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Mathematics Assessment Probes

To differentiate instruction effectively, teachers need diagnostic assessment strategies to gauge their students' prior knowledge and uncover their understandings and misunderstandings. By accurately identifying and addressing misunderstandings, teachers prevent their students from becoming frustrated and disenchanted with mathematics, which can reinforce the student preconception that "some people don't have the ability to do maths". Diagnostic strategies also allow for instruction that builds on individual students' existing understandings while addressing their identified difficulties. The Mathematics Assessment Probes in this book allow teachers to target specific areas of difficulty as identified in research on student learning. Targeting specific areas of difficulty – for example, the transition from reasoning about whole numbers to understanding numbers that are expressed in relationship to other numbers (decimals and fractions – focuses diagnostic assessment effectively (National Research Council, 2005, p. 310).

Mathematics Assessment Probes represent one approach to diagnostic assessment. The probes specifically elicit prior understandings and commonly held misconceptions that may or may not have been uncovered during an instructional unit. This elicitation allows teachers to make instructional choices based on the specific needs of students. Examples of commonly held misconceptions elicited by a Mathematics Assessment Probe include ideas such as "an equals sign means *the answer follows*" and "to add fractions, add the numerators and then add the denominators". It is important to make the distinction between what we might call a silly mistake and a more fundamental one, which may be the product of a deep-rooted misunderstanding. It is not uncommon for different students to display the same misunderstanding every year. Being aware of and eliciting common misunderstandings and drawing students' attention to them can be a valuable teaching technique (Griffin & Madgwick, 2005) that should be used no matter what particular curriculum program a teacher uses, be it commercial, region developed or teacher developed.

The process of diagnosing student understandings and misunderstandings and making instructional decisions based on that information is the key to increasing students' mathematical knowledge.

To use the Mathematics Assessment Probes for this purpose, teachers need to

- determine a question;
- use a probe to examine student understandings and misunderstandings;
- use links to cognitive research, national curriculum and maths education resources to drive next steps in instruction;
- implement the instructional unit or activity; and
- determine the impact on learning by asking additional questions.

The probes and the above process are described in detail in this chapter. The Teacher Notes that accompany each of the Mathematics Assessment Probes in Chapters 3 to 6 include information on research findings and instructional implications relevant to the instructional cycle described above. Detailed information about the information provided in the accompanying Teacher Notes is also described in detail within this chapter.

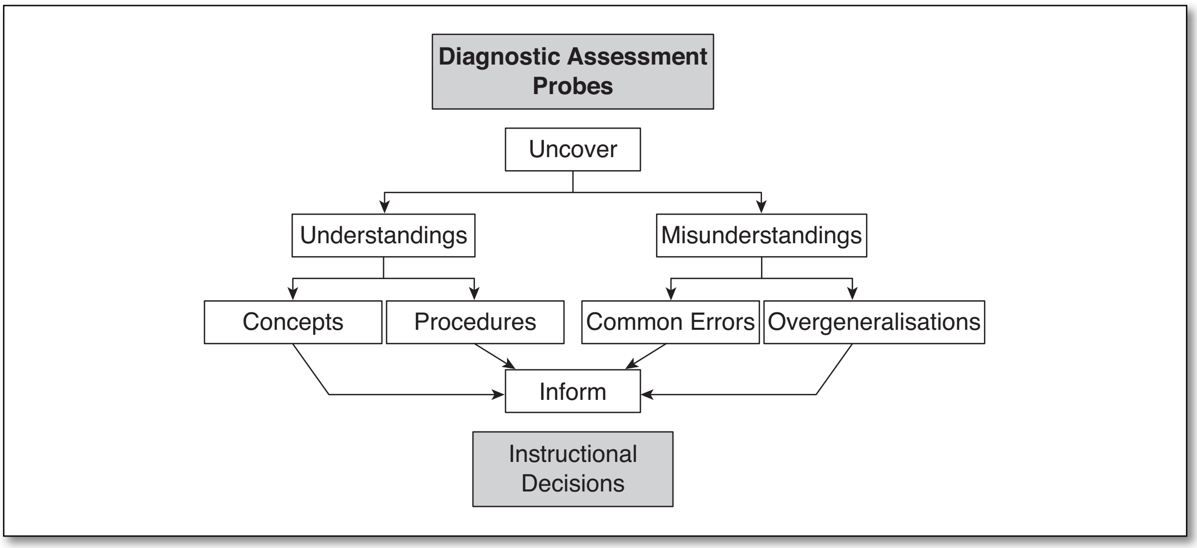
WHAT TYPES OF UNDERSTANDINGS AND MISUNDERSTANDINGS DOES A MATHEMATICS ASSESSMENT PROBE UNCOVER?

Developing understanding in mathematics is an important but difficult goal. Being aware of student difficulties and the sources of those difficulties, and designing instruction to diminish them, are important steps in achieving this goal (Yetkin, 2003). The Mathematics Assessment Probes are designed to uncover student understandings and misunderstandings; the results are used to inform instruction rather than make evaluative decisions. As shown in Figure 1.1, the understandings include both conceptual and procedural knowledge, and misunderstandings can be classified as common errors or overgeneralisations. Each of these is described in the following in more detail.

Understandings: Conceptual and Procedural Knowledge

> Research has solidly established the importance of conceptual understanding in becoming proficient in a subject. When students understand mathematics, they are able to use their knowledge flexibly. They combine factual knowledge, procedural facility and conceptual understanding in powerful ways (National Council of Teachers of Mathematics [NCTM], 2000).

Figure 1.1 Diagnostic Assessment Probes



Source: Rose, C., Minton, L. & Arline, C. (2007).

Conceptual Understanding

Students demonstrate conceptual understanding in mathematics when they

- recognise, label and generate examples and non-examples of concepts;
- use and interrelate models, diagrams, manipulatives and so on;
- know and apply facts and definitions;
- compare, contrast and integrate concepts and principles;
- recognise, interpret and apply signs, symbols and terms; and
- interpret assumptions and relationships in mathematical settings.

Procedural Knowledge

Students demonstrate procedural knowledge in mathematics when they

- select and apply appropriate procedures;
- verify or justify a procedure using concrete models or symbolic methods;
- extend or modify procedures to deal with factors in problem settings;
- use numerical algorithms;
- read and produce graphs and tables;
- execute geometric constructions; and
- perform noncomputational skills, such as rounding and ordering.

Source: From US Department of Education, 2003, Chapter 4.

The relationship between understanding concepts and being proficient with procedures is complex. The following description gives an example of how the Mathematics Assessment Probes elicit conceptual or procedural understanding. The What Is the Value of the Digit? probe (see Figure 1.2) is designed to elicit whether students understand place value beyond being able to procedurally connect numbers to their appropriate places.

Students who choose B, *There is a 2 in the ones place*, and E, *There is a 1 in the tenths place*, understand the value of the place of digits within a number. By also choosing C, *There are 21.3 tenths*, and H, *There are 213 hundredths*, these students also demonstrate a conceptual understanding of the relationship between the value of the places and the number represented by the specific combination of digits.

Figure 1.2 What Is the Value of the Digit? (see page 71, Probe 6, for more information on this probe)

Teacher prompt: "Circle all of the statements that are true for the number 2.13."

Statement	Explanation (why circled or not circled)
A) There is a 3 in the ones place.	
B) There is a 2 in the ones place.	
C) There are 21.3 tenths.	
D) There are 13 tenths.	
E) There is a 1 in the tenths place.	
F) There is a 3 in the tenths place.	
G) There are 21 hundredths.	
H) There are 213 hundredths.	

Misunderstandings: Common Errors and Overgeneralisations

In *Hispanic and Anglo Students' Misconceptions in Mathematics,* Jose Mestre (1989) describes misconceptions as follows:

> Students do not come to the classroom as "blank slates". Instead, they come with theories constructed from their everyday experiences. They have actively constructed these theories, an activity crucial to all successful learning. Some of the theories that students use to make sense of the world are, however, incomplete half-truths. They are misconceptions.
>
> Misconceptions are a problem for two reasons. First, they interfere with learning when students use them to interpret new experiences. Second, students are emotionally and intellectually attached to their misconceptions because they have actively constructed them. Hence, students give up their misconceptions, which can have such a harmful effect on learning, only with great reluctance. (para. 2–3)