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Introduction

Building the 21st-Century Mathematics Classroom

Imagine yourself as a year two student. In mathematics, you're adding, subtracting, multiplying, comparing fractions, and reading some basic graphs. Someone asks you, "Do you like maths?" What would you say? Flash forward to year six. The mathematics you're learning certainly has advanced, but so has your mind. How do you think you'd answer the question, "Do you like maths?" Now, jump another four years. It's year ten, and you're working your way through geometric proofs. Once again, you're asked that simple question, "Do you like maths?" What's your answer this time?

The fact is, studies show a disturbing trend in which "students in secondary school become increasingly less positive with regard to their attitude toward mathematics and their beliefs in the social importance of mathematics" (Wilkins & Ma, 2003, p. 58). For many students, this negative attitude becomes full-blown "maths anxiety," an almost compulsive dislike of mathematics and mathematics instruction that emerges around year four, reaches its peak in secondary school (Scarpello, 2007), and sounds like this:

> Nothing made sense, I felt sick to my stomach, and I could feel the blood draining from my face. I had studied so hard, but it didn't seem to make any difference – I barely even recognized the math problems on the page. When the bell rang and my quiz was still blank, I wanted to disappear into my chair. I just didn't want to *exist.* (McKellar, 2008, p. xv)

These are the words of Danica McKellar, the actress who played Winnie Cooper on television's *The Wonder Years* and the author of *Math Doesn't Suck: How to Survive Middle School Math Without Losing Your Mind or Breaking a Nail* (2008). While McKellar may be unique in that she became a famous actress before she was a teenager, her experiences as a secondary mathematics student are, sadly, all too common. For example, the classroom research that I have conducted with teachers and students over the last several years indicates that in years three and four, almost 80% of students have positive attitudes toward mathematics and feel confident in their ability to succeed in mathematics. But as the mathematics curriculum becomes more difficult, more abstract, and more algebraic in the middle years, the numbers change dramatically. By year seven in secondary school, almost 50% of all students have developed an aversion to mathematics; they don't like it, they don't believe they're good at it, and many of them are proud to declare that they plan on taking the fewest number of mathematics courses possible in secondary school and beyond. As the secondary school mathematics curriculum progresses through algebra, geometry and trigonometry, the numbers get worse. This means that well over half of our students leave secondary school entertaining the dangerous idea that mathematics is a special realm for mathematicians and engineers, inscrutable to the average person and unnecessary for success in life.

This idea should give secondary school teachers of mathematics the shivers. We know that mathematics is at the heart of so many things that affect everyone, from economics to technology, from the complexities of global marketing to the simple act of purchasing groceries. Mathematics, as Howard Gardner (1983, 1999, 2006) has shown us, is a vital form of human intelligence. Mathematics opens up career paths, empowers consumers, and makes all kinds of data meaningful – from football statistics to political polls to the latest trends in the stock market. Quite simply, we cannot afford to have so many secondary students who dread maths class. We cannot allow the majority of our students to walk into a fast-moving, technological society looking to avoid confrontations with mathematics. For if we send an army of maths-haters out into today's competitive global culture, we are shortchanging millions of students by severely limiting their chances of future success.

And yet, I have met many teachers of mathematics who are wondering openly if students really can be successful. "These kids hardly know basic mathematics. How can they be expected to do well in advanced algebra?" is a common refrain from teachers in our middle years. So, what is the truth? Do we believe our students can be successful in mathematics, or is the situation hopeless?

The good news is that research and experience both show that students' attitudes toward maths and their problem-solving abilities are not fixed in place. In my 35 years of work in schools across America, I have seen some truly remarkable changes in the way secondary school students perceive mathematics and their ability to succeed in it. For example, I recently had the pleasure of working with a group of mathematics teachers in Old Bridge, New Jersey. Together, we crafted a different kind of pre-algebra unit on three-dimensional figures. Instead of a test, we decided to build the unit around a summative assessment task (see Figure i.1 on page 3). This task required students to demonstrate just about everything they learned during the unit while also encouraging them to apply mathematics creatively. And in designing an instructional sequence to build the knowledge and skills students would need to succeed on the summative assessment task, we employed a variety of research-based strategies – strategies that we selected specifically for their power to pique students' curiosity, actively engage students in learning, and speak to different styles of learners in the classroom.

When the teachers implemented their units in the classroom, the change in students' attitudes was palpable. Students were curious. They asked questions. They pursued difficult problems with vigour. Best of all, more students succeeded. In three of the four classrooms where the strategies were used, test scores rose by a significant amount. By taking the time

Final Task: A Monument to Learning

MathsCorp has commissioned you to design and sketch a monument for a new maths garden. The garden will have different sections, including sections devoted to important mathematicians and famous number sequences. The section of the garden they have asked you to design is the three-dimensional figure section.

Your task is to design and sketch a monument for the garden. The monument will be constructed of solid marble and must meet the following criteria:

1. In your design, you may only use the three-dimensional figures we learned about during our unit: *triangular prism, rectangular prism, triangular pyramid, rectangular pyramid, cylinder* and *cone*.
2. You must include at least one of each of these three-dimensional figures in your design.
3. You must calculate the *volume* of your monument and show your work.
4. You must identify the total number of *bases, faces, edges* and *vertices* within your monument.
5. You must include with your sketch a brief explanation of the thinking that went into your design. In your explanation, you must include at least 10 critical vocabulary words from our investigation into three-dimensional figures and their volume.

FIGURE i.1 Summative Assessment Task

Source: Thoughtful Education Press. (2009). *Math Tools for Three-Dimensional Figures.* (Curriculum guide designed for the teachers of Old Bridge, New Jersey).

to engage students in the mathematics, the students were charged with learning; we also improved their comprehension, retention and achievement levels.

This kind of change, of course, comes from teachers. And on this point, the research is sparklingly clear. A recent study tracking 3000 year seven students through secondary school, for example, demonstrates that "teachers' choices of activities and mathematics problems can have a strong impact on the values that are portrayed in the classroom and on how students view mathematics and its usefulness" (Wilkins & Ma, 2003, p. 59).

So, how do secondary school mathematics teachers use this critical time to engage and motivate more students to meet the new and higher demands of the 21st century, not to mention the challenges of expanding curriculums, state and national standards, school report cards, and greater expectations from universities, government and the public? The answer can be summed up in two simple but deep principles that drive this book and Ed Thomas's, John Brunsting's and Pam Warrick's work in mathematics in general:

Effective mathematics instruction is strategic.

Effective mathematics instruction engages *all styles* of learners.

PRINCIPLE ONE: EFFECTIVE MATHEMATICS INSTRUCTION IS *STRATEGIC*

In what are two of the most comprehensive studies of the research behind various teaching strategies and their impact in the classroom, Robert Marzano (2007) and Robert Marzano, Debra Pickering and Jane Pollock (2001) demonstrate conclusively that teaching strategies have a real and pervasive effect on student learning. Indeed, the evidence is clear: classroom strategies like comparing and contrasting, developing and testing hypotheses, working cooperatively, creating visual representations, organising information graphically, and using higher-order questions result in better performance and deeper learning among students. But as most teachers know, asking students to compare and contrast two different types of chemical mixture problems, for example, or having students work cooperatively to solve a particularly rigorous problem may not result in the kinds of deep learning the research points to. It is in moments like these – when we apply research-based techniques only to experience a roomful of blank faces when what we were expecting was active engagement – that the gap between research and practice seems wider than ever. So, the question becomes, "How can I put this research

IN THE CLASSROOM, PART I

Situation: Bonnie Cruz has been teaching her students how to solve quadratic equations for the past week. Each day, Bonnie reviews the process, answers questions, provides in-class practice time and assigns appropriate homework. She believes there is not much more she can do. Yet, when her students return to class, Bonnie finds they are still making many of the same mistakes. She is ready to test, move to the next unit, and admit that some of her students will never become fully proficient in solving quadratic equations.

Applying a strategy: If Bonnie had a working knowledge of how and when to use teaching strategies for mathematics, she might have incorporated the Convergence Mastery strategy into her teaching. This strategy applied to Bonnie's situation would work as follows.

Once Bonnie realised that her students had reached an apparent plateau of proficiency, she would inform her students that they were going to participate in an engaging activity. She would prepare a series of five short quizzes on solving quadratic equations using three different methods (see Figure i.2). Before each quiz, students would work cooperatively for 5 minutes to review and perfect the different ways of solving quadratic equations. Then, all students would be required to take the first quiz.

Quiz 1	Quiz 2	Quiz 3
Solve the equation by graphing.	*Solve the equation by graphing.*	*Solve the equation by graphing.*
1. $d^2 + 6d + 8=0$	1. $z^2 + 4z + 3=0$	1. $c^2 + 5c + 4=0$
Solve each equation by factoring.	*Solve each equation by factoring.*	*Solve each equation by factoring.*
2. $y^2 - y=12$	2. $m^2 - 5m =6$	2. $p^2 + p=20$
3. $3t^2 + 4t =0$	3. $18u^2 - 3u =1$	3. $3x^2 - 5x =2$
Solve each equation by completing the square.	*Solve each equation by completing the square.*	*Solve each equation by completing the square.*
4. $n^2 + 8n - 84=0$	4. $p^2 +10p +9=0$	4. $n^2 + 6d + 8=0$
5. $z^2 + z - 3=0$	5. $t^2 + 5t + 3=0$	5. $b^2 - 3b - 1=0$

Quiz 4	Quiz 5
Solve the equation by graphing.	*Solve the equation by graphing.*
1. $n^2 - 3n=0$	1. $x^2 - 2x - 3=0$
Solve each equation by factoring.	*Solve each equation by factoring.*
2. $r^2 + r=30$	2. $d^2 - 3d =4$
3. $bc^2 - c=2$	3. $2w^2 - 3w =0$
Solve each equation by completing the square.	*Solve each equation by completing the square.*
4. $x^2 - 6x + 5=0$	4. $a^2 - 4a - 12=0$
5. $r^2 - 3r + 1=0$	5. $z^2 + 3z - 8=0$

FIGURE i.2 Five Short Quizzes

At the end of the first quiz, students would cooperatively mark their solutions with Bonnie's help. Students who scored 100% would become permanent tutors and helpers and would exit the quiz-taking portion of the activity. Students who scored less than 100% would work cooperatively with the tutors and helpers to find their mistakes, correct them and prepare for the next quiz. This process would continue until all five quizzes were taken. Since 100% success on a quiz is equivalent to an A in the grade book, students are highly motivated to communicate with each other, work cooperatively and work hard to eliminate errors so they can take advantage of the immediate help and "retake" opportunities. As students progress through this process, they converge toward mastery.

into classroom practice so that it leads to a positive change in student learning?" To answer this question, let's look in on a classroom.

What effect do you think Convergence Mastery would have in your classroom? Do you think students' mastery of the equation-solving process would improve as a result of the strategy?

Let's look in on another classroom where the students are having a different kind of problem.

IN THE CLASSROOM, PART II

Situation: Robert Gould is trying to curb his students' impulsivity as problem solvers. Too often, when Robert's students are faced with difficult word problems, they will jump to solutions rather than engage in quality, pre-solution thinking and planning. This is especially worrisome to Robert because he knows that nearly one half of the items on his state's mathematics test are problems that students need to set up themselves.

Applying a strategy: Robert selects the strategy known as Math Notes because it is designed specifically to help students:

1. Identify the facts of the problem;
2. Determine exactly what the problem is asking;

(Continued)

(Continued)

3. Represent the problem visually; and
4. Plan out the steps that need to be taken to solve the problem.

He begins by presenting a classic word problem whose solution seems obvious at first glance:

> *"Bookworm Problem"*
>
> *Volumes One and Two of a two-volume set of math books are next to one another on a shelf in their proper order (Volume One on the left, Volume Two on the right). Each front and back cover is ¼ inch thick and the pages portion of each book is 2 inches thick. If a bookworm starts at page one of Volume One and burrows all the way through to the last page of Volume Two, how far will the bookworm travel?*

Next, he asks students to take a minute and try to solve the problem as they normally do. As Robert suspects, nearly all the students answer impulsively, coming up with either 5 inches (2½ inches for each book times 2) or 4½ inches (2½ inches for each book minus ½ inch for the front and back cover). That's when Robert introduces and models Math Notes. Using the same problem, Robert shows students how he thinks through and sets up the problem on a Math Notes organiser (Figure i.3).

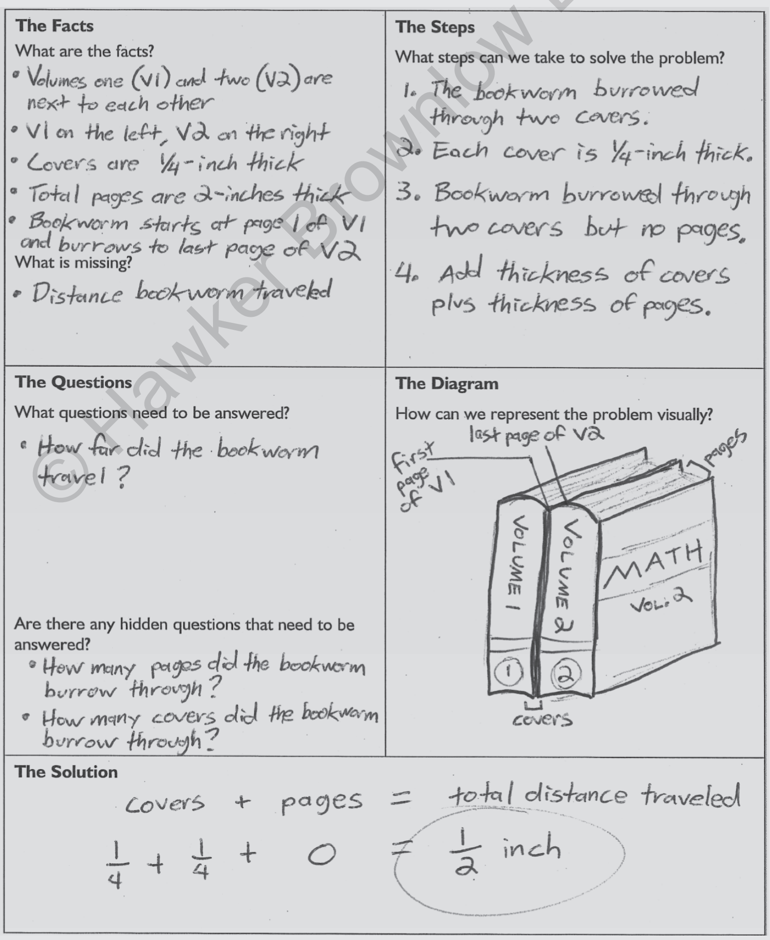

FIGURE i.3 Completed Math Notes Organiser

What students see very clearly as a result of Robert's use of Math Notes is that, without a strategy for breaking down, attacking and visualising difficult word problems, they are likely to miss essential information or misinterpret what the problem is asking them to do.

"Now," Robert tells his students, "let's try this strategy out on the uniform-motion problems that many students have been struggling with."

Over the course of the year, students keep a notebook of problems they've solved using Math Notes. This way, they can refer back to their notebooks and look for models they can use whenever they come across new problems.

Convergence Mastery and Math Notes are only 2 of the 21 research-based teaching strategies that Ed Thomas, John Brunsting and Pam Warrick lay out in this book. Convergence Mastery is, as its name suggests, a Mastery strategy – a strategy focused on helping students remember mathematical procedures and practise their computational skills. But mathematics, of course, is about more than memory and practice. It is also about asking questions, making and testing hypotheses, thinking flexibly, visualising concepts, working collaboratively and exploring real-world applications. To accommodate this cognitive diversity, the strategies in this book are broken up into five distinct categories. Four of these categories – Mastery, Understanding, Self-Expressive and Interpersonal – develop specific mathematical skills. The fifth category, Multistyle strategies, contains strategies like Math Notes, strategies that foster several kinds of mathematical thinking simultaneously. The following map (Figure i.4) explains these five categories.

Mastery strategies help students ***remember*** mathematical content and ***procedures*** and ***practise*** their ***computational skills.***

Interpersonal strategies help students ***discuss*** mathematical ideas, ***collaborate*** to solve problems and ***explore the human connections*** to mathematical content.

Multistyle strategies ***combine*** the thinking of Mastery, Understanding, Self-Expressive, and Interpersonal strategies to help students become ***complete, multifaceted problem solvers.***

Understanding strategies help students ***uncover*** and ***explain*** the ***principles*** and ***big ideas*** behind the mathematics they study.

Self-Expressive strategies help students ***visualise*** mathematics and ***think flexibly*** and ***creatively*** to solve ***non-routine problems.***

FIGURE i.4 Map of Mathematical Strategies

Each of the strategies in these five categories represents a different kind of thinking, a different way of interacting with mathematical content, a different opportunity to grow as a learner and problem solver. Take just one of these ways of thinking away, and you really don't know mathematics. Think about it: If you can't compute accurately (Mastery), explain mathematical concepts (Understanding), find ways to solve nonroutine problems (Self-Expressive) or explore and discuss real-world applications with fellow problem solvers (Interpersonal), then you don't have the complete picture; and without a complete picture, you don't *really* know mathematics. This simple but often-overlooked idea – that mathematical learning and problem solving require the cultivation of different kinds of thinking – brings us to the second way that this book will help you and your students achieve higher levels of success: *learning styles.*

PRINCIPLE TWO: EFFECTIVE MATHEMATICS INSTRUCTION ENGAGES *ALL STYLES* OF LEARNERS

Let's listen in on two secondary students who were asked the same question:

"Who was your favourite mathematics teacher and why?"

> **Alisha:** *My favourite maths teacher so far has definitely been Ms Tempiano. She really taught, and by that I mean she was very clear about explaining what we were learning and always showed us exactly how to do it. Whenever we learned a new skill or a new technique, not only would she review the steps, she would work with us to develop a way to help us remember how to apply the steps, like the acronym "Please Excuse My Dear Aunt Sally" for remembering the order of operations. Once we knew the steps, she would let us practise the steps with different problems. Sometimes we practised alone, and sometimes we practised in groups, but Ms Tempiano always walked around the room and worked with us like a coach. I loved getting feedback right away. That really helped me when she would walk around and watch what we were doing and help us with any problems we were having.*
>
> **Ethan:** *I didn't really think I liked or was good at maths before I had Mr Hollis for Algebra. He did this thing called "Problem Solving Fridays." Every Friday, we focused on what he called "nonroutine" problems, which were basically these really cool problems about things like building bridges or developing a new lottery game, problems that didn't have simple answers. So, we had to experiment, try different things out – you know, get creative – to see how we might be able to find a solution. Actually, I knew I would like Mr Hollis on the first day of class.*
>
> *I was in year seven and maths was first period. I walked in expecting the same old thing: worksheets, the odd problems, quizzes. But instead, Mr Hollis spent the first day on metaphors! He challenged us to create a metaphor for the problem-solving process. I showed how each step in the problem-solving process was like one of the stages in human digestion. It was really cool – I showed how you "chew" and "breakdown" and "process" both equations and food. The class loved it. And you know what else? I never forgot the steps in solving equations after that.*

Almost immediately, we can see that Alisha and Ethan treat mathematics very differently. Alisha is attracted to problems that have clear solutions.