Uncovering STUDENT THINKING in MATHEMATICS

25 Formative Assessment Probes

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Preface

Mathematics Questions to Uncover and Explore Student Thinking

Overview

With mandates from the No Child Left Behind Act and other state-driven assessment initiatives, substantial amounts of educator time and energy are being spent on developing, implementing, scoring, and analyzing summative assessments of students' mathematical knowledge. Although the importance of summative assessment is recognized, findings point to *formative* assessment as an important strategy in improving student achievement in mathematics.

Formative assessment informs instruction. It takes many forms, the purpose of which is determining students' prior knowledge of a learning target and using that information to drive instruction, moving each student toward understanding of the targeted concepts and procedures. Questioning, observation, and student self-assessment are examples of instructional strategies that educators can incorporate to gain insight into student understanding. These instructional strategies become formative assessment if the results are used to plan and implement learning activities designed to address the specific needs of the students.

This resource focuses on using diagnostic questions, called Mathematics Assessment Probes, to elicit prior understandings and commonly held misconceptions. This elicitation allows the educator to make sound instructional choices based on the specific needs of a particular group of students.

> Diagnostic assessment is as important to teaching as a physical exam is to prescribing an appropriate medical regimen. At the outset of any unit of study, certain students are likely to have already mastered some of the skills that the teacher is about to introduce, and others may already understand key concepts. Some students are likely to be deficient in prerequisite skills or harbor misconceptions. Armed with this diagnostic information, a teacher gains greater insight into what to teach. (McTighe & O'Connor, 2005, p. 14)

The Mathematics Assessment Probes provided in this resource are tools teachers can use to gather these important insights.

Audience

The collection of Mathematics Assessment Probes and the accompanying Teachers' Notes are designed for the busy K–12 classroom teacher who understands that a growing body of research discusses students' learning difficulties and that thoughtful use of this research in developing and selecting diagnostic assessments promises to enhance the efficiency and effectiveness of mathematics instruction.

Background

The probes are designed to uncover student understandings and misunderstandings based on research findings, and they have been pilot-tested and field-tested with teachers and students.

Because the probes are based on cognitive research, examples exist in multiple resources, but no previous publication had collected them and designed them for the specific purpose of action research in the classroom. In the actual research reports, the questions used in these mathematics assessment probes are spread throughout the material and are not ready for classroom use. In this book, the probes were developed using the process described in *Mathematics Curriculum Topic Study: Bridging the Gap Between Standards and Practice* (Keeley & Rose, 2006). They were originally piloted with the Maine, New Hampshire, and Vermont teachers participating in the National Science Foundation (NSF)-funded Northern New England Co-Mentoring Network. The use of the probes was expanded to include teacher leaders and mathematics specialists who are Fellows in the Maine Governor's Academy for Mathematics and Science Education Leadership, a State Mathematics and Science Partnership Project: Mathematics: Access and Teaching in High School (MATHS), and various other mathematics professional development programs offered through the Maine Mathematics and Science Alliance.

Organization

This book is organized to provide readers with the purpose, structure, and development of the Mathematics Assessment Probes, as well as to support the use of applicable research and instructional strategies in mathematics classrooms.

Chapter 1 provides in-depth information about the process and design of the mathematics probes along with the development of the QUEST cycle. Chapter 2 highlights instructional implications and images from practice to illuminate how easily the probes can be used in mathematics classrooms and how many ways they can be employed. Chapters 3 to 5 are the collections of probes categorized by content strands, within grade spans, with accompanying Teachers' Notes that provide the specific research and instructional strategies designed to speak directly to the mathematics involved specific to the probe.

TEACHERS' NOTES: FRACTIONAL PARTS

Grade Level for "Fractional Parts" Probe

Grades K–2	3–5	6–8	9–12

Questioning for Student Understanding

What do students understand about part-whole relationships in identifying fractional parts?

Uncovering Understandings

Fractional Parts Content Standard: Number and Operations
Variation: Fraction ID

Examining Student Work

The distracters may reveal *overgeneralizations* regarding application of whole number properties to working with fractions.

- *The correct answer is 1/4.* Students who answer 1/4 most likely consider the size of B as it relates to the whole circle. They are using knowledge of part-whole relationships. (See Students 5 and 6 in *Student Responses* section.)
- *Students who answered 1/5.* Students who answer 1/5 considered the total number of pieces of the circle without relating to the size of the pieces within the circle. They most likely see all pieces as the total, much as in whole numbers, without considering the varying size of pieces in the circle. (See Students 1 and 2 in *Student Responses* section.)
- *Students who answered "Other."* Students who chose other fractions or whole numbers displayed difficulty defining a part-whole relationship. (See Students 3 and 4 in *Student Responses* section.)

Seeking Links to Cognitive Research

> Of all the ways in which rational numbers can be interpreted and used, the most basic is the simplest—rational numbers are numbers. The fact is so fundamental that [it] is easily overlooked. A rational number like 3/4 is a single entity just as the number 5 is a single entity. Each rational number holds a unique place (or is a unique length) on the number line. Further, the way common fractions are written (e.g., 3/4) does not help students see a rational number as a distinct number. Research has verified what many teachers have observed, that students continue to use properties they learned from operating with whole numbers even

though many whole number properties do not apply to rational numbers. With common fractions, for example, students may reason that 1/8 is larger than 1/7 because 8 is larger than 7. Or they may believe that 3/4 equals 4/5 because in both fractions the difference between numerator and denominator is 1. (NRC, 2001, p. 235)

One of the primary characteristics of students' informal knowledge of fractions is that students' informal solutions involve separating units into parts and dealing with each part as though it represents a whole number, as opposed to dealing with each part as a fraction (Mack 1990). For example, consider the following problem: If you have 5/6 of a cake and I eat 2/6 of the cake, how much of the cake do you have left? Students often refer to the fractions in the problem in terms of the "number or pieces" (e.g., five pieces or five pieces of six). However, the use of fraction names (e.g., five-sixths) refers to the fractions as specific parts of a whole. (NCTM, 2002, p. 137)

Fractions make it possible to represent numbers between whole numbers. Fractions express numbers as an indicated division of two whole numbers. A fraction bar indicates the division. Some of the main ways that fractions are used in elementary school are as part of the whole, as quotient representations of ratios, as measures, as individual numbers on a number line, and in computation. (Bay Area Mathematics Task Force, 1999, p. 59)

In grades K–2, in addition to work with whole numbers, young students should also have some experience with simple fractions through connections to everyday situations and meaningful problems, starting with the common fractions expressed in the language they bring to the classroom, such as "half." At this level it is more important for students to recognize when things are divided into equal parts than to focus on fraction notation. (NCTM, 2000, p. 82)

In grades 3–5, students should build their understanding of fractions as parts of a whole and as division. They will need to see and explore a variety of models of fractions, focusing primarily on familiar fractions such as halves, thirds, fourths, fifths, sixths, eighths, and tenths. By using an area model in which part of a region is shaded, students can see how fractions are related to a unit whole, compare fractional parts of a whole, and find equivalent fractions. They should develop strategies for ordering and comparing fractions, often using benchmarks such as 1/2 and 1. (NCTM, 2000, p. 150)

In grades 6–8, students should deepen their understanding of *fractions*, decimals, percents, and integers, and they should become proficient in using them to solve problems. Students should have learned to generate and recognize equivalent forms of *fractions*, decimals, and percents, at least in some simple cases in grades 3–5. (NCTM, 2000, p. 215)

Teaching Implications

To support a deeper understanding for students in elementary school in regard to fractions, specifically with identifying the size of fractional parts, the following are ideas and questions to consider in conjunction with the research.

Focus Through Instruction

- Students should make comparisons between numbers by using their understanding of equivalence
- Students need experience considering that fractions have precise locations and values in our number system
- Students should see various meanings and models of fractions; how they are related to each other and to the unit whole and how they are represented
- Models such as the number line and thermometer allow students to consider numbers less than zero
- Use of an area model allows students to “see” the part-to-whole relationship
- Students should develop strategies for ordering and comparing fractions
- Students should learn to use benchmarks, for example, 1/2 and 1, to compare size
- Students should become familiar with equivalent fractions

Questions to Consider . . . *when working with students as they grapple with the ideas of fractions*

- Do students use what they know about whole numbers when working with fractions?
- Can students make connections about relative size of fractions in relation to whole numbers?
- Do students use their knowledge of division in working with fractions?
- Are students using models to represent their thinking as proof of equivalence and to demonstrate their understanding of fractions?

Teacher Sound Bite

“I believed that my Grade 5 students would have no difficulty correctly identifying B as 1/4. I was disappointed in my assumption when I realized that more than half of my students chose 1/5 along with some 2/6, 1/3, and various nonfraction representations. Giving this probe and seeing my students’ explanations, I was able also to see that some students who correctly said 1/4 did not have solid reasoning for their decision, and this prompted me to design some instruction related to the findings of the probe.”

Additional References for Research and Teaching Implications

Bay Area Mathematics Task Force (1999), *A Mathematics Source Book for Elementary and Middle School Teachers*, pp. 59–70.

NCTM (1993a), *Research Ideas for the Classroom: Early Childhood Mathematics*, pp. 11–15.

NCTM (2000), *Principles and Standards for School Mathematics*, pp. 82, 150, 215.

NRC (2001), *Adding It Up: Helping Children Learn Mathematics*, p. 235.

Wearne & Hiebert (2001), *Putting Research Into Practice in the Elementary Grades*, p. 137.

Fractional Parts

Curriculum Topic Study

Related CTS Guide: Fractions

STUDENT RESPONSES TO "FRACTIONAL PARTS"

Sample Responses: 1/5

Student 1: There are five blocks, and B is one out of the five so you make a line and put five on the bottom and one on the top.

Student 2: There are five sections, and B represents one section filled in so therefore it is 1/5.

Sample Responses: Other

Student 3: I think it is one and a half fifth because it takes up that much of the circle.

Student 4: 1/2 out of 5/5 because B and C are a little bit bigger than A, D, and E.

Sample Responses: 1/4

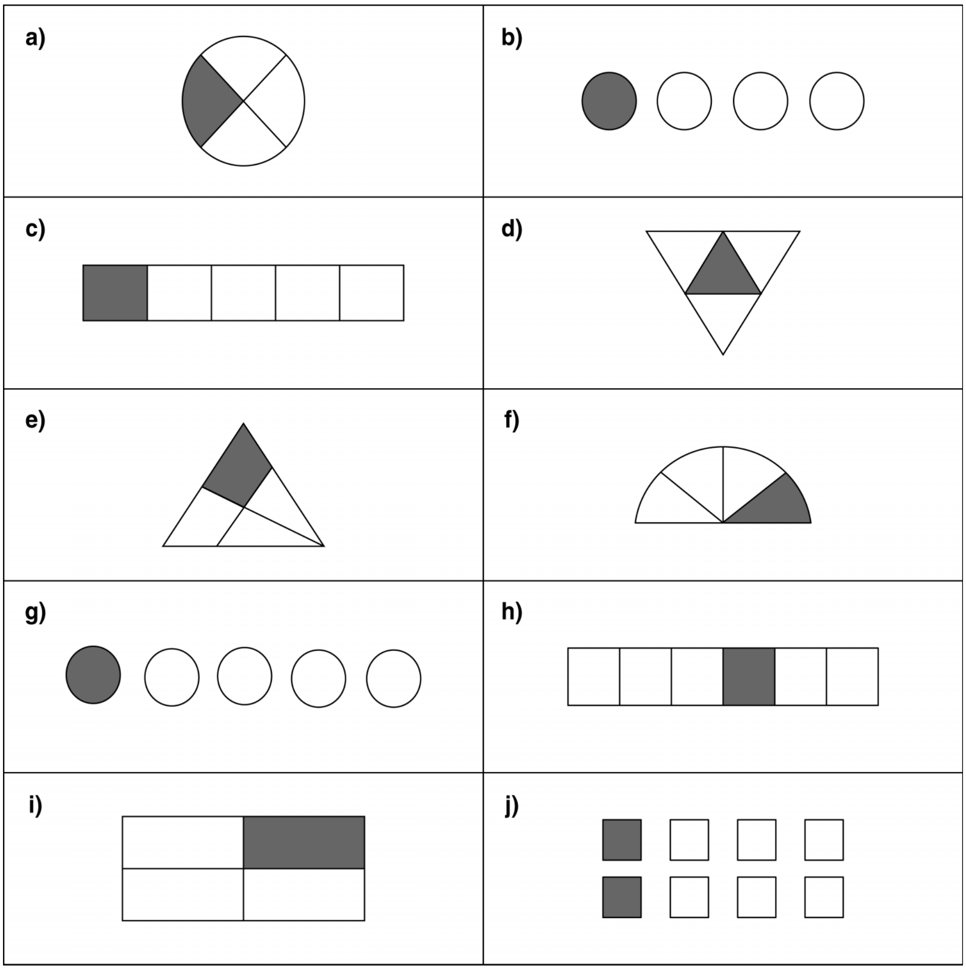
Student 5: 1/4 because if you split the circle into 4 groups B is 25% of the whole and 1/4 equals 25%.

Student 6: B is 1/4 of the circle because half of a half of the circle is B.

Probe 4a

VARIATION: FRACTION ID

Circle the items that show ¼ of the whole.



How did you decide if the item showed ¼ of the whole?