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What Is Visible Thinking?

Visible thinking, the focus of this book, may be described as clarity and transparency in one's cognitive processes. Visible thinking requires overt, conscious and deliberate acts by both students and teachers. When thinking is visible, participants are aware of their own thoughts and thought processes, as well as those of the individuals with whom they are working. With visible thinking, there is a heightened level of awareness both individually and collectively. There is also a heightened degree of productivity referred to as synergy. Visible thinking occurs routinely in effective business communities during dialogues and discussions, brainstorming sessions, collaborative group situations and crisis-management scenarios. Effective communication is the basis for effective visible thinking. Ideas are formulated, expanded and refined through sharing. Acquiring this vital skill should not be left to chance.

True mathematical learning, as identified in numerous reports by the National Council of Teachers of Mathematics (NCTM; 2000) and the National Research Council (NRC; 2000, 2001, 2005), requires visible thinking. Research shows that, in the mathematics classroom, visible thinking is the key to mathematics learning and success. Evidence of visible thinking is apparent during mathematical discussions, explanations, demonstrations, drawing, writing and other ways that ideas are conveyed.

Students and teachers must think, have awareness of their thinking, organise and clarify their thinking, and then share their thinking. Visible thinking is intentional and manifests itself within classrooms in multiple ways:

- Teachers explain their thinking out loud.
- Students orally articulate their thinking.
- Students listen to other students articulate their thinking.
- Students engage in discussions while forming their understanding.

- Students consciously activate their inner dialogue
 - when reading for understanding and
 - when studying mathematics.
- Students record their thinking by
 - solving problems,
 - keeping journals
 - completing projects.
- Students demonstrate their thinking through use of technology, manipulatives or mathematical tools.

Visible thinking occurs within group settings as well as in individual settings. Experts in a field of study are very aware of their knowledge and are very adept at comparing their knowledge with the needs of a situation or problem. “In research with experts who were asked to verbalize their thinking as they worked, it was revealed that they monitored their own understanding carefully, making note of when additional information was required for understanding, whether new information was consistent with what they already knew, and what analogies could be drawn that would advance their understanding” (NRC, 2004, p. 18). These skills and self-monitoring processes used by experts are the very same ones students need to learn and understand mathematics.

When visible thinking is present in classrooms, students are consciously aware of their current understanding of the mathematical concepts being discussed. They are also aware of these concepts in relation to their previous learning and understanding. When thinking is visible, discrepancies and dissonance are obvious to the students. If classroom conditions support visible thinking through safe, open discussion and discourse, these misunderstandings are also readily apparent to teachers. Immediacy is a very important factor in visible thinking. When the discrepancies are apparent to teachers, the teachers have the information they need to take action – and they can clarify the misunderstandings on the spot.

Yet thinking is all too often invisible in schools, and successful learning depends on reversing this trend (Perkins, 2003). “Fostering thinking requires making thinking visible” (Ritchhart & Perkins, 2008, p. 58). By increasing thinking, motivation to learn is also increased. Visible thinking improves the ability to learn, and the increased ease of mastering a skill, in turn, provides motivation to continue learning.

UNDERSTANDING MATHEMATICAL CONCEPTS

The problem $3 + 4 = \square$ is not a challenge for adults and is certainly not difficult for the educators reading this book. Nonetheless, this problem is

Larry: Well, that makes sense, but I don't see what we did.

Beth: I think we didn't get the whole numbers right. Do you agree that $1 \frac{1}{8}$ cups of mix is right?

Larry: Let me see, we had 4 cups and $\frac{1}{2}$ cup. So 4 cups divided by 4 would give us 1 cup, and we are pretty sure about the $\frac{1}{2}$ cup divided by 4 gives us $\frac{1}{8}$ cup. Yeah, $1 \frac{1}{8}$ cups of mix is right.

Beth: Well, what about the milk? We have $1 \frac{1}{2}$ cups of milk divided by 4. Let's take the 1 cup. If we divide 1 cup by 4 we get $\frac{1}{4}$ cup, right? We know the $\frac{1}{2}$ cup divided by 4 is $\frac{1}{8}$, so we should add $\frac{1}{4} + \frac{1}{8}$.

Larry: Good, $\frac{1}{4} + \frac{1}{8}$ is $\frac{3}{8}$. We need $\frac{3}{8}$ cup of milk, not 1 cup like we first figured.

Beth: We know the oil is right, so our answer is $1 \frac{1}{8}$ cups of mix, $\frac{3}{8}$ cup of milk and $\frac{1}{8}$ cup of oil.

How did the teacher use visible thinking to intervene and correct students' misconceptions?

In this example, the teacher did not intervene in the usual manner. Instead, she provided an opportunity for students to collaborate. Collaboration allowed time for students to think and reason about the problem. Students worked together to self-correct their work and their thinking. They realised that initial estimations for the first two ingredients would give them amounts that are closer to their final answers. This process increased their understanding. By working together, their combined experience and knowledge allowed them to talk through the problem and realise something in their early calculations was not correct. In all likelihood, this would not take place if the students were working in isolation. By working in pairs and with sufficient time to understand the problem presented, students were able to make their thinking visible to themselves. They found their errors in thinking and solved the problem correctly.

As a direct result of this strategy, and an open, safe environment, students were able to self-correct their own thinking and misunderstandings.

VISIBLE THINKING SCENARIO 6: PLACE VALUE

Year two students had been working on writing and counting numbers to 100. The teacher felt the students needed time to reflect on place value and devised a version of Naughts and Crosses and Bingo for the students to play.

Problem

Students were formed into pairs and given two 6-sided die, plus two different coloured highlighters. One of the dice was blue, representing the 10s digit, and the other dice was yellow, representing the 1s digit. Students were also given a 5×5 number grid with 25 numbers randomly selected from the number set 11 to 66.

Students were to take turns rolling the die. Then, a student would align the die with the blue on the left and the yellow on the right, read the combined number, and then see if it was located on the grid the pair was given. For example, a 5 rolled on the blue die and a 3 on the yellow die would equal the number 53. If there was a match for 53 on the card, the student would highlight it with a coloured texta. The object was to be the first student of the pair to connect the colour across the grid, top to bottom or side to side. The colour squares did not have to be in a straight line, only touching at a side or a corner (see Figure 5.4).

Figure 5.4 Winner's Number Grid

Mathematics Within the Problem

The teacher wanted to reinforce the idea of place value with the students, involving them in an activity beyond the use of base 10 blocks.

24	15	56	16	26
51	46	25	62	13
43	23	61	31	11
52	22	36	64	55
34	53	21	32	33