being required to more effectively meet struggling students' many and varied needs in the context of their daily instructional lessons. As researchers, we support the implementation of evidence-supported practices to the maximum extent possible and focus our recommendations in this book accordingly. As parents, we applaud the teachers who do the same in their classrooms.

Response to Intervention in Maths provides educators with an understanding of the components of effective instructional design and delivery for students with diverse needs in the area of mathematics. Specifically, readers will learn procedures for teaching mathematics using systematic and explicit instruction as an approach to assessment, instructional planning and evaluation. The instructional recommendations found in this book are aligned with the recommendations put forth by the NMAP's *Final Report* (2008; www.ed.gov/mathpanel), the IES Practice Guide written by Gersten and Colleagues (2009), and the research base on effective mathematics instruction, albeit relatively small compared to research available for reading.

The authors would also like to note that there is no one "thing" or "waving of a magic mathematics wand" that will address the many and varied issues impacting the learning or lack of learning in mathematics. Specific student differences are so widely diverse and often very complex, it is unlikely that the ideas in this book will address every student issue appearing in classrooms. As such, none of the recommendations in this book should be interpreted as "absolutes", but rather as starting points for consideration in the context of your mathematics program and specific characteristics of your students. Moreover, we advocate that only a concerted effort at all levels and by all educators, both general and special education teachers, is an effective and efficient approach that will ultimately capitalise on efforts to improve your school's curriculum and instruction in the area of mathematics. Without unilateral support for student learning and improvements in mathematics, an RTI maths effort is premature.

CHAPTER OVERVIEW

Chapter 1, "What Is RTI, and Why Is It Important?" provides an overview of response to intervention (RTI) in general and how it specifically relates to teaching mathematics. Topics covered in this chapter include an overview and description of RTI practices and procedures and common components in models of RTI, and it concludes with a brief overview of key research supporting RTI in mathematics.

Chapter 2, "The RTI Process for Maths", provides a description of the essential components to consider when designing and implementing an RTI model in maths, more detailed description of the standard protocol model and problem-solving model, progress monitoring, and the importance of the core mathematics program.

Chapter 3, "A Tiered Approach to More Effective Mathematics Instruction", differentiates different levels of instruction and intervention necessary for implementing RTI. Additionally, through a series of detailed self-studies of curriculum, instructional delivery and interventions, along with some classroom examples, it becomes evident whether a school is ready to initiate RTI in mathematics.

Chapter 4, "Mathematics Interventions Overview", describes who requires interventions and how to define the necessary interventions per each student's needs. Details about building an appropriate environment for interventions as well as choosing effective curriculum and instructional delivery are explained as well as setting the time frame for intervention and developing interventions. The chapter includes a list of mathematics interventions and programs to consider.

Chapter 5, "Number Sense and Initial Maths Skills", details the basic components of number sense and early numeracy as defined by educational programs and related assessments. More important, from number recognition, to magnitude, to counting strategies for basic facts, instruction delivery and interventions are described, with illustrations that may be used to teach number sense to students who are struggling in mathematics.

Chapter 6, "Building Students' Proficiency With Whole Numbers", provides a rationale for the importance of teaching students to proficiency with basic whole number operations. Instructional strategies will be provided for building understanding, relationships and fluency with whole numbers. Peer-assisted learning strategies (PALS) in maths are also described for prep and years one to six.

Chapter 7, "Fractions and Decimals", acknowledges the major struggles that students have with fractions, decimals and percentages. These struggles are worse for those with maths difficulties. The failure to succeed in fractions has an ill effect on performance in secondary mathematics, particularly algebra. In this chapter, year-level expectations are set along with illustrated ways on how to instruct and intervene with the teaching of fractions.

Chapter 8, "Teaching Problem Solving Strategically", will present teaching problem solving strategically through three problem-solving programs that have been used as Tier 2 instructional programs.

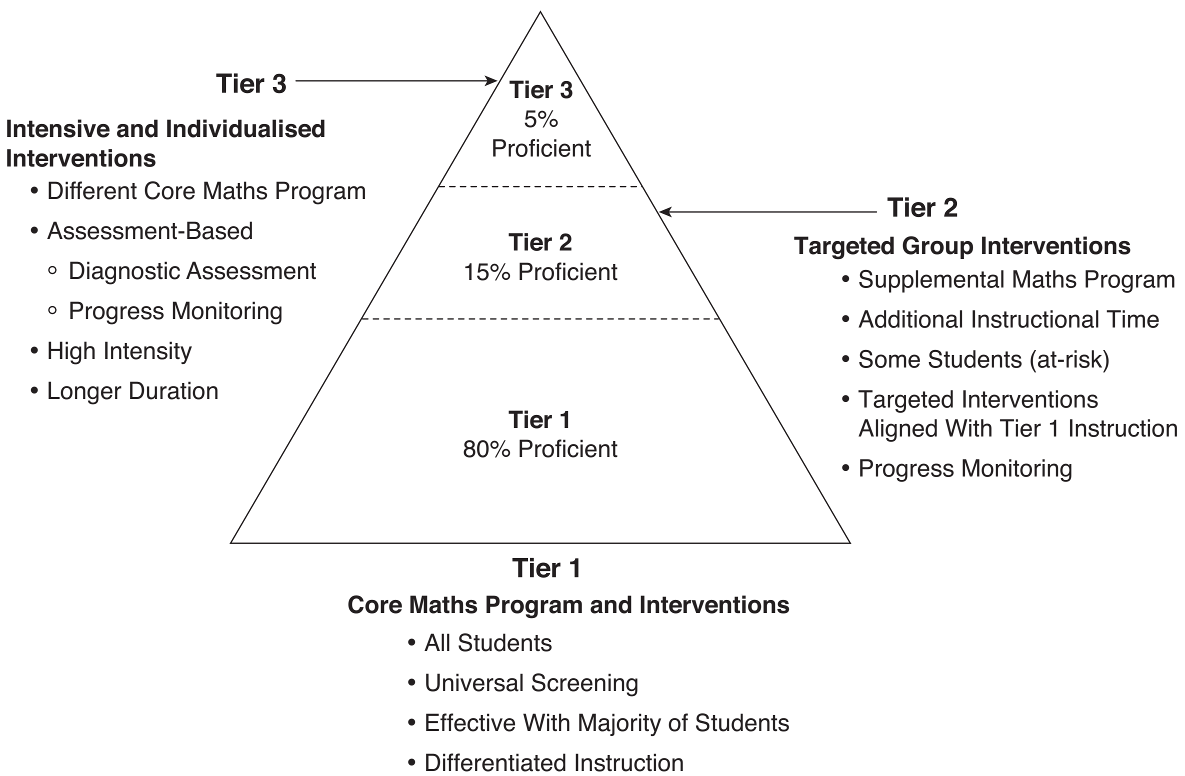
Chapter 9, "The Importance of Teaching Mathematical Vocabulary", focuses exclusively on mathematical vocabulary and how it influences mathematical proficiency. Five general guidelines for teaching vocabulary and seven maths-specific recommendations for teaching mathematical vocabulary are described. Additionally, five instructional activities to facilitate deeper understanding are described and how to assess student's vocabulary knowledge.

Chapter 10, "Next Steps in the RTI Process", explores the next steps as models continue to be refined and expanded to secondary settings, other student groups such as gifted and talented, and implications for changing systems. Additionally, an alternative approach to Tier 1 instructional programs is described for future consideration.

time are found in the research, we recommend that Tier 1 instruction be a minimum of 50–60 minutes per day with an additional 20–30 minutes in Tier 2 for students who are struggling.

Tier 3 is generally described as the most intensive instruction and is reserved for students who have received evidenced-based instruction and various levels of intensity in Tiers 1 and 2 but have still not made adequate progress or have made no progress. At this point, the RTI team may decide that the students in the very top of the instructional pyramid require a different core program, one that is much more intensive, systematic and explicit. If the effectiveness of Tier 1 and Tier 2 instruction is maximised, no more than 5% of the student population should require services in Tier 3 (see Figure 1.2).

Figure 1.2 Example RTI Model for Maths



Note: The dashed lines separating the instructional tiers indicate that students can move back and forth from each instructional tier depending on student progress. Generally, Tier 2 instructional supports are in addition to Tier 1. In other words, students receiving Tier 2 instruction remain in the Tier 1 core maths program. Also, three tiers of instructional supports are most common, but some RTI models have four tiers.

KEY RESEARCH SUPPORT FOR RTI AND MATHEMATICS

As educators continue to refine and expand RTI to mathematics, it is important that decisions are guided by the best available research. Unfortunately, to date

These three general beliefs about learning are vital to a successful RTI model in any content area; however, specific to RTI in mathematics, teachers must have four common core beliefs: (1) All students can be mathematically proficient; (2) all students need a high-quality mathematics program; (3) effective mathematics instructional programs must teach conceptual understanding, computational fluency, factual knowledge and problem-solving skills; and (4) effective instruction matters and significantly impacts student learning and achievement in mathematics. A brief discussion of each core belief is presented below.

> "All students can and should be mathematically proficient in Grades pre-K through 8."
>
> —*Adding It Up: Helping Children Learn Mathematics (NRC, 2001), p. 10*

Core Belief #1: All Students Can Be Mathematically Proficient

Mathematics difficulties, unlike reading problems, are sometimes overlooked or attributed to the idea that not everyone can be proficient in mathematics. Even parents will often excuse their own children's problems in maths as "not that important" or justify problems by saying, "I wasn't very good at maths when I was a kid". This mind-set towards learning maths is very different than that towards reading. It should be an accepted belief that all students can be mathematically proficient starting in prep and continuing all the way through high school (National Mathematics Advisory Panel [NMAP], 2008; National Research Council, 2001). If teachers, parents and students lack the fundamental goal that all students can become proficient in mathematics, an effective RTI model is likely to be less effective. The goal of proficiency for all students is directly related to the next two core beliefs.

> "All students need access to a high-quality mathematics program."
>
> —*National Mathematics Advisory Panel, 2008*

Core Belief #2: All Students Need a High-Quality Mathematics Program

For all students to reach mathematical proficiency, students need a high-quality mathematics program designed to prepare them for more advanced mathematics courses as well as for post-secondary school and/or various careers. The emphasis on high-quality mathematics programs is sometimes pursued by schools enacting higher standards of learning. This is important, but it does not guarantee a high-quality mathematics program. Standards are the goals for learning; an effective mathematics program must have high standards but also include effective delivery (i.e. instruction) of the standards. We will not go into detail regarding standards, but rather we will discuss later in this chapter the importance of core mathematics programs.

> "Effective mathematics instructional programs must teach conceptual understanding, computational fluency, factual knowledge, and problem-solving skills."
>
> —*National Mathematics Advisory Panel, 2008*

To explain the need for number sense, let's look at two students: one who understands place value, Caroline, and one who does not, Jason. For a problem like 32 + 13, Jason, who is still counting addition on his fingers, will have to count up from 32, thirteen times. However, Caroline, who understands place value but is still adding on her fingers, will add 3 tens + 1 ten and 2 ones + 3 ones. They both will conclude the answer is 45, but Caroline is quicker at obtaining the accurate solution and has a better grasp of base 10.

Demonstrating Number Sense With Addition

32	3 tens + 2 units
+ 13	+ 1 ten + 3 units
	4 tens + 5 units

In a subtraction problem with borrowing such as 43 – 15, Caroline's strength in place value will give her a profound advantage over Jason. While Jason will try to count out the difference between the two numbers or with a decent strategy count down from 43, Caroline will approach numbers by decomposing each number and organise the numbers based on their place value.

Demonstrating Number Sense With Subtraction

43	4 tens + 3 units	3 tens + 13 units
– 15	–1 ten – 5 units	– 1 ten – 5 units
		2 tens + 8 units = 28

In this example, neither student has a profound grasp of number sense (i.e. counting on fingers does not show strong number sense; Jordan, Kaplan, Ramineni & Locuniak, 2008). However, Caroline's understanding of the place value component can lead to a more fluent problem-solving procedure. This chapter will focus on what to look for in assessing number sense and some possible interventions for teaching number sense.

ASSESSMENTS OF NUMBER SENSE

With such importance placed on number sense, it is important to implement a high-quality number sense assessment battery as early as prep in order to determine who requires intervention. Additionally, Gersten, Clarke and Jordan (2007) recommend using progress monitoring for number sense to track students longitudinally. Such progress monitoring works well within the framework of an RTI model. Number sense assessments should occur until at least the end of year three and most preferably until year five in order to help teachers adjust for student learning. Areas of number sense that should be tracked include but are not limited to the following: