100 Commonly Asked Questions in Maths Class

Answers That Promote Mathematical Understanding, Years 6–12

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1 General Questions

1. WHY DO I HAVE TO LEARN MATHEMATICS?

The "why" question is perhaps the one encountered most frequently. It is not a matter of if but when this question comes up. And after secondary school, it will come up again in different formulations. (Why should I become a mathematics major? Why should the public fund research in mathematics? Why did I ever need to study mathematics?) Thus, it is important to be prepared for this question. Giving a good answer is certainly difficult, as much depends on individual circumstances.

First, you should try to find an answer for yourself. What was it that convinced *you* to study and teach mathematics? Why do *you* think that mathematics is useful and important? Have *you* been fascinated by the elegance of mathematical reasoning and the beauty of mathematical results? Tell your students. A heartfelt answer would be most credible. Try to avoid easy answers like "Because there is a test next week" or "Because I say so," even if you think that the question arises from a general unwillingness to learn.

It is certainly true that everybody needs to know a certain amount of primary mathematics to master their life. You could point out that there are many everyday situations where mathematics plays a role. Certainly one needs mathematics whenever one deals with money – for example, when one goes shopping, manages a bank account or makes a monthly budget.

Nevertheless, we encounter the "why" question more frequently in situations where the everyday context is less apparent. When students ask this question, it should be interpreted as a symptom indicating that they do not appreciate mathematics. This could have many reasons, but the common style of instruction certainly has an influence. Are we really doing our best to make sure that education in the classroom becomes an intellectually stimulating and pleasurable experience? If lessons can be attended without fear of failure and humiliation, it is more likely that mathematics gets a positive image. Try to create an atmosphere where curiosity will be rewarded and where errors and mistakes are not punished but rather are welcomed as a necessary part of acquiring competence. It is most important that you always try to enrich your lessons with interesting facts, examples and problems showing that mathematics is fascinating and intriguing, and well worth the effort.

It is wrong to say that studying mathematics should always be fun and entertaining. Mathematics, like most worthwhile things, requires a great deal of effort to master. Mathematics is the science of structured thinking, logical reasoning and problem solving. It requires commitment and time to acquire these skills. To solve a given problem, students have to concentrate on a task, devote attention to details, keep up the effort for some time and achieve understanding. But the students will be rewarded: Practising precise logical thinking as well as learning many problem-solving techniques will be useful for many different situations in many different aspects of one's life.

Often, the "why" question originates in a basic misunderstanding of what mathematics is about. The basic fact is that mathematics is useful because it solves problems. In fact, it has been developed for at least 4000 years to solve problems of everyday life. In early times, mathematics was needed for trading, managing supplies, distributing properties, and even describing the motion of the stars and planets to create calendars and predict seasons for agricultural and religious activities.

Over time, the scope of the problems that can be solved by mathematics has widened considerably, and presently it encompasses all fields of human knowledge. Mathematics is not only useful for measurements and statistics; it also, in particular, is needed to formulate and investigate the laws of nature. With the help of computer technology, mathematics can deal even with very complicated real-world problems. Therefore, mathematical models provide us with useful and vital information about climate change, economic trends and predictions, financial crises, movements of the planets and the workings of the human body, to name a few. Mathematics has played a major role in many technical developments; a few more recent examples are space exploration, CD players, mobile phones, Internet technologies (e.g. the compression algorithms used for storing music, pictures and movies), and global positioning system technology for navigation.

So, one of the main reasons to learn mathematics is that it is useful. Today it is more useful than ever before, and it is of importance to more fields of knowledge than ever before. Correspondingly, mathematics is used in many different jobs by scientists, engineers, computer programmers, investment bankers, tax accountants and traffic planners, to name just a few.

Refusing to learn mathematics would mean closing off many career opportunities. Students will perhaps understand that it is important to keep their options open.

However, the comparatively simple mathematics problems prevailing in school are bound to create a wrong impression of the importance of mathematics in the modern world. For students, it may be impossible to understand what calculus has to do with meteorology or risk analysis or automotive engineering, unless you make some attempts to explain these connections. You should point out, for example, that the derivative of a function could be used to describe a rate of change. Differential calculus would be needed where we need precise information about the rate of change of some observable quantity. It should therefore become clear that even the basics of science and technology cannot be understood without a solid background in maths.

There is a final, and perhaps the most important, reason why one should learn maths, although it is difficult to communicate: Mathematics is a huge, logically and deductively organised system of thought, created by countless individuals in a continuous collective effort that has lasted for several thousand years and still continues at breathtaking pace. As such, mathematics is the most significant cultural achievement of humankind. It should be a natural and essential part of everyone's general education.

2. IS THERE A LANGUAGE CONNECTION BETWEEN MATHEMATICAL TERMS AND COMMON ENGLISH WORDS?

Many mathematical terms are seen by students as words whose definitions must be memorised. Students rarely see applications of these words outside their mathematical context. This is akin to having someone learn words from another language simply for use in that language and then avoiding tying the words back to their mother tongue, even when possible. To learn the meanings of the frequently used mathematical terms without connecting them back to common English usage deprives students of a genuine understanding of the terms involved and keeps them from appreciating the richness and logical use of the English language.

The term *perpendicular*, which everyone immediately associates with geometry, is also used in common English, meaning "moral virtue, uprightness, rectitude". The rays that emanate from the centre of a circle to its circumference take on a name closely related, *radius*.

The rectangle is a parallelogram that stands erect. And the right angle, whose German translation is *rechter Winkel,* shows us the connection to the word rectangle. By itself, the word *right* has a common usage in the word *righteous,* meaning "upright, or virtuous".

The word *isosceles* typically is used in connection with a triangle or a trapezium, as in *isosceles triangle* and *isosceles trapezium*. The word *isosceles* evolved from the Greek language, with *iso* meaning "equal" and "skelos" meaning "leg". The prefix *iso* is also used in many other applications, as in isomorphism (meaning equal form or appearance, and in mathematics, two sets in a one-to-one onto relation that preserves the relation between elements in the domain), *isometric* (meaning equal measure), or *isotonic* (in mathematics sometimes used as isotonic mapping, referring to a monotonic mapping, and in music meaning that which is characterised by equal tones), just to name a few.

The word *rational* used in the term *rational number* (one that can be expressed as a ratio of two integers) means "reasonable" in our common usage. In a historical sense, a rational number was a "reasonable number". Numbers such as $\sqrt{2}$ were not considered as "reasonable" in the very early days of our civilisation, hence the name irrational. The term *ratio* comes from the Latin *ratio,* meaning "a reckoning, an account, a calculation" from where the word rate seems to emanate.

It is clear, and ought to be emphasised, that natural numbers were so named because they were the basis for our counting system and were "nature's way" to begin in a study or buildup of mathematics. Real numbers and imaginary numbers also take their mathematical meanings from their regular English usage.

Even the properties called *associative, commutative* and *distributive* have the meaning that describes them. The associative property "associates" the first pair of elements together (out of three elements) and then associates the second pair of these three elements. The commutative property changes position of the first element of two to the second position, much as one "commutes" from the first position (at home) to the second position (at work), and at the end of the workday, reverses the "commute" from work back to home. The distributive property "distributes" the first element to the two other elements.

It is clear that when we speak of *ordinal numbers,* we speak of the position or order of a number, as in first, second, third and so on. The *cardinal numbers* refer to the size or magnitude of a number – that is, its importance. In this sense, one might say that the larger the number is, the more "important" it is. In English, we refer to something of "cardinal importance".

From the word *complete,* we get the term *complementary,* as a complementary item completes something. In its most common usage, an angle is the complement of another if it "completes" a right angle with it.

Once the prefix *con* meaning "with" is understood, terms such as *concentric* (circles having the same centre), *coplanar* (in the same plane) or *concyclic* (points lying on the same circle) are easily defined. So just pointing out to students these small hints gives the mathematics terms more meaning.

The word *cave,* a hollow in a mountain, is related to the term *concave*, and one can draw a similar mental picture. The term *convex*, stemming from the Latin *convexus,* meaning "vaulted, arched, . . . drawn together to a point," refers to the opposite of concave.

Some words speak for themselves, such as a *bisector*, which sections something into two (equal) parts. The use of the prefix *bi* in mathematics is usually clear. For example, we have *biconditional* (conditional in two directions), *binomial* (having two names or terms) or *bilateral* (two sided). The word *triangle* also is self-descriptive: "three-angled polygon".

When one considers the definition of *factor* in its common English usage (i.e. "one of the elements contributing to a particular result or situation"), the mathematical definition of factor becomes clear (i.e. one of the numbers multiplied to get a product).

If we stretch the imagination just a bit, the term *fraction* also makes sense. It comes from the Latin *fractio,* meaning "a breaking in pieces", which is what a fraction represents: a piece. In some languages, such as German, a language with roots similar to those of English, the word for *fraction* is "Bruch", which also means "a break or piece", just as the Latin derivation of fraction does. When we fracture a bone, we break the bone.

In making students aware of the words used in mathematics, you also should make them aware of the prefixes that indicate magnitude, such as poly, bi, semi, tri, quad, pent, hex, sept, oct, non, dec, dodec and so on. These prefixes, when combined with suffixes such as gon, hedron and so on, allow students to determine a word's meaning.

Not to be overlooked is the term *prime number*, since it stems from the true definition of the word prime. As in the word *primitive,* referring to the basic elements, in a mathematical sense a prime number is one of the basic numbers from which, through multiplication, we build the other numbers.

Whenever a new word or term is introduced in mathematics, it should be related back to common English usage. This may require having a good dictionary at ready reference. The time it takes to tie mathematical terms back to ordinary English will help strengthen the mathematical understanding as well as enlarge a student's regular vocabulary. It is time well spent!

3. HOW MANY LEAVES ARE ON A TREE?

At first sight, this question does not seem to have much to do with mathematics. But it is about counting, and this is where mathematics starts. There is no need for special knowledge in biology, but it could be helpful to join forces with the biology teacher when attacking a problem like this in class.