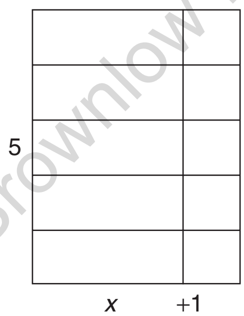
Materials: Algebra tiles

Procedure: In recent years, the use of algebra tiles has enhanced the teaching of abstract concepts and has helped to make working with polynomials more concrete. Depending on the abilities of your students, you may need an entire lesson to introduce the different algebra tiles and to familiarize your students with them. There are commercially made algebra tiles; however, you can make your own by duplicating the following diagrams on colored paper and cutting them out. Each student should have 10 of each type of tile. Here the nonshaded tiles represent positive numbers and the shaded tiles represent negative numbers.

The number associated with a rectangular tile represents its area and color. For example, the +5 tile has dimensions 5 by 1, and thus its area is 5. Where it is unshaded (clear), we use the number +5 to identify it. The –5 tile has a similar situation, except that it is shaded, and therefore we use the number –5 to identify it. Likewise, the $+x$ tile has dimensions x by 1, and its area is, therefore, x. Note that because we are *not* given the length of this tile, we denote it with an x. Note also that an integral number of units do not fit the length.



Pose the question, "How can we multiply $(x + 1)$ by 5 using algebra tiles?" If a rectangle is to be created with area $(x + 1) \cdot 5$, it should have length $(x + 1)$ and width 5. Have the students build such a rectangle with their tiles as follows:

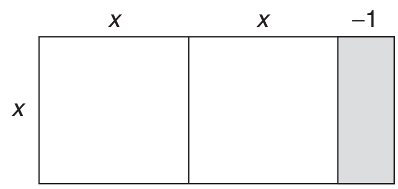


After building such a rectangle, have the students count the algebra tiles to find that the area associated with the rectangle is $5x + 5$. This shows that

$$(x + 1) \cdot 5 = 5x + 5$$

This is equivalent to the distributive law.

Now consider the product $(2x - 1) \cdot x$. The resulting rectangle is



Note that the tile on the right must be a $-x$ tile because the -1 is one side length. Adding the results represented by the three tiles gives

$$x^2 + x^2 + (-x) \text{ or } 2x^2 - x$$

It may be appropriate to show how the distributive law applies here as well.

Before expanding the demonstration to the product of binomials, have the students practice using the technique, especially with respect to signed numbers. Have students use algebra tiles to multiply

1. $(x - 3) \cdot 4$
2. $(2x - 2) \cdot (-1)$
3. $(3 - 2x) \cdot (3)$
4. $(-x + 5) \cdot (-5)$
5. $(x^2 - 3) \cdot (-4)$
6. $(x + 2x - 3) \cdot (3)$

Ask students which extra tiles are needed to consider products that involve the variables x and y.

ALGEBRA IDEA 3

Multiplying Binomials (Using Algebra Tiles)

Objective: To examine the multiplication of binomials by binomials geometrically

Materials: Algebra tiles

Procedure: The basis for this unit is to use the previous work on algebra tiles (see Algebra Idea 2) to multiply binomials. Students should be able to represent algebraically the area of a rectangle, whose sides represent binomials.

Consider the product $(x + 3)^2$. Some students may incorrectly give the answer $x^2 + 9$. To see the correct answer, suggest that students use their algebra tiles to construct a square, each of whose sides has length $x + 3$. The resulting square should look like