SECOND EDITION

DESIGNING ASSESSMENT FOR MATHEMATICS

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EDUCATION

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Introduction

THE STATE OF MATHEMATICS

In today's classroom, mathematics teachers need to teach mathematics differently from the way they were taught mathematics. The world has changed in many ways in recent years. The use and availability of computers and computer technology has multiplied, the Internet has become a common research and communication tool, and videos and video games have become popular. As a result, students have grown accustomed to various forms of brain stimulation and have come to welcome such forms of stimulation. Teachers lecturing while students take notes and complete routine problems from a textbook does not produce mathematics proficiency in all students. Students are not making the mathematical gains they deserve by working in isolation to complete worksheets or watching the teacher complete problems on the board; these practices do not reach all students (Black, 1994). As stated by O'Brien (1999), students need to be active thinkers and not simply memorise the thinking of others. Mathematics needs to make sense to students, and the best way for this to occur is through hands-on learning. Practices must change and new strategies must be employed to help today's students be successful in mathematics.

In recent times, suggestions for new educational practices and techniques have flourished. Research shows that the classroom teacher and the strategies employed can and do affect the learner. New strategies (e.g. actively involving students through the use of performance tasks and problems that provide real-world applications) help students gain a deeper understanding of concepts and allow them to be better able to transfer that knowledge to new situations. Metacognitive methods and reflective writing are strategies that assist students in developing a new level of understanding because students learn to evaluate their thinking and the processes they use to arrive at solutions. Assessment results can be used to positively drive instruction, and gains in student achievement can be made when students take an active role in the learning process. However, to help students be successful, teachers must be willing to adjust, adapt, try new things, and teach in ways that they probably were not taught. Teachers need to step outside their comfort zones to seek practices that will affect their students.

For educators, increasing student achievement must be a primary goal. Creating a nation of thinkers and active problem solvers who can use their skills in ways that demonstrate true understanding is crucial for student success outside the classroom. Relating schoolwork and textbooks to real-world situations helps students see a purpose for their education. Active engagement in the learning process helps students make the mathematical connections necessary for increased learning and understanding. Rubrics are a necessary tool for promoting student achievement because they provide students with a clear road map that shows how to achieve success by allowing learners to assess their work against criteria for various levels of performance.

ABOUT THIS BOOK

This book offers methods, techniques and tasks that can be used to engage learners in the hands-on learning of mathematics. This book establishes the importance of using standards and benchmarks as educational targets to help students understand expected outcomes, and it stresses the importance of rubric development and the assessment process. The performance tasks actively engage the learners while allowing them to have fun in the process. Reflection opportunities provide students with the opportunity to evaluate their own performance and use that evaluation to grow and advance as maths students. Data use is promoted to provide feedback to students and teachers about the strengths and challenges of students.

Each chapter concentrates on a different aspect of rubrics and the assessment process. Strategies and suggestions are based on the research and opinions of top educators and organisations.

The first section of every chapter lays the foundation for suggested practices. The second section, the "Rubric/Mathematics Application", suggests a performance task that relates to the information covered in the first section. A complete explanation of the application is provided, as well as the resources (e.g. task explanations, suggested procedures, rubrics, graphic organisers, charts and assessments) necessary to implement the task in a classroom. An overview page (see Figure Intro. 1) is included in each of these applications. This page encapsulates all the important information needed to carry out the performance task. The steps for each task are accompanied by thumbnail versions of the figures that make up the data charts, reflection pages, assessments and rubrics for the task.

The mathematical application in each chapter is designed to allow students to demonstrate their level of understanding of the mathematics standards embedded in the application. Each application states a problem or task to be solved using mathematical skill, reasoning and communication. The application also includes suggested procedures. These can be used to give direction to students who are not as familiar with performance tasks. (Performance tasks are opportunities for students to apply

Rubric/Mathematics Applications Overview

Page Explanation

Standards:	Each project is linked to an example mathematics content standard. To see how each state or territory present these standards, see the table on p. 203.
Mathematics concepts:	The concepts related to the performance are listed.
Year level:	Suggested year levels are stated, although they can be extended in both directions.
Related curricular areas:	Any curricular areas the project relates to in addition to mathematics are listed.
Materials:	This section lists any necessary supplies. It also states whether computers are needed.
Task:	The problem is a situation presented to students that sets the stage for a maths connection related to the real world.
Suggested student procedures:	The procedures outline what students need to do to complete the task.
Teacher resources:	This is a list of the resource pages included in the chapter. The list indicates the pages that can be used with students.
Internet resources:	Each application includes several websites that are excellent resources for maths teachers and students.

Figure Intro.1

what they have learned in a real-life application.) For tasks with several steps or for more complex tasks, a task explanation and suggested procedures page are provided and can be handed out to students. Each chapter offers Internet resource links that can be used to promote technology research in the classroom, and figures are included in each application and can be copied for classroom use. Each application also contains information about time requirements. (Though activities in the book are based on 45-minute class periods, this time estimation may vary according to the level of the students, the use of calculators or computers, and the incorporation of task extensions.)

All of the mathematics applications in this book have been used with students and have produced outstanding results. Students who need to be challenged have found that the applications allowed them to grow and extend their capabilities, and students who find mathematics difficult have successfully completed the applications and developed a deeper understanding of mathematical concepts and their use. Extensions and simplifications are provided so each application can be differentiated for individual classrooms, students and year levels. In this way, low achievers,

as well as gifted students, are able to achieve beneficial results. Expansions are also offered that help extend the performance tasks into other curricular areas, such as English, fine arts and humanities. Such expansions are important as they incorporate an integrated, multiple-intelligences (Gardner, 1983) approach to learning. Technology is incorporated throughout the book in many ways—as a reference tool (to locate answers), as a construction tool (to build charts, graphs or graphic organisers) and as a research tool (to find data).

Teachers can make a difference in the achievement of their students by providing them with experiences that have an impact on their learning and performance. Students who become engaged in learning through active involvement develop an in-depth understanding of mathematics, how it relates to them, and how it influences the world. The strategies and applications provided throughout this book can help teachers assist students in this process.

1

Using Rubrics in Mathematics

THE RUBRIC

According to the dictionary, a rubric is a heading, title, class or category. In education, however, it means so much more. A rubric is a scoring guide that can be used to make reliable judgments about a student's product or performance. It is a tool that distinguishes teaching and learning by clearly stating criteria and describing levels of quality.

According to Schmoker (1996), rubrics allow individuals to assess and track numerical data in an expanded way. Schmoker explained that rubrics can assess thinking skills, student understanding, and students' ability to apply their knowledge to mathematical tasks. The rubric gives teachers, students and parents a reliable picture of what students should know and be able to do as well as to what extent they are able to demonstrate their knowledge. Schmoker also stated that a rubric—a set of written criteria used to analyse the outcomes of a student's learning—is "one of the most promising developments in assessment" (p. 70). Rubrics are based on educational standards and expectations and address not only what a student is expected to know and be able to do but also the level of quality he or she is expected to achieve.

According to Burke (2006), rubrics serve to assist students in improving their own performance. Rubrics provide students with the feedback necessary to set goals and demonstrate understanding at a higher level.

Rubrics are built on a rating scale. Scales vary, often ranging from 1 to 4, 0 to 3, or 1 to 6. The use of a specific scale is often a personal preference.

The scale of 1 to 4 is probably the most common because it is small, which helps keep students from being overwhelmed by descriptors, yet broad enough to encompass a wide range of quality descriptors. Typically, the scale of 0 to 3 is used when the rubric creator wants to use a 0 to represent no progress being made by a student toward a specific criterion. Larger scales, such as 1 to 6, are used when products are more involved and there is opportunity for a more widespread level of performance.

Rating scales contain concise descriptions for each level of progress. These criteria, or performance indicators, clearly state the level of performance expected at each numerical value on the rubric. The criteria stated within the rubric can help the assessor accurately evaluate students' work. Students too can use this tool to self-assess and improve their level of performance. The purpose of the rubric is not only to evaluate but also to help students increase their level of performance by outlining a vision of success.

There are two types of rubrics: analytic and holistic. The analytic format uses multiple descriptors for each criterion evaluated within the rubric (see Figure 1.1). In essence, a student's product has multiple opportunities to be evaluated within the same rubric. In an analytic rubric, "a performance is assessed several times, using the lens of a separate criterion each time" (McTighe & Wiggins, 1999, p. 273).

Example of Analytic Rubric

	1	2	3	4
Graph Title	Partially or inaccurately titled	Title unrelated to graph	Title related to graph	Title related to graph Perfect mechanics
Axes Title	Incomplete or illegible titles	Titles present but unrelated to axes data	Titles present and related to axes data	Titles present and related to axes data Perfect mechanics
X-axis Information	Incomplete or illegible information	Labels or numbers present partial inaccuracies	Labels or numbers present and accurate	Labels or numbers present, accurate and easy to read
Y-axis Information	Incomplete or illegible information	Labels or numbers present partial inaccuracies	Labels or numbers present and accurate	Labels or numbers present, accurate and easy to read
Key	Incomplete	Complete with inaccuracies	Complete and accurate	Complete, accurate and easy-to-read
Graphed According to Key	No correspondence to key	Much graphed inaccurately	Graphed with minor error	Graphed according to key
Accuracy of Graph	Graph mostly inaccurate	Some inaccuracies	One inaccuracy	Graph completely accurate

Figure 1.1

A holistic rubric (see Figure 1.2) has one performance expectation description at each numerical level on the rubric. The product or performance is evaluated as a whole and often given a single score. The holistic rubric is, then, "a rubric used to obtain the overall impression of the quality of a performance or product" (McTighe & Wiggins, 1999, p. 277).

Analytic rubrics are best used as a part of the formative process. These rubrics are specifically designed as tools to improve student achievement through their use during any task or performance. Students are able to rely on the rubric descriptors to increase their level of performance. Holistic rubrics, in turn, are summative in nature. This type of rubric is most often used to evaluate work only at the end of a process. All rubrics used within this book are analytic rubrics, as these are the types of rubrics that best help to increase student performance.

WHY USE RUBRICS IN MATHEMATICS?

A myth surrounding the teaching of mathematics is thought to be true by too many: If the curriculum is covered, students will learn. Of course, all good mathematics teachers realise this is not the case, but often it is difficult for teachers to know what to do to better help students internalise the material. Teachers often wonder how students make mathematics their own and how they can help students personalise and make sense of mathematical concepts.

Student learning and understanding are often directly related to the variety of experiences within the mathematics classroom. According to Battista (1999), traditional methods in mathematics instruction are often ineffective and fail to help students make connections to other situations where a mathematical formula might apply. Students must be given numerous opportunities to form solid mathematical connections by solving real-world problems and making real-life applications.

In the traditional mathematics classroom, student work solely involves calculations. Answers are right or wrong, and, as a result, grading is fairly simplistic. These techniques, however, do not fulfill students' mathematical needs. Studies have shown that a high percentage of students are unable to understand mathematics at a basic level. Adults often admit that they do not understand, and never will understand, mathematics. These indicators demonstrate that traditional mathematics methods may have done a good job of teaching calculations but a poor job of teaching thinking and application. Therefore, the focus in mathematics today has become one of active involvement, where students are able to calculate as well as think, apply and explain their actions and processes. Cawelti (2004) indicated that students can achieve competence in procedures and calculations through the use of strategies that promote conceptual understanding and the construction of knowledge.

According to Bryant and Driscoll (1998), "Open-ended questions and other alternative assessments go further, which allows for richer responses from students and better data about students' mathematical content knowl-

Example of Holistic Rubric

ORAL PRESENTATION RUBRIC

Name: _____ Date: _____

Subject: _____ Final Grade: _____

5	The subject is addressed clearly. Speech is loud enough and easy to understand. Eye contact is good. Visual aid is used effectively. Presentation is well organised.
4	Subject is addressed adequately. Speech has appropriate volume. Eye contact is intermittent. Visual aid helps presentations. Organisation is good.
3	Subject is addressed adequately. Speech volume is erratic. Student reads notes and has erratic eye contact. Visual aid does not enhance speech. Speech gets off track in places.
2	Speech needs more explanation. Speech is difficult to understand at times. Adequate eye contact is lacking. Visual aid is poor. Organisation is lacking.
1	Speech does not address topic. Speech cannot be heard. Very little eye contact is made. No visual aid is shown. No organisation is apparent.

Scale: 5 = A; 4 = B; 3 = C; 2 = D; 1 = Not Yet
General Comments:

Figure 1.2

Adapted from *How to Assess Authentic Learning* by Kay Burke © 2008 Hawker Brownlow Education.

edge and problem solving skills. This information helps to inform teachers about where students are and what instruction they need to concentrate on. In order to be useful for the teacher, those data must be skillfully interpreted" (p. 27). Rubrics are the necessary tool for this type of skillful interpretation. Rubrics assess student outcomes and encourage student growth, enhanced performance and increased levels of understanding. Through the use of clear, precise descriptors, student output can be categorised according to level of performance or understanding and a point value can be assigned to that level. Students can use rubrics prior to project completion to identify levels of quality and can use rubric descriptors to enhance their performance.

Increased use of a variety of performance tasks and analytic rubrics can be the key to increased student understanding and achievement because rubrics give students prior notice of the expectations for a given task or performance. Teachers should keep in mind that they need to share rubrics with students before beginning a task. Discussing the criteria with students is important so that rubrics do not become a top-down system like most assessments. If the indicators of success are not shared before the assignment begins, the rubric loses its effectiveness in aiding student success. Rubrics encourage quality by giving students access to clearly defined criteria and levels of performance. Rubrics also make students aware of the effort and accomplishment required to complete a task successfully. Through the use of various tasks and performances, students internalise mathematical concepts by engaging in the process of learning instead of simply memorising the process used to reach an answer. Students see the relationship between a concept and its real-world applications and demonstrate a level of understanding far beyond a page of calculations.

Regarding the benefits of using rubrics, Schmoker (1996) stated that by clearly defining a performance, students see it as achievable. The rubric also provides students and parents with feedback that clearly identifies strengths and weaknesses—a characteristic that is not a strong point of traditional assessment practices.

Rubrics help promote the goal of a sound education in mathematics, which includes developing "powerful mathematical thinking in students, [and] instruction [that] must focus on, guide, and support their personal construction of ideas. Such instruction encourages students to invent, test, and refine their own ideas" (Battista, 1999, p. 430). Rubrics are the evaluation tool necessary to aid in this task because they provide students with a guide to success and encourage them to reach for their goals.

The performances and tasks that encourage student success found throughout this book are called Rubric/Mathematics Applications. Each Rubric/Mathematics Application section outlines a mathematical task that puts rubrics into practice. The application for this chapter is titled "You're Right on Target!"

RUBRIC/MATHEMATICS APPLICATION

YOU'RE RIGHT ON TARGET!

This application was created to enhance skills in coordinate graphing. The application is based on three mathematics standards: geometry and spatial sense, communication, and problem solving. These standards, which are highlighted on the overview page, are addressed during completion of the task.

This Rubric/Mathematics Application is written at three levels of difficulty: primary, middle and secondary school. Each figure provided is adapted to meet these three levels. These levels can also be used as a way to differentiate within a multilevel or multiability classroom. Students who need to be challenged can work at Level 3, students who are capable can work at Level 2, and students who need more foundational skills might function best at Level 1.

The application creates a problem for students to solve that requires the use of a variety of mathematical and communication skills. The problem may be presented to students with or without the suggested procedures listed within the overview. Students who are inexperienced in dealing with performance tasks or those who require more guidance will benefit from knowledge of the procedures. In classrooms where problem-based learning is commonplace, the task alone can be supplied.

For best results with mathematics applications, review all written information, teaching tools and resources prior to presenting the task to students. Flexibility and creativity are encouraged, and throughout the unit suggestions are made to assist in restructuring this task to fit the needs of individual classroom populations and teachers.

The teacher resources within this chapter include instructions for the students and explanations for the teacher. All applications have answer keys. If teachers need additional resources or more information about coordinate graphing, the Internet is an excellent tool. A few websites are listed in the overview (see Figure 1.3) to get teachers started.

Performance Task Explanation

Students need to understand the meaning of coordinates and coordinate graphing before completing this task. During the completion of the task, students demonstrate an awareness and knowledge of graph numbering. During the task they locate the *x*- and *y*-axis and place stickers at the intersections of lines on the graph paper. Then, they accurately identify the coordinates at which the stickers are placed. This task takes about two to four 45-minute class periods depending on the use of expansion activities.

The task is designed with three levels of difficulty. Younger students use large grid paper and 12 stickers. They use six stickers to identify coordinates and six stickers on their Student Answer Record Charts. Students in the middle years place 30 stickers on more detailed graph paper and list 20 coordinates. Students in the upper years use four-quadrant graph paper and place 30 stickers on the paper, making certain that at least 6 stickers are placed in each quadrant. Students in every level identify the coordinates of all the stickers. The activity may be done as a class, in cooperative groups, or independently.

Figures 1.4 to 1.6 contain task sheets for all three levels, and Figures 1.7 to 1.9 contain the record charts for all three levels. To complete the activity, students should be given copies of the Student Task Sheet and record chart. The record chart serves as a graphic organiser to help students organise their data and list the coordinates. Students

Unit Overview

You're Right On Target!

Standards:	Geometry and Spatial Sense, Communication, Problem Solving
Mathematics concepts:	Coordinate graphing
Year levels:	2 to 10
Related curricular areas:	humanities—map skills; English—writing directions; art—design packaging
Materials:	Assorted stickers, grid paper (blackline master included)
Task:	Quadrant Brothers Toy Company is in the process of creating a new strategy game. The game consists of locating hidden targets on a graph by guessing the location's coordinates. Players are given hints relating to the location of the hidden targets. The game contains hundreds of different graphs. The company needs your help to create the graphs and target locations as well as to provide answer keys for the players.

Suggested Student Procedures

1. Choose stickers and place them at various intersections on a grid (piece of graph paper).
2. Label and number the *x*-axis (horizontal axis).
3. Label and number the *y*-axis (vertical axis).
4. Create a title for the graph.
5. Create an answer key for the graph that lists the coordinates of the stickers. Place the coordinates on the chart provided.

Teacher Resources

Student Task Sheet for three levels

Student Answer Record Chart for three levels

Rubric for three levels

Graph checklist for all levels

Checkup and key for three levels

Reflections for all levels

Internet Resources

Maths Matters: Contains information on combating mathematics anxiety, such as information on brain research and metalearning, as well as tips and strategies on helping students feel more comfortable with mathematics. (http://www.mathmatters.net/matips.htm)

Quick Maths Site: Provides help to mathematics questions in various areas, including graphing. (for higher levels; www.quickmath.com)

Figure 1.3