

HOW THE

BRAIN LEARNS

MATHEMATICS

SECOND EDITION

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Contents

ABOUT THE AUTHOR	vii
INTRODUCTION	1
Everyone Can Do Mathematics	1
What Is Mathematics?	1
Why Is Learning Mathematics So Hard?	2
Responses From Mathematics Educators	4
About This Book	4
Questions This Book Will Answer	5
Chapter Contents	5
Other Helpful Tools	7
Who Should Use This Book?	7
Assessing Your Current Knowledge of How We Learn Mathematics	8
What's Coming?	9
CHAPTER 1—Developing Number Sense	11
Babies Can Count	11
What Is Number Sense?	12
Animals Also Have Number Sense	13
Why Do We Have Number Sense?	14
Learning to Count	14
Subitizing	14
Counting	16
How Language Affects Counting	18
The Mental Number Line	21
Expanded Notions of Number Sense	24
Can We Teach Number Sense?	25
Quantities to Words to Symbols	28
Gardner's Logical/Mathematical Intelligence	29
What's Coming?	30
Questions and Reflections	31
CHAPTER 2—Learning to Calculate	33
Development of Conceptual Structures	33
Structures in 4-Year-Olds	35
Structures in 6-Year-Olds	35

HOW THE BRAIN LEARNS MATHEMATICS

Structures in 8-Year-Olds	36
Structures in 10-Year-Olds	36
Dealing With Multiplication	37
Why Are Multiplication Tables Difficult to Learn?	37
Multiplication and Memory	37
Is the Way We Teach the Multiplication Tables Intuitive?	38
The Impact of Language on Learning Multiplication	41
Do the Multiplication Tables Help or Hinder?	42
What's Coming?	43
Questions and Reflections	44
CHAPTER 3—Reviewing the Elements of Learning	45
Learning and Remembering	45
Memory Systems	46
Impact of Technology on Attention and Memory	48
Rehearsal Enhances Memory	50
The Importance of Meaning	52
How Will the Learning Be Stored?	55
When Should New Learning Be Presented in a Lesson?	57
Does Practice Make Perfect?	58
Include Writing Activities	62
Fixed and Growth Mind-Sets in Mathematics	62
Gender Differences in Mathematics	63
Consider Learning Styles	65
Consider Teaching Styles	67
How Do You Think About Mathematics?	68
Motivating Students in Mathematics	70
Motivation Surveys	71
Strategies for Motivating Students in Mathematics	71
What's Coming?	73
Questions and Reflections	74
CHAPTER 4—Teaching Mathematics to the Preschool and Kindergarten Brain	75
Should Preschoolers Learn Mathematics at All?	76
Assessing Students' Number Sense	77
Preschoolers' Social and Emotional Behavior	78
What Mathematics Should Preschoolers and Kindergartners Learn?	78
Preschool and Kindergarten Instructional Suggestions	80
General Guidelines	80
Suggestions for Teaching Subitizing	81
Learning to Count	85
An Easier Counting System	87
Teacher Talk Improves Number Knowledge	87
Questioning	87
Developing Sorting and Classifying Skills	89
What's Coming?	93
Questions and Reflections	94

CONTENTS

CHAPTER 5—Teaching Mathematics to the Preadolescent Brain	95
What Is the Preadolescent Brain?	95
How Nature Influences the Growing Brain	96
Environmental Influences on the Young Brain	98
Teaching for Meaning	99
What Content Should We Be Teaching?	102
Common Core State Standards for Mathematics	103
Teaching Process Skills	105
Does the Lesson Enhance Number Sense?	105
Does the Lesson Deal With Estimation?	108
From Memorization to Understanding	112
Multiplication With Understanding	117
Does the Lesson Develop Mathematical Reasoning?	119
Using Practice Effectively With Young Students	123
Graphic Organizers	124
Taking Advantage of Technology	125
What’s Coming?	125
Questions and Reflections	126
CHAPTER 6—Teaching Mathematics to the Adolescent Brain	127
What Is the Adolescent Brain?	127
Overworking the Frontal Lobes	128
The Search for Novelty	130
Learning Styles and Mathematics Curriculum	133
Qualitative Versus Quantitative Learning Styles	133
Developing Mathematical Reasoning	134
Instructional Choices in Mathematics	135
Graphic Organizers	143
Interpreting Word Problems	149
Making Mathematics Meaningful to Teenagers	151
What’s Coming?	156
Questions and Reflections	157
CHAPTER 7—Recognizing and Addressing Mathematics Difficulties	159
Detecting Mathematics Difficulties	160
Determining the Nature of the Problem	160
Diagnostic Tools	161
Environmental Factors	165
Student Attitudes About Mathematics	165
Fear of Mathematics (Math Anxiety)	166
Neurological and Other Factors	173
Dyscalculia	173
Addressing Mathematics Difficulties	179
Research Findings	179
Basic Guidelines	181
The Concrete–Representational–Abstract Approach	181
Numeracy Intervention Process	185

HOW THE BRAIN LEARNS MATHEMATICS

Students With Nonverbal Learning Disability	185
Students With Both Mathematics and Reading Difficulties	187
English Language Learners	190
What's Coming?	194
Questions and Reflections	195
CHAPTER 8—Putting It All Together: Planning Lessons in PreK–12 Mathematics	197
Questions to Ask When Planning Lessons	197
Is the Lesson Memory Compatible?	198
Does the Lesson Include Cognitive Closure?	198
Will the Primacy–Recency Effect Be Taken Into Account?	199
What About Practice?	201
What Writing Will Be Involved?	201
Are Multiple Intelligences Being Addressed?	203
Does the Lesson Provide for Differentiation?	203
Integrating the Arts	206
From STEM to STEAM	207
Examples of How to Integrate the Arts in Mathematics Lessons	208
Places Where Arts Integration in Mathematics Is at Work	210
Simplified Instructional Model	210
Remember Action Research	212
Conclusion	212
Questions and Reflections	214
GLOSSARY	215
REFERENCES	219
RESOURCES	235
INDEX	241

Introduction

Numbers rule the universe.

—Pythagoras

EVERYONE CAN DO MATHEMATICS ■

Human beings are born with some remarkable capabilities. One is language. In just a few years after birth, toddlers are carrying on running conversations without the benefit of direct instruction. Over the next few years, their sentences become more complex and their vocabulary grows exponentially. By the age of 10, they understand about 10,000 words and speak their native language with 95 percent accuracy.

Another innate talent is number sense—the ability to determine the number of objects in a small collection, to count, and to perform simple addition and subtraction, also without direct instruction. Yet by the age of 10, some of these children are already saying, “I can’t do math!” But you never hear them saying, “I can’t do language!” Why this difference?

One reason is that spoken language and number sense are survival skills; abstract mathematics is not. In elementary schools we present complicated notions and procedures to a brain that was first designed for survival in the African savanna. Human culture and society have changed a lot in the past 5,000 years, but the human brain has not. So how does the brain cope when faced with a task, such as multiplying a pair of two-digit numbers, for which it was not prepared? Thanks to modern imaging devices that can look inside the living brain, we can see which cerebral circuits are called into play when the brain tackles a task for which it has limited innate capabilities. That the human brain *can* rise to this challenge is testimony to its remarkable ability to assess its environment and make calculations that can safely land humans on the moon and send a space probe into orbit around a planet hundreds of millions of miles away.

Children often say, “I can’t do math!” But you never hear them say, “I can’t do language!” Why this difference?

WHAT IS MATHEMATICS? ■

To most people, mathematics is about calculating numbers. Some may even expand the definition to include the study of quantity (arithmetic),

space (plane and solid geometry), and change (calculus). But even this definition does not encompass the many areas where mathematics and mathematicians are found. A broader definition of mathematics comes from W. W. Sawyer (1982). In the 1950s, he described mathematics as the “classification and study of all possible patterns” (p. 12). He explained that *pattern* was meant “to cover almost any kind of regularity that can be recognized by the mind” (p. 12).

Other mathematicians who share Sawyer’s view have shortened the definition even further: Mathematics is the science of patterns. Devlin (2000) not only agrees with this definition but has used it as the title of one of his books. He explains that patterns include order, structure, and logical relationships, and go beyond the visual patterns found in tiles and wallpaper to those that occur everywhere in nature. For example, patterns can be found in the orbits of the planets, the symmetry of flowers, how people vote, the spots on a leopard, the outcomes of games of chance, the relationship between the words that make up a sentence, and the sequence of sounds we recognize as music. Some patterns are numerical and can be described with numbers, such as voting patterns of a nation or the odds of winning the lottery. Other patterns, such as a leopard’s spots, are visual designs not connected to numbers at all.

Mathematics can be defined simply as the science of patterns.

Devlin (2000) further points out that mathematics can help make the invisible visible. Two-thousand years ago, the Greek mathematician Eratosthenes was able to calculate the diameter of Earth with considerable accuracy and without ever stepping foot off the planet. The equations developed by 18th-century mathematician Daniel Bernoulli explain how a jet plane flying overhead stays aloft. Thanks to Isaac Newton, we can calculate the effects of the unseen force of gravity. More recently, linguist Noam Chomsky has used mathematics to explain the invisible and abstract patterns of words that we recognize as a grammatical sentence.

If mathematics is the science of patterns and if visible and invisible patterns exist all around us, then mathematics is not just about numbers but about the world we live in. If that is the case, then why are so many students turned off by mathematics before they leave high school? What happens in those classrooms that gives students the impression that mathematics is a sterile subject filled with meaningless abstract symbols? Clearly, educators have to work harder at planning a mathematics curriculum that is exciting and relevant and at designing lessons that carry this excitement into every day’s instruction.

I will leave the discussion of what content to include in a preK–12 mathematics curriculum to experts in that area, especially now that the Common Core State Standards for Mathematics (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010) have been released and adopted by many states. My purpose here is to suggest how the research in cognitive neuroscience can be used to plan lessons in mathematics that are more likely to result in learning and retention.

Why Is Learning Mathematics So Hard?

Succeeding in high school mathematics is still no easy feat. Take a look at Table I.1. The results of the 2013 National Assessment of Educational

INTRODUCTION

Table I.1 Proficiency Levels for Grades 4, 8, and 12 in Mathematics on NAEP, 2005–2013

Year	Grade 4				Grade 8				Grade 12			
	<i>Below Basic</i>	<i>At Basic</i>	<i>Proficient</i>	<i>Advanced</i>	<i>Below Basic</i>	<i>At Basic</i>	<i>Proficient</i>	<i>Advanced</i>	<i>Below Basic</i>	<i>At Basic</i>	<i>Proficient</i>	<i>Advanced</i>
2013	17	41	34	8	26	38	27	9	35	39	23	3
2011	18	42	34	7	27	39	26	8	—	—	—	—
2009	18	43	33	6	27	39	26	8	36	38	23	3
2007	18	43	34	6	29	39	25	7	—	—	—	—
2005	20	44	31	5	31	39	24	6	39	38	21	2

SOURCE: NAEP (2013).

Progress (NAEP) mathematics tests of twelfth-grade students revealed that 23 percent were considered proficient in mathematics skills. This was the same percentage found in the 2009 assessment. No educator or parent can feel reassured by results showing such a low percentage of high school seniors performing at this proficiency level in mathematics. For fourth graders, 34 percent were proficient, compared with 33 percent for 2009 and 34 percent for 2011. As for the eighth graders, 27 percent scored proficient in 2013, compared with 26 percent for both the 2009 and 2011 assessments. The improvement was not significant (NAEP, 2013). Despite all the attention and high-stakes testing focused on mathematics instruction in recent years, achievement results have barely moved.

Explanations for this lackluster performance abound. Some say that learning mathematics is difficult because it is so abstract and requires more logical and ordered thinking. Others say that the various symbols used in mathematics make it similar to tackling a foreign language. Education critics maintain that only a few students are really developmentally incapable of handling mathematics and that the poor performance stems mainly from inadequate instruction. They cite the so-called “math wars” as hindering major progress in mathematics curriculum development, similar to what the “reading wars” did to reading instruction during the 1990s.

Impact of Teacher Preparation

Another potential factor affecting students’ success in mathematics is the content knowledge of their teachers. Numerous studies have shown that middle and high school students learn more when their teachers have certifications or degrees in mathematics (e.g., Wayne & Youngs, 2003). Although states have been increasing the course requirements for individuals to be licensed to teach mathematics in secondary schools, problems persist. A recent study of 115 prospective middle school mathematics teachers at a large U.S. public university revealed that a substantial number of them had a limited knowledge of algebra for teaching (Huang & Kulm, 2012). The students made numerous mistakes when solving quadratic equations

and in algebraic reasoning and manipulation. They also had difficulties with connecting algebraic and graphical representations.

A survey by the U.S. Department of Education showed that only 63 percent of the nation's nearly 144,000 high school mathematics teachers have both a college major and certification in mathematics (Hill, 2011). Nearly 26 percent have only a major or only certification in mathematics, and about 11 percent have neither a major nor certification in mathematics. This last group is referred to as out-of-field teachers. Other surveys find that out-of-field teachers are more likely to be found in high-poverty schools or in schools where they are assigned to the most challenging students (e.g., Kalogrides, Loeb, & Betelle, 2011).

Responses From Mathematics Educators

The National Council of Teachers of Mathematics (NCTM) published the *Principles and Standards for School Mathematics* in 2000, proposing five process standards and five content standards for preK–12 mathematics instruction. Since then, interpretation of the standards in the elementary and middle school grades became so broad that NCTM decided to refocus the curriculum at those grade levels.

In 2006, NCTM released *Curriculum Focal Points for Mathematics in Prekindergarten Through Grade 8*, which identified three important mathematical topics at each level, described as “cohesive clusters of related knowledge, skills, and concepts” that form the necessary foundation for understanding concepts in higher-level mathematics. The publication was intended to bring more coherence to the very diverse mathematics curricula currently in use. It provided a framework for states and districts to design more focused curricular expectations and assessments for preK–8 mathematics curriculum development. Shortly thereafter, the National Mathematics Advisory Panel (2008) published its final report, making recommendations for curriculum changes, teacher education, instructional practices and materials, and assessment. In the meantime, the National Governors Association and the Council of Chief State School Officers launched an effort to develop standards in mathematics and English/language arts. They were finally published in 2010 as the Common Core State Standards for Mathematics, and have been adopted by most of the states. We will discuss these standards further in Chapters 4 and 5.

Whether this new effort will succeed in improving student achievement in mathematics remains to be seen. In the meantime, teachers enter classrooms every day prepared to help their students feel confident enough to master mathematics principles and operations. One thing seems certain: Students who are poor in mathematics in their early years remain poor in mathematics in their later years.

■ ABOUT THIS BOOK

I am often asked to give specific examples of how the fruits of scientific research have made an impact on educational practice. That question is a lot easier to answer now than it was 20 years ago because recent discoveries in cognitive neuroscience have given us a deeper understanding of the