HOW THE BRAIN LEARNS MATHEMATICS

SECOND EDITION

DAVID A. SOUSA

Hawker Brownlow EDUCATION

Contents

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Introduction

Numbers rule the universe.

—Pythagoras

EVERYONE CAN DO MATHEMATICS

Human beings are born with some remarkable capabilities. One is language. In just a few years after birth, toddlers are carrying on running conversations without the benefit of direct instruction. Over the next few years, their sentences become more complex and their vocabulary grows exponentially. By the age of 10, they understand about 10,000 words and speak their native language with 95 percent accuracy.

Another innate talent is number sense—the ability to determine the number of objects in a small collection, to count, and to perform simple addition and subtraction, also without direct instruction. Yet by the age of 10, some of these children are already saying, "I can't do math!" But you never hear them saying, "I can't do language!" Why this difference?

Children often say, "I can't do math!" But you never hear them say, "I can't do language!" Why this difference?

One reason is that spoken language and number sense are survival skills; abstract mathematics is not. In elementary schools we present complicated notions and procedures to a brain that was first designed for survival in the African savanna. Human culture and society have changed a lot in the past 5,000 years, but the human brain has not. So how does the brain cope when faced with a task, such as multiplying a pair of two-digit numbers, for which it was not prepared? Thanks to modern imaging devices that can look inside the living brain, we can see which cerebral circuits are called into play when the brain tackles a task for which it has limited innate capabilities. That the human brain *can* rise to this challenge is testimony to its remarkable ability to assess its environment and make calculations that can safely land humans on the moon and send a space probe into orbit around a planet hundreds of millions of miles away.

WHAT IS MATHEMATICS?

To most people, mathematics is about calculating numbers. Some may even expand the definition to include the study of quantity (arithmetic), space (plane and solid geometry), and change (calculus). But even this definition does not encompass the many areas where mathematics and mathematicians are found. A broader definition of mathematics comes from W. W. Sawyer (1982). In the 1950s, he described mathematics as the "classification and study of all possible patterns" (p. 12). He explained that *pattern* was meant "to cover almost any kind of regularity that can be recognized by the mind" (p. 12).

Other mathematicians who share Sawyer's view have shortened the definition even further: Mathematics is the science of patterns. Devlin (2000) not only agrees with this definition but has used it as the title of one of his books. He explains that patterns include order, structure, and logical relationships, and go beyond the visual patterns found in tiles and wallpaper to those that occur everywhere in nature. For example, patterns can be found in the orbits of the planets, the symmetry of flowers, how people vote, the spots on a leopard, the outcomes of games of chance, the relationship between the words that make up a sentence, and the sequence of sounds we recognize as music. Some patterns are numerical and can be described with numbers, such as voting patterns of a nation or the odds of winning the lottery. Other patterns, such as a leopard's spots, are visual designs not connected to numbers at all.

Mathematics can be defined simply as the science of patterns.

Devlin (2000) further points out that mathematics can help make the invisible visible. Two-thousand years ago, the Greek mathematician Eratosthenes was able to calculate the diameter of Earth with considerable accuracy and without ever stepping foot off the planet. The equations developed by 18th-century mathematician Daniel Bernoulli explain how a jet plane flying overhead stays aloft. Thanks to Isaac Newton, we can calculate the effects of the unseen force of gravity. More recently, linguist Noam Chomsky has used mathematics to explain the invisible and abstract patterns of words that we recognize as a grammatical sentence.

If mathematics is the science of patterns and if visible and invisible patterns exist all around us, then mathematics is not just about numbers but about the world we live in. If that is the case, then why are so many students turned off by mathematics before they leave high school? What happens in those classrooms that gives students the impression that mathematics is a sterile subject filled with meaningless abstract symbols? Clearly, educators have to work harder at planning a mathematics curriculum that is exciting and relevant and at designing lessons that carry this excitement into every day's instruction.

I will leave the discussion of what content to include in a preK–12 mathematics curriculum to experts in that area, especially now that the Common Core State Standards for Mathematics (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010) have been released and adopted by many states. My purpose here is to suggest how the research in cognitive neuroscience can be used to plan lessons in mathematics that are more likely to result in learning and retention.

Why Is Learning Mathematics So Hard?

Succeeding in high school mathematics is still no easy feat. Take a look at Table I.1. The results of the 2013 National Assessment of Educational

Table I.1 Proficiency Levels for Grades 4, 8, and 12 in Mathematics on NAEP, 2005–2013

Year	*Grade 4*				*Grade 8*				*Grade 12*			
	Below Basic	*At Basic*	*Proficient*	*Advanced*	*Below Basic*	*At Basic*	*Proficient*	*Advanced*	*Below Basic*	*At Basic*	*Proficient*	*Advanced*
2013	17	41	34	8	26	38	27	9	35	39	23	3
2011	18	42	34	7	27	39	26	8	—	—	—	—
2009	18	43	33	6	27	39	26	8	36	38	23	3
2007	18	43	34	6	29	39	25	7	—	—	—	—
2005	20	44	31	5	31	39	24	6	39	38	21	2

SOURCE: NAEP (2013).

Progress (NAEP) mathematics tests of twelfth-grade students revealed that 23 percent were considered proficient in mathematics skills. This was the same percentage found in the 2009 assessment. No educator or parent can feel reassured by results showing such a low percentage of high school seniors performing at this proficiency level in mathematics. For fourth graders, 34 percent were proficient, compared with 33 percent for 2009 and 34 percent for 2011. As for the eighth graders, 27 percent scored proficient in 2013, compared with 26 percent for both the 2009 and 2011 assessments. The improvement was not significant (NAEP, 2013). Despite all the attention and high-stakes testing focused on mathematics instruction in recent years, achievement results have barely moved.

Explanations for this lackluster performance abound. Some say that learning mathematics is difficult because it is so abstract and requires more logical and ordered thinking. Others say that the various symbols used in mathematics make it similar to tackling a foreign language. Education critics maintain that only a few students are really developmentally incapable of handling mathematics and that the poor performance stems mainly from inadequate instruction. They cite the so-called "math wars" as hindering major progress in mathematics curriculum development, similar to what the "reading wars" did to reading instruction during the 1990s.

Impact of Teacher Preparation

Another potential factor affecting students' success in mathematics is the content knowledge of their teachers. Numerous studies have shown that middle and high school students learn more when their teachers have certifications or degrees in mathematics (e.g., Wayne & Youngs, 2003). Although states have been increasing the course requirements for individuals to be licensed to teach mathematics in secondary schools, problems persist. A recent study of 115 prospective middle school mathematics teachers at a large U.S. public university revealed that a substantial number of them had a limited knowledge of algebra for teaching (Huang & Kulm, 2012). The students made numerous mistakes when solving quadratic equations and in algebraic reasoning and manipulation. They also had difficulties with connecting algebraic and graphical representations.

A survey by the U.S. Department of Education showed that only 63 percent of the nation's nearly 144,000 high school mathematics teachers have both a college major and certification in mathematics (Hill, 2011). Nearly 26 percent have only a major or only certification in mathematics, and about 11 percent have neither a major nor certification in mathematics. This last group is referred to as out-of-field teachers. Other surveys find that out-of-field teachers are more likely to be found in high-poverty schools or in schools where they are assigned to the most challenging students (e.g., Kalogrides, Loeb, & Betelle, 2011).

Responses From Mathematics Educators

The National Council of Teachers of Mathematics (NCTM) published the *Principles and Standards for School Mathematics* in 2000, proposing five process standards and five content standards for preK–12 mathematics instruction. Since then, interpretation of the standards in the elementary and middle school grades became so broad that NCTM decided to refocus the curriculum at those grade levels.

In 2006, NCTM released *Curriculum Focal Points for Mathematics in Prekindergarten Through Grade 8,* which identified three important mathematical topics at each level, described as "cohesive clusters of related knowledge, skills, and concepts" that form the necessary foundation for understanding concepts in higher-level mathematics. The publication was intended to bring more coherence to the very diverse mathematics curricula currently in use. It provided a framework for states and districts to design more focused curricular expectations and assessments for preK–8 mathematics curriculum development. Shortly thereafter, the National Mathematics Advisory Panel (2008) published its final report, making recommendations for curriculum changes, teacher education, instructional practices and materials, and assessment. In the meantime, the National Governors Association and the Council of Chief State School Officers launched an effort to develop standards in mathematics and English/language arts. They were finally published in 2010 as the Common Core State Standards for Mathematics, and have been adopted by most of the states. We will discuss these standards further in Chapters 4 and 5.

Whether this new effort will succeed in improving student achievement in mathematics remains to be seen. In the meantime, teachers enter classrooms every day prepared to help their students feel confident enough to master mathematics principles and operations. One thing seems certain: Students who are poor in mathematics in their early years remain poor in mathematics in their later years.

■ ABOUT THIS BOOK

I am often asked to give specific examples of how the fruits of scientific research have made an impact on educational practice. That question is a lot easier to answer now than it was 20 years ago because recent discoveries in cognitive neuroscience have given us a deeper understanding of the