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Prologue

Effective mathematics teaching has, as its beginning, thoughtful preparation and, as its end, student mastery of the mathematics content. In the moment of instruction, such teaching requires that a teacher, taking into account the current mathematics development of their students, actively transform their plans and goals into student learning of substantial mathematics. However, there is nothing more difficult than imagining how such teacher actions might best be accomplished without seeing it being demonstrated. The purpose of this book is to give the reader a glimpse – through vignettes, abbreviated stories of teaching and learning, featuring the work of exemplary primary mathematics teachers – of the research-based strategies successful primary mathematics teachers use when they teach in a manner that is consistent with teaching practices and learning experiences, for example, those recommended by the National Council of Teachers of Mathematics (NCTM) in their *Principles and Standards for School Mathematics (PSSM).*[1]

Such teaching can be challenging work. The NCTM recommends teaching that, taking into account children's mathematical development, encourages mathematical questions, conjectures and explanations; ensures procedural competency; and develops deep mathematical understandings. However, many primary teachers and primary teacher candidates have not experienced such teaching or learning. This, as the following vignette[2] illustrates, may cause a cognitive disjunction[3] when teachers are called upon by their school region to teach a curriculum that requires many of the teaching practices and incorporates many of the learning experiences suggested by resources like the *PSSM:*

> Two year three teachers, Ms Kim and Mr Jackson, are in their first months of using a new mathematics textbook that incorporates many of the suggestions of the *PSSM.* For example, the lessons set forth have the potential to encourage students to make sense of, construct and compute their own addition problems and, furthermore, have the potential to intellectually engage students in mathematics problem solving.
>
> One morning Ms Kim is working on a coin problem from the new textbook with her class. Ms Kim asks the class, "I have 10 cent

coins, 20 cent coins and one dollar coins in my pocket. Suppose that I pull out three coins. How much money might I have?" One boy shouts out, "That's easy. Thirty cents!" "Good, Raymond," says Ms Kim, "although I don't want people to shout out answers. Now, put that in your notebook, and then think: What other solutions can you find?" The children set to work, finding solutions. After about 20 minutes, Ms Kim calls the class together and conducts a discussion of the problem. She records the different solutions that the students propose in a chart, ordered from the least amount to the most, at the same time modelling the invention of notation (10-10-20 for 10 cent, 10 cent, 20 cent; 10-20-20 for 10 cent, 20 cent, 20 cent; etc.) to organise and record the different coin combinations for each amount.

A few days later, Mr Jackson is on the same lesson. He assigns the same problem and gets, as the text suggests, his students working independently. A few students ask how many coins were in the person's pocket. Mr Jackson peers at the text. "I guess they don't tell you," he remarks. Several ask him for help, and he carefully shows them that they can combine coins to see what the amount is. "There are a few different answers," he tells his class, "so everyone should have, at least, two." As the children finish, they turn in their papers. He glances at several. They have answers written on them – different amounts of money on some, combinations of coins on others. He hands back a paper that has only one answer. "I told you two," he says to Danny. He stacks a practice addition worksheet next to the finished workbasket so that the students who finish more quickly will have something to do.

Ms Kim chooses to conduct her lesson in a manner similar to that recommend by *PSSM*. Mr Jackson, it appears, chooses to do the lesson in a different manner. This does not mean that Mr Jackson lacks mathematics content knowledge or pedagogical skills. However, as with many prep–5 teachers we have mentored and taught, Mr Jackson may not have had some of the relevant experiences that would prepare him to effectively teach a lesson of this sort.

We have found, in our own teaching and mentoring, that there are a number of learning experiences that might prepare teachers and teacher candidates to realise the suggestions of the *PSSM* and similar strategies in actual classroom practice. For example, there is a deepening engagement in the mathematics of prep–5; observation in exemplary prep–5 mathematics classrooms and video of such classrooms; and discussion of relevant case studies and education research. However, while observation of the teaching of mathematics often has one of the most significant impacts on teaching dispositions, there are often a limited number of prep–5 classrooms in any school or region that might be considered appropriate

exemplars of the relevant standards. In such instances, the thoughtful analysis of mathematics vignettes – that is, short reinterpretations of actual mathematics teaching and learning in the classroom – can provide a useful supplement.

One of the first collections of such vignettes was the Professional Standards for Teaching Mathematics[4]; however, the intervening years have seen a number of other collections, of which the *PSSM* is perhaps the most well known. A primary goal in writing this book was to consolidate certain collections to further illustrate how a teacher might teach prep–5 mathematics that exemplifies teaching practices that accord with the relevant standards. The book is designed to be an easy and ready reference for the prep–5 mathematics teacher. It consists of ten chapters that mirror much of the content and structure of the *PSSM*, but can be easily adapted to any relevant structure and standards. Each chapter presents a collection of teaching strategies organised into sub-chapters by year level bands – Years Prep–2 and Years 3–5 – concisely presented in a friendly format:

Year-Level-Band Introduction

We begin each year-level-band section with a short discussion of the chapter topic for that year level band. For example, we begin the prep to year two band of the chapter titled "Reasoning and Proof" by noting that students need to be encouraged by teachers to make sense of mathematics and to be guided by teachers to test whether their mathematical generalisations are correct. We begin the year three to five band of that chapter with a short discussion of students' prior understandings of proof and how teachers might use these understandings to guide students in the construction of acceptable forms of mathematical explanation.

Strategy

We give a simple and crisp statement of the teaching strategy being recommended.

Example Standard/s

We give an exact statement of a relevant example standards.

What the Research Says

We give, in most cases, a brief restatement of what the *PSSM* says about a particular example standard together with references to relevant research. This section is intended to simply give the teacher some confidence in, and a deeper understanding of, the particular teaching strategy being addressed.

Classroom Application

We present a vignette chosen to exemplify the recommended teaching strategy. These vignettes come from a variety of sources: NCTM publications (most particularly the *PSSM*), accounts of mathematics research, units for mathematics instruction, and published observations of mathematics classrooms. In many cases these vignettes are adaptations of the original source material (moderate adaptations are usually marked by pseudonyms that differ from those originally used). The vignettes necessarily lack much of their original context – planning, resources, layout of the classroom, and follow-up are, most usually, not depicted – and have been purposively refocused on the teaching strategy. However, every effort has been taken to preserve the substance of mathematics done by teacher and students and the character of student explanations and discussions.

Although the majority of the vignettes are set in classrooms, the first vignettes of the chapters on Numbers and Operations and Algebra purposefully feature the early learner child and purposefully are set in the home. There are two primary reasons for this choice. First, a young child's early experiences of mathematics significantly influence his or her later engagement with mathematics in the primary years.[5] Interactions with peers, siblings and caregivers are especially formative. Primary teachers need to be aware of the possibility of such experiences as they can use these for further in-school mathematics instruction and as a means of reinforcing and validating current and later mathematical experiences in the child's larger world.

Second, the mathematics that an early learner child needs to master can often be taken for granted by caregivers and teachers (including prep teachers). It is no minor task for a child to realise that the squiggle 9 somehow comes right after the squiggle 8 or to realise that there might be some benefit to arranging objects neatly so that they may be purposively counted. Further, it is important to realise that in the early years, such notions as cardinality – the notion that, beginning with one, the last sequential number tagged might be the total number of objects in a set – are often far from obvious.

Precautions and Possible Pitfalls

We specify, for each vignette, some of the possible pitfalls that need to be taken into consideration when implementing or extending that teaching strategy.

Sources

We provide references to relevant research and curriculum implementations so that readers can further investigate reasons for, and implementations of, a particular standard or strategy. Many of these references come from the *PSSM*. Where we felt these might not be sufficient, we have provided references from recent mathematics education research. We note that many of these later references are to articles in *A Research Companion to* Principles and Standards for School Mathematics,[6] which was explicitly written to accompany the *PSSM*.

As you read through the vignettes offered in this book, we hope you will thoughtfully reflect on the mathematics featured, the explanations and comments of the students, and the manner in which a teacher implements a particular strategy. We recommend, for example, that you do the featured mathematics. Mathematics is not a spectator sport – especially that of young children. You may also wish to compare what you read with your own experiences as a mathematics learner and as a mathematics teacher. Think about what you found then, or now see, as difficult, and remember that much of the mathematics that now seems obvious to you was, most likely, somewhat challenging in your primary school years. Above all, note that "Excellence in mathematics education requires equity – high expectations and strong support of all students."[7]

Notes

1. NCTM. (2000). *Principles and standards for school mathematics.* Reston, VA: NCTM.
2. Slightly adapted from Cohen, D. K., & Ball, D. L. (2001). Making change: Instruction and its improvement. *Phi Delta Kappan, 83,* 73–74.
3. See, for example, Feiman-Nemser, S., & Remillard, J. (1995). *Perspectives on learning to teach.* East Lansing, MI: Michigan State University.
4. NCTM. (1991). *Professional standards for teaching mathematics.* Reston, VA: NCTM.
5. NCES. (2000). *The kindergarten year.* Washington, DC. [On-line]. Available: http://nces.ed.gov
6. Kilpatrick, J., Martin, W. G., & Shifter, D. (Eds). (2003). *A research companion to* Principles and standards for school mathematics. Reston, VA: NCTM.
7. NCTM. (2000). *Principles and standards for school mathematics.* Reston, VA: NCTM.