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Preface

Over the years, problem solving has emerged as one of the major concerns at all levels of school mathematics. In fact, "learning to solve problems is the principal reason for studying mathematics" (NCSM 1977, 1). In more recent years, it has been noted of the upper primary years, "The goal of school mathematics should be for all students to become increasingly able and willing to engage with and solve problems" (NCTM 2000, 182). We are in complete agreement! In fact, we would go one step further—we feel that problem solving is not only a skill to be engaged in mathematics, but also a skill that will be carried over to everyday issues and serve a person well throughout life.

In many cases, students seem to feel that a mathematics problem can be solved in only one way, specific to the *type* of problem being taught (i.e. age problems, motion problems, mixture problems, and so on). Students often feel that some computational procedure or formula is the only approach that will *work.* Where does this misconception come from?

In fact, it is often the teachers themselves who are not aware of the many problem-solving strategies that can be used to provide efficient and elegant solutions to many problems. As Cindy, a prospective primary school teacher, puts it,

> One thing I realised was that in high school we never learned the theories behind our arithmetic. We just used the formulas and carried out the problem solving. For instance, the way I learned permutations was just to use the factorial of the number and carry out the multiplication. (Ball 1988, 10)

Cindy and her peers unconsciously convey to their students the notion that problems can be solved using only one particular computational approach. Although we would agree that the standard algorithms are powerful tools, unless students can solve problems, such tools are of little use (NCTM 2000). This book is a result of our many years of efforts to make teachers and students aware of this most important aspect of teaching mathematics. The book is designed for the classroom teacher who has a sincere desire to help students succeed as problem solvers both in mathematics and beyond. This is not to say that the book cannot be used by students directly; quite the contrary! However, its *tone* is directed to the teacher, who "can help students become problem solvers by selecting rich and appropriate problems, orchestrating their use, and assessing students' understanding and use of strategies" (NCTM 2000, 185).

In this book, we examine a number of the strategies that are widely used in problem solving, in both mathematics and real-life situations. In the mathematics classroom, these strategies provide an alternate plan for resolving many problem situations that arise within the curriculum. We have selected, within year-level bands, several examples to illustrate each strategy, realising that teachers will wish to apply these strategies to their regular instructional program. To do this, we recommend a careful review and study of the examples provided for each strategy so that the strategy eventually becomes a genuine part of the teacher's thinking processes or, one might say, a part of his or her arsenal of problem-solving tools.

Although it is true that many of these examples can be solved using some formula, such a purely *mechanical* approach often masks some of the efficiency, beauty and elegance of the mathematics. In many cases, the problem-solving strategies presented make the solution of a problem much easier, much *neater,* much more understandable, and thereby enjoyable!

Throughout the book, we try to show how each of these strategies occurs and ought to be used consciously in real-life situations. Many people already make use of these strategies without realising it. This carryover into life outside of the school adds importance to the mathematics our students study, and ultimately will improve their everyday performance. We believe that you and your students alike can profit from a careful reading of (and working along with) this book. As you examine each problem, take the time to solve it in any way you wish, or perhaps in a variety of ways. Compare your solutions to those provided. (Naturally, we welcome any clever alternatives to those in the book.) Most importantly, try to absorb the impact of the application of the problem-solving strategies and how they contribute to the beauty and power of mathematics. All the better if you can carry over this motivated feeling to your students.

Understand our feeling that problem solving must be the cornerstone of any successful mathematics program. Then try to infuse this same enthusiastic feeling and attitude in your daily teaching. This concentrated effort will make you a better problem solver and in turn help your students to become better problem solvers also. Not only will their attitude toward mathematics improve, so will their skills and abilities. This is our ultimate goal.

Problem 4.9 (Years 6–7)

Mrs Edwards' class was given the following problem:

In parallelogram *ABCD*, a point, *E*, is selected anywhere on side *CD*. Line segments *AE* and *BE* are drawn as shown in Figure 4.2.

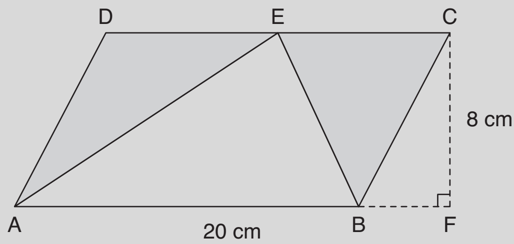

Figure 4.2

Side *AB* equals 20 cm and the altitude *CF* = 8 cm.
Which has the greater area, Triangle *ABE* or Triangle *AED* + Triangle *BEC?*

Solution: Let's look at this problem and see if we can make it simpler and yet equivalent. Because the location of point *E* was not specified, other than that it must lie on *CD,* we can place it anywhere we wish. Let's move it to coincide with point *C* (see Figure 4.3).

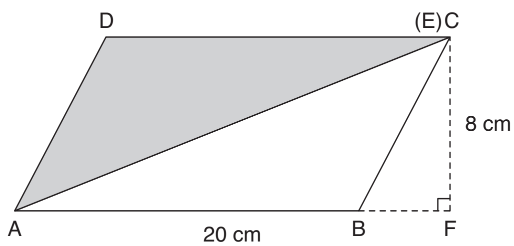

Figure 4.3

Now the two shaded triangles become one, and segment *AE* is a diagonal. Thus, the shaded triangles are half of the original parallelogram, or 40 square centimetres.

Some students may attempt to find the area of the three triangles. This will lead to a correct answer, but it depends on the student realising that the bases of triangles *DEA* and *CEB* add up to the side *CD* or 20. Furthermore, the altitude of each of the three triangles = 8 because it lies between parallel line segments. Other students may find the area of the parallelogram and the area of triangle *AEB* and subtract. Either solution, although more complicated than that given above, would be correct.