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User's Guide

This book is about strategies for using the four modes of language – reading, writing, listening and speaking – to facilitate and enrich the teaching of mathematics. Although these strategies will work for teaching mathematics at any level, most of the examples and specific references to content focus on years three to nine.

The Introduction gives a rationale for creating a language-rich mathematics classroom in plain English. This rationale is supported by the Common Core State Standards in Mathematics in the US, however can be applied globally. These CCSSM consist of eight Standards, each of which depends, to an extent, on the teacher's skillful use of language in the classroom. By language in the classroom, I do not mean the teacher's language alone. The thesis of this book is that students – especially those whose preferred learning style is verbal – need to talk and write their ways into understanding. Verbal learners need to speak, read, write and listen to each other to internalise the mathematics they are expected to learn.

Most of this book consists of a detailed explanation of ten classroom-ready strategies for incorporating plain English activities in a mathematics class. I expect you to find that you are already using at least some of these strategies to some extent. I'm hoping that you will embrace and refine more of them in your repertoire of teaching skills.

The two appendices offer useful lists about word components and word connectedness. These lists are not to be served up as gobs of information to be memorised by rote. Rather, they are offered to enrich your own explanations about how the words of mathematics are connected to familiar words in students' vocabularies and lives.

The polarisation of mathematics and English is false and unnecessary. The flip side of the "I'm no good in maths" coin should not be "But I'm good in English". Being good in English is an advantage that can be leveraged to become good in mathematics. Please read this book with those language-loving learners in mind. Make them believe that mathematical language can become plain English to them.

Strategy 1

Teaching Mathematical Words Explicitly

Excellent teachers of mathematics use plain English to teach the words of mathematics *explicitly*. They understand that etymology illuminates meaning. They help students make connections between strange-sounding mathematical words (*polygon, trinomial, denominator*) and familiar words (*tricycle*), as well as words that students learn (or will learn) in other disciplines (*polytheism*). Learning about how words are related through etymology is a fascinating life-long pursuit that excellent teachers of mathematics include in their explanations about terminology.

When I ask mathematics teachers what the biggest stumbling block is in "reading the maths", they invariably reply that it is vocabulary. The words of mathematics tend to be long, unfamiliar sounding and unfamiliar to look at – that is, having letters that are rare in conversational vocabulary: a lot of *x, y, z,* and strange consonant combinations: *rh, ph, gn*. The sheer unfamiliarity of the appearance of the "math words", combined with the daunting length of the words creates a stumbling block between the reader and the text.

In what we will call ordinary text (as opposed to technical mathematical language), unfamiliar words are often nestled in enough context that the reader can figure out the meaning, if not on the first exposure, then perhaps through subsequent exposures. With mathematical terminology, the words cannot be figured out through contextual clues. Chances are, the new words are introduced with explicit definitions at the outset of a chapter. After that, you're on your own. "We told you once what congruent angles are, now here's a bunch of congruent angles that go about having all kinds of adventures and getting into all kinds of scrapes and problems that you have to figure out."

Many mathematical terms have more than one personality. Words like *value, property, associate, solve* and even *find* are used broadly in conversation, but very narrowly in the world of mathematics. It's as if they have a "home" personality, which the student knows, but then they adopt a whole new personality when they go to work for maths. For maths, they suit up and put on a maths "game face". This maths game face does bear a resemblance to the familiar, conversational meaning, but not all students can deepen their understanding of a mathematical term by connecting it to its vernacular meaning. We must teach students to make these connections if we want them to understand mathematical terms on a deeper level.

Unfortunately, many teachers, in their desire to get students to use mathematical terms precisely, deny the conversational meaning altogether! "Forget what you think you know about the word *ray* (or *value, property, area, square,* etc.). It doesn't mean what you think it means. In mathematics, it means something different." The problem with this approach is that it strips the student of the most valuable resource for learning that they have: *background knowledge*. The better strategy is to acknowledge and draw from background knowledge about words, open the door to connectedness and then narrow the word from its familiar meaning to its mathematical meaning in a way that integrates the mathematical meaning into the student's schema. (Appendix 1, page 101, gives detailed etymology that will help you illuminate these connections.)

Then, there are those mathematical phrases that need to be treated as single words. The student needs to process phrases like *base ten system*, *side of an angle* and *greatest common factor* as units – immediately recognisable codes in the language of mathematics.

The words of mathematics, then, fall roughly into three categories:

- *Mathematics only:* Words that we are likely to encounter only in the world of mathematics: *milligram, frustum, radian, rhombus, quotient, etc.*
- *Multiple meaning:* Words that have a specific meaning when used in the world of mathematics, but that have another meaning when used in other *academic* fields or in conversation: *function, line, point, evaluate, improper, etc*.
- *Phrases of mathematics:* Words that, when put together, mean more than the sum of their parts; phrases are to be understood as a whole, as if they were single words: *common denominator, sum of the squares, lowest common multiple, linear equation, etc*.

Teaching the Words of Mathematics and the Academic Words Surrounding Them

How do words – any words – get learned and stay learned? And, what is specific about the words of mathematics – math words, we'll call them – that might deserve special consideration?

To shed light on how words get learned and stay learned, I will refer to the theories of Stephen Krashen (2004), whose work in the field of second language acquisition can be applied to learning technical terminology in one's native language. In other words, let's think about what works well in expanding language capacity in general, and then apply those principles to expanding our students' language capacity to include the language of the world of mathematics.

Krashen sets forth five hypotheses about second language acquisition. Let's look at each and consider its applicability to learning the words of mathematics.

Acquisition-Learning Hypothesis

This theory posits that we learn a second language in two ways. The first is naturally, by being exposed to the target language in a meaningful context. In the process of attempting to communicate in the target language, a person picks up both the words and the grammar of that language. The learner is concentrating on purpose-driven communication, not language acquisition, but language acquisition results naturally and unconsciously.

The second means of learning a language is through direct, orderly instruction in the target language.

As this applies to mathematics class, we already know what the direct, orderly instruction looks like. And we already rely heavily on this means: Picture the teacher – supplemented by her mathematics textbook – defining and giving examples and illustrations of the mathematical terms. This type of instruction could be made better by incorporating the etymologies of the mathematical terms so that they may be linked to other words already in the students' vocabulary. (See Appendix 1, page 101, for an annotated list of common word components used in mathematical language.) The more connections and associations that are made to *any* new terminology, the more memorable and meaningful the learning of the terms will be.

For teachers to improve their direct instruction in the vocabulary of mathematics, they need to learn more about how words of mathematics are connected to familiar words and expand their explanations accordingly. In so doing, they are building the habit in their students of making similar kinds of connections.

Now let's expand the model to incorporate natural language acquisition. When students are given opportunities to communicate in authentic problem-solving situations about mathematics, when they are listening to their teachers not as strictly information givers but as co-solvers of the problem, when students strive to express themselves mathematically, cued by a person fluent in the language of mathematics, we have what Krashen refers to as *acquisition*.

To illustrate, picture yourself in a cooking class. The master chef explains the key terms to be used in a recipe that you are about to prepare. These terms include terms about process (verbs): *stir, mix, separate, blend, blanche*. The terms refer to the ingredients and tools (nouns): *cilantro, shallots, Chinese eggplant, wok*. If you were not fluent in the language of cooking, you would learn the verbs by watching and doing the processes, and you would learn the nouns by seeing them. And, in the context of authentic communication, you would hear key words repeatedly. But you would hear not only these key words: you would hear a set of supportive words that are often used in the cooking field: words of sequence, words about temperature, words about the condition and texture of food. You would absorb more words than you realised. Some of these words would come into your full control; others, you would learn to a lesser degree, and you may come into full control of them as you continue your experiences and communication about cooking. By communication, we mean more than just listening to the master chef. To learn a language, we need to use the language as novices. A language learner needs to repeat directions, ask questions, create analogies, ask for clarification, rephrase information.

As it is with any specialised vocabulary, so it is with mathematical words! The more your students are given opportunities to engage in authentic communication in mathematics, the more their mathematical vocabulary will grow in depth and scope.

Monitor Hypothesis

Krashen's second theory is called the *monitor hypothesis*: This facet of language learning may be described as the development of intuition about what sounds right/what sounds wrong in the language and how to correct it. To develop the internal monitor that allows us to edit and correct our own language, we need to be steeped in the language. As students hear and read the language of mathematics, they develop their internal monitors only if they are given ample opportunities to use all four language capacities: listening and reading (receptive capacity), speaking and writing (productive capacity).

The interplay of all four language capacities is extremely important if we want students to become proficient in mathematics. The "old model" of teaching maths relies on listening as the main language capacity. Reading may be done as part of homework, word problems or using the textbook as a reference. Speaking is actually done very little, with a few students voicing questions and answers. Cooperative learning, which involves speaking among peers, is often regarded as "taking too long", "being disorganised" or "being disorderly". Writing is done rarely, except in geometric proofs or the labelling of steps when students are required to "show their work."

We can surmise that the sheer lack of experience in the four language capacities has a lot to do with the lack of development of the self-monitoring abilities that result from constant practice in the target language. To fix that, we need to weave listening, reading, speaking and writing into mathematics classes, even if it "takes too much time". The time that it takes is a wise investment. In short, if students in mathematics class are too quiet, they may be too quiet to learn.

Natural Order Hypothesis

Krashen's third theory, the *natural order hypothesis*, has to do with the order in which speakers of a second language are likely to become grammatically proficient. Mathematical language has its own patterns, and those patterns form a kind of grammar. Although "maths grammar" follows the patterns of English grammar (maths grammar is not a *separate* kind of grammar) it does have its own preferred sentence structures, as follows:

- *Commands:* Many mathematical sentences begin with a verb that tells you what to do. In addition to the four operations – add, subtract, multiply, divide – we also encounter such commands as *find, evaluate, compare, solve, round, estimate, regroup, explain, undo, write, simplify, replace, reduce* and *check.*
- *Definitions:* Definitions are not always presented as sentences. A definition begins with a noun or a noun phrase that is then placed in a genus (general category, such as *shape, process, condition*) and then refined into a species (*having three sides, of adding like numbers, in which all angles are congruent, etc*.).
- *Word problems:* Word problems usually begin with a simple statement that expresses some kind of condition: *Kyle has a half a tank of petrol in his car*. This sentence is followed by details about the condition, expressed as a simple sentence, grammatically. The third sentence is likely to pose the problem in the form of a question. So, in a typical word problem, the student has to process the first two sentences to derive the information necessary to answer the question.
- *Syllogisms:* The famous *if…then* statement, known as a syllogism, is ever present in mathematics text.

Generally, sentences of mathematics have a simple grammatical structure that can be characterised as having short sentences and a clipped style. The syntax (sentence structure) of the mathematical language that we would find in a textbook is, in of itself, not unfamiliar or inaccessible to most native speakers. The syntax that they encounter in their English classes – stories, dialogue, poetry – is far more complex. Strategy 3 will go further into mathematical syntax and its implications for reading comprehension.

Input Theory

Krashen's fourth theory, the *input* theory, posits that the learner will advance in the target language if given sufficient "comprehensible input" so that the learner can *use what the learner knows* to go to the next level. What this means in mathematics class is that enough of the mathematical language has to be nestled in familiar language so that the learner is challenged without being totally lost in new language. Comprehensible input can manifest as short, grammatically simple sentences; familiar vocabulary; easily accessible metaphors; internal "translations" or "in other words" statements. With enough of these, targeted mathematical terms may be understood. Everyone whose job it is to initiate the novice into the professional conversation needs to keep the input theory in mind as they speak.

Affective Filter Theory

Finally, Krashen's theory of the *affective filter* educates us about the deleterious effect that negative emotions have on learning. Negative emotions such as fear, nervousness, anger, resentment, feelings of inferiority and a defeatist attitude impede the learner's ability to accept and remember new information. The wise and skillful mathematics teacher is keenly aware of how anxiety over the ability to learn and perform mathematically creates a mental block. They do everything possible to reduce maths anxiety: re-explains; creates a positive and nonthreatening classroom climate; differentiates instruction; celebrates small moments of success; acknowledges progress; connects the new to the known.

You can see how all of these five theories of second language acquisition overlap and interlock. In summary, we can say that new language/new terminology of mathematics is greatly helped by:

- Integrating the four language capacities: listening and reading (receptive capacities) and speaking and writing (productive capacities).
- Connecting new language to known language.
- Using simple, predictable grammatical structures.
- Providing comprehensible input so that new language is nestled in enough familiar language to make the new language accessible.