Contents

Preface

Mathematics Assessment Probes

OVERVIEW

Formative assessment informs instruction and supports learning through a variety of methods and strategies aimed at determining students' prior knowledge of a learning target and using that information to drive instruction that supports each student in moving toward understanding of the learning target. Questioning, observation and student self-assessment are examples of instructional strategies educators can incorporate to gain insight into student understanding. These instructional strategies become *formative assessment* if the results are used to plan and implement learning activities designed specifically to address the specific needs of the students.

This book focuses on using short sets of diagnostic questions, called Mathematics Assessment Probes (Probes). The Probes are designed to elicit prior understandings and commonly held misunderstandings and misconceptions. This elicitation allows the educator to make sound instructional choices targeted at a specific mathematics concept and responsive to the specific needs of a particular group of students.

> Diagnostic assessment is as important to teaching as a physical exam is to prescribing an appropriate medical regimen. At the outset of any unit of study, certain students are likely to have already mastered some of the skills that the teacher is about to introduce, and others may already understand key concepts. Some students are likely to be deficient in prerequisite skills or harbor misconceptions. Armed with this diagnostic information, a teacher gains greater insight into what to teach. (McTighe & O'Connor, 2005, p. 68)

The Mathematics Assessment Probes provided in this resource are tools that enable teachers in Years 3 to 5 to gather important insights in a practical way and that provide immediate information for planning purposes.

AUDIENCE

The first collection of Mathematics Assessment Probes and the accompanying Teacher Notes were written for the busy classroom teacher eager for thoughtful, research-based, diagnostic assessments focused on learning difficulties and aimed at enhancing the effectiveness of mathematics instruction. Since the publication of the first three *Uncovering Student Thinking in Mathematics Resources* books (Rose & Arline, 2009; Rose, Minton & Arline, 2007; Rose Tobey & Minton, 2011), we have continually received requests for additional Probes. Both teachers and education leaders have communicated the need for a collection of research-based Probes that focus on a narrower year-level span. In addition to additional Probes for each year-level span, educators were eager for an alignment of the Probes to mathematics curriculum standards. In response to these requests, we set to work writing, piloting and field testing a more extensive set of Probes for primary teachers with a focus on targeting mathematics concepts within the new standards. This book is one in a series of *Uncovering* resources for the F–2, 3–5, 6–8 and 9–12 year-level spans.

ORGANISATION

This book is organised to provide readers with an understanding of the purpose, structure and development of the Mathematics Assessment Probes as well as to support the use of applicable research and instructional strategies in mathematics classrooms.

Chapter 1 provides in-depth information about the process and design of the Mathematics Assessment Probes along with the development of an action-research structure we refer to as a QUEST Cycle. Chapters 2 to 6 contain the collection of Probes categorised by concept strands with accompanying Teacher Notes to provide the specific research and instructional strategies for addressing students' challenges with mathematics. Chapter 7 highlights instructional considerations and images from practice to illuminate how easily and in how many varied ways the Probes can be used in mathematics classrooms. This chapter also highlights how use of the Probes can support students' proficiency within the Australian Curriculum: Mathematics framework.

THE AUSTRALIAN CURRICULUM

In recent years, the Australian Federal Government has been working closely with state and territory educational offices in an effort to implement a national curriculum for all Australian schools. This Australian Curriculum sets consistent national standards, in an effort to improve learning outcomes for all students, as well as laying the foundations for future learning, growth and active participation in the community.

The Australian Curriculum: Mathematics aims to ensure that students:

- are confident, creative users and communicators of mathematics, able to investigate, represent and interpret situations in their personal and work lives and as active citizens
- develop an increasingly sophisticated understanding of mathematical concepts and fluency with processes, and are able to pose and solve problems and reason in Number and Algebra, Measurement and Geometry, and Statistics and Probability
- recognise connections between the areas of mathematics and other disciplines and appreciate mathematics as an accessible and enjoyable discipline to study. (ACARA, 2015)

The Australian Curriculum content descriptions found in *Uncovering Student Thinking About Mathematics in the Australian Curriculum* are taken from Years 4–7, but it should always be assumed that students engage in appropriate prerequisite work prior to those year levels and meaningful review and extensions subsequent to those year levels. The included activity sheets can also be used as remediation activities for struggling students in the higher year levels or for extra practice for students who are excelling in the lower year levels.

While it is recommended that teachers use the mathematical content in this book with their Australian Curriculum: Mathematics lessons, the activities featured in *Uncovering Student Thinking About Mathematics in the Australian Curriculum, Years 4–7* can just as easily be used with other educational frameworks at the state or institutional level. For a full overview of the Australian Curriculum please visit http://www.australiancurriculum.edu.au/.

1

Mathematics Assessment Probes

To differentiate instruction effectively, teachers need diagnostic assessment strategies to gauge their students' prior knowledge and uncover their misunderstandings. By accurately identifying and addressing areas of difficulties, teachers can help their students avoid becoming frustrated and disenchanted with mathematics and can prevent the perception that "some people just aren't good at maths". Diagnostic strategies also support instruction that builds on individual students' existing understandings while addressing their identified difficulties. From infancy to preschool, children develop a base of skills, concepts and misconceptions about numbers and mathematics (National Research Council [NRC], 2005, p. 157). Understanding and targeting these specific areas of difficulty enables teachers to perform focused and effective diagnostic assessment. The Mathematics Assessment Probes in this book allow teachers to target specific areas of difficulty as identified in research on student learning.

The Probes typically include a prompt or question and a series of responses designed specifically to elicit prior understandings and commonly held misunderstandings that may or may not be uncovered during an instructional unit. In the example in Figure 1.1, students are asked to choose from a selection of responses and write about how they determined their answer choice.

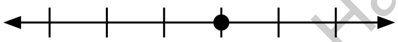
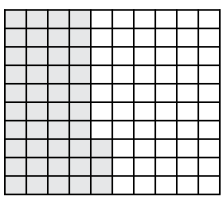
This combination of selected responses and further explanation helps to guide teachers in making instructional choices based on the specific needs of their students. Since not all Probes follow the same format, we will discuss the varying formats later in this chapter. If you are wondering what other kinds of Probes are included in this book, take a few moments now to review two or three additional Probes from Chapters 2–6 before continuing reading. But we suggest that you return to read the rest of this chapter before beginning to use the Probes with your students.

At this point, you may be asking; "What is the difference between Mathematics Assessment Probes and other assessments?" Comprehensive

Figure 1.1 Example of a Probe: Decimal Division Estimates

Representing Decimals

Which of the following are equivalent to **0.43**?

	Circle Yes or No:	Explain your answer:
A. **.04 + 0.3**	Yes No	
B.  0.4 0.5	Yes No	
C.  □ = $\frac{1}{100}$	Yes No	

diagnostic assessments for mathematics such as AIMSweb (Pearson) as well as the many state- and federal-developed assessments can provide information important for finding entry points and current levels of understanding within a defined progression of learning for a particular mathematics sub-domain such as operations and algebraic thinking. Such assessments will continue to play an important role in schools because they allow teachers to get a snapshot of student understanding across multiple sub-domains, often at intervals throughout the year depending on the structure of the assessment.

Are you wondering about the Probes? If you are, we suggest reviewing the following Probes as initial examples:

- Rounding Decimals, p. 80
- Equivalent Fractions Card Sort, p. 102
- Naming the Perimeter, p. 159

Consider the following vignette:

Are They Equivalent?

In an intermediate classroom, the teacher uses a Probe to uncover students' explanations of how to determine equivalent and non-equivalent products and quotients of various two-digit numbers (for example, is 16 × 24 equal to 24 × 16? Is 36 ÷ 12 equal to 12÷ 36?). By creating a column graph of students' responses to anonymously display students' ideas, the teacher and the class can see that many students believe that both products and quotients are equivalent regardless of the order of the numbers. Knowing that this is a common misunderstanding cited in the research literature and seeing that the data from her own class mirror that misunderstanding, the teacher designs a lesson that involves the students in using visual models to model the multiplication and division of various two-digit numbers. After students experience modelling the operations, they revisit their original ideas and have an opportunity to revise them. The next day, students are given the task of defining the commutative property of multiplication. They work in small groups to demonstrate the property and explain why there is no commutative property for division. At the end of the lesson, students are asked to reflect on their original thinking on the Probe about whether the products and quotients were or were not equivalent. (Adapted from Keeley & Rose Tobey, 2011, p. 2)

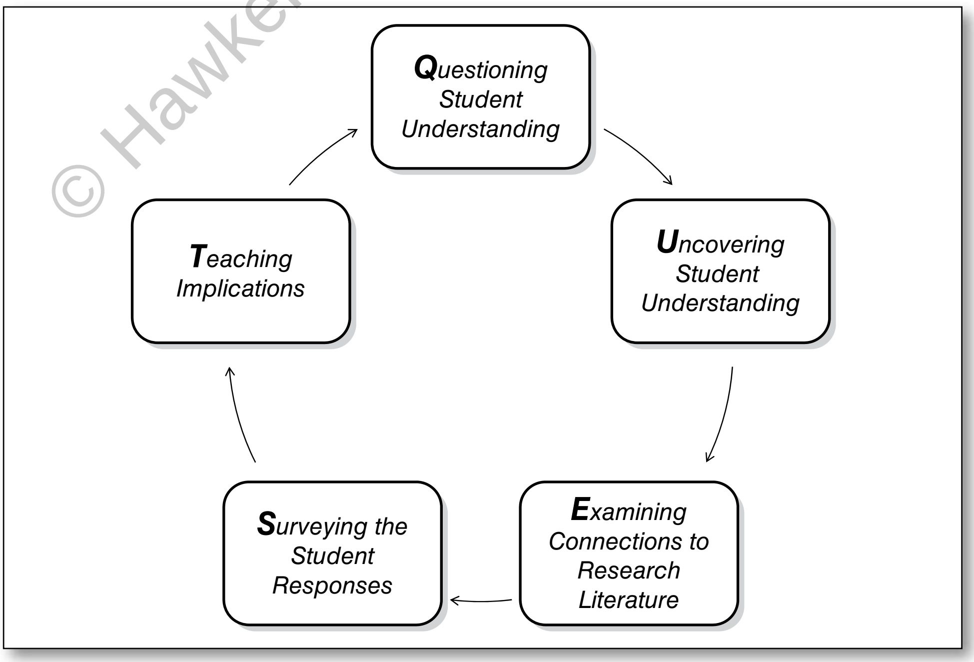
The Probe used in this vignette, the Are They Equivalent? Probe, serves as a diagnostic assessment at several points during the two-day lesson. The individual elicitation allows the teacher to diagnose students' current understanding; the conversation and practice around modelling the operations both builds the teacher's understanding of what students are thinking and creates a learning experience for students to further develop their understanding of the commutative property. The individual time allotted for reflection allows the teacher to assess whether students are able to integrate this new knowledge with former conceptions or whether additional instruction or intervention is necessary.

Rather than addressing a variety of maths concepts, Probes focus on a particular subconcept within a larger mathematical idea. By pinpointing one subconcept, the assessment can be embedded at the lesson level to address conceptions and misconceptions while learning is under way, helping to bridge from diagnostic to formative assessment.

Helping all students build understanding in mathematics is an important and challenging goal. Being aware of student difficulties and the sources of those difficulties, and designing instruction to diminish them, are important steps in achieving this goal (Yetkin, 2003). The process of using a Probe to diagnose student understandings and misunderstandings and then responding with instructional decisions based on the new information is the key to helping students build their mathematical knowledge. Let's take a look at the complete Probe implementation process we call the ***QUEST*** *Cycle*:

- **Q**uestioning student understanding: Determine the key mathematical understandings you want students to learn.
- **U**ncovering student understanding: Use a Probe to uncover understandings and areas of difficulties.
- **E**xamining connections to research and educational literature: Prepare to answer the question, In what ways do your students' understandings relate to those described in the research base?
- **S**urveying the student responses: Analyse student responses to better understand the various levels of understanding demonstrated in their work.
- **T**eaching implications: Consider and follow through with next steps to move student learning forward.

Figure 1.2 Quest Cycle



Source: Adapted from Rose, Minton & Arline (2007).