

# Using Open-Ended Maths Tasks High School

## Thinking Skills in Secondary Mathematics

Using Open-Ended Tasks  
Using Open-Ended Questions  
Using Creative and Critical Thinking  
Using MI Theory  
Using Bloom's Taxonomy  
Using Ryan's Thinkers Keys

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# Introduction

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## What are open-ended tasks?

While there are many different theories and tools available to enhance learning, methods of incorporating them into a traditional classroom and understanding the direct impact they can have on learning is not always easy. *Open-Ended Maths Tasks* has been written for maths teachers looking to include a range of different strategies into their teaching without dramatically altering the way they teach. It looks at how to write short, innovative tasks, how to incorporate them into a traditional lesson structure, and the impact they can have on maths learning.

*Open-Ended Maths Tasks* has been broken into four chapters, each covering a different learning theory – Creative and Critical Thinking, Multiple Intelligences, Bloom’s Taxonomy and Ryan’s Thinkers Keys. Each chapter provides a background on the learning theory and its different categories, and suggestions are provided for developing tasks for the maths classroom using each category. The additional benefits these theories can bring to students’ learning are outlined and suggestions are given for how tasks from each learning theory could be placed into a traditional curriculum to maximise their effectiveness.

The second part of the book is a set of task cards written with the four learning theories. As well as containing a feature of learning theories, each task incorporates one of four mathematical content learning areas – Structure, Number, Space, and Measurement, Chance and Data – for lower secondary (Years 7 and 8) or middle secondary (Years 9 and 10). Furthermore, every task in the book is “open-ended”. These tasks are included to be available for immediate use, as well as to demonstrate how tasks may be written to incorporate a range of features, to assist teachers to produce more of their own.

## Using Open-Ended Maths Tasks

The focus of this book is on using the four different learning theories to generate short tasks to complement traditional maths work. These tasks are designed to assist students to develop thinking skills not usually focused on during a normal maths class, as well as enhance comprehension of their maths skills. Most of the tasks take between 5 and 20 minutes and, with most of the learning theories designed to be accessible to all students, are ideal as an introduction or conclusion to a lesson in a normal maths class, or as a homework task. While most tasks are designed to take only a short time, there is also scope within each task for the teacher who wishes to further explore some issues that the task may raise. Advanced students completing work early may use their time to further challenge their understanding and develop extra skills by completing more of these tasks. Also, this book may be a guide for a teacher ready to put in some extra preparation by developing an extended task or project to incorporate extra skills while working with the maths

content. Furthermore, for the teacher or school looking to transform the traditional maths curriculum, these tasks could form the cornerstone of a more adventurous program.

Using the tasks outlined in this book to develop maths and thinking skills is obviously only the first step. Encouraging students to use these skills to improve performance on traditional maths assessment tasks will be the focus of a subsequent publication. It will look at identifying the specific locations in traditional maths assessment tasks requiring these skills, and assisting students to use these skills to perform better and more thoroughly on a range of traditional assessment tasks including basic skills tests, analysis tests, problem solving tasks, worded problems and research projects.

## Victorian Essential Learning Standards

As Victorian teachers, we acknowledge the importance and significance of changes in attitude to teaching and student learning in recent directive documents. The inclusion of Thinking as part of Interdisciplinary learning acknowledges that the ever-changing world we live in requires a population of competent thinkers. Students need to use existing knowledge to produce new understandings in creative ways. VELs states

*“Our world and the world of the future demand that all students are supported to become effective and skilful thinkers.”*

We believe that approaching mathematics with tasks that provide for a wide number of creative solutions assists in the development of skilful thinkers.

In developing this set of cards we acknowledge the different models of strategies, practices and taxonomies suggested by many informed people to improve student thinking. Gardner, Blooms, Costa and Kallick are a few. The VELs Teaching and Learning Resource, Support Materials lists a great variety of “learning theory, principles and strategies for teachers.” We too have included many of these ideas in the development of these task cards. The VELs Thinking document states

*“Students need to be supported to move beyond the lower-order cognitive skills of recall and comprehension to the development of higher-order processes required for creative problem solving, decision making and conceptualising.”* We hope that these cards do just that!

The three dimensions listed for consideration at each level documented in the VELs Thinking curriculum reinforces the need to provide for more flexible teaching and learning. The first, Reasoning, processing and inquiry, points out the need for critical thinking in organising information and making judgments. Creativity, the second dimension, documents the need for students to use their imagination to be innovative problem solvers. Thirdly, Reflection, evaluation and metacognition encourages learners to think about their thinking and to use learned information or skills for future learning. These dimensions are applicable at each level with increasing complexity as students advance through the levels.

We believe that by allowing our students to solve mathematical tasks that are meaningful, interesting and challenging, and to provide them with appropriate and varied means of solving them, we will develop students who automatically use their higher-order thinking to attempt to solve all problems that life presents them in the future. It is pleasing to see this acknowledged in VELS material also.

### **References and suggested further reading**

Anderson, L. W. and D. R. Krathwohl (eds), (2001). *A Taxonomy for Learning, Teaching and Assessing: A Revision of Bloom's Taxonomy of Educational Objectives*. New York: Longman.

# 1

# Creative and critical thinking

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Creative and Critical Thinking tasks can complement students' maths skills practice to develop the creative and analytical thinking skills required for problem solving and analysis tasks.

## What is Creative and Critical thinking?

Creative and Critical thinking is a set of analytical and lateral thinking skills. The model used for "Creative and Critical Thinking" in this book was outlined in the title, *Activities for Developing Thinking Skills* (Wellner and Yoder, 2005). In this model, four key components to both creative and critical thinking were identified. Their approach incorporates the four creative thinking skills in the cognitive domain identified by Frank E. Williams (1970). Creative thinking includes the components of fluency, flexibility, elaboration and originality, and tends to be required in tasks that are spatial, flexible, spontaneous, analogical and divergent. This is 'right brain' thinking. Critical thinking includes evaluating, forecasting, planning and decision making, and tends to be required in tasks that are logical, rational, sequential, analytical and convergent. This is 'left brain' thinking.

## Using "Creative and Critical thinking

In traditional maths assessment tasks, strong Creative and Critical thinking skills are required to complete analysis and problem solving tasks. Students can often struggle on these tasks despite having a solid grasp of the maths content required to complete the task. Developing Creative and Critical Thinking skills along with subject specific skills can assist students in their performance on analysis and problem solving tasks.

As a lesson introduction, conclusion or homework task, short creative and critical thinking tasks allow students to intersperse their maths skills practice with thinking skills practice. As demonstrated in the tasks included in this chapter, these tasks can also be written to contain appropriate maths skills to have the students warming up or challenging the maths skills, while simultaneously developing their thinking ability. An open-ended task including one of the four "creative" components is particularly well suited to acting as an informal pre- or post-test also.

Students should be encouraged to tackle tasks from different categories of creative and critical thinking each time. Each component plays an important role in different problem solving and analysis contexts, and students should be encouraged to develop their skills in all eight components.

# 2

# Multiple Intelligences

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Multiple Intelligences tasks can provide all students with the opportunity to learn and express their understanding of a concept in a context which makes most sense to them.

## What are Multiple Intelligences?

Howard Gardner is the Professor of Cognition and Education at the Harvard Graduate School of Education. As an educational psychologist, Gardner is probably most highly recognised for his contribution in the area of intellectual development with his theory of Multiple Intelligences (MI). Gardner's research revolved around his desire to move away from tests and correlations among tests, looking instead at more naturalistic sources of information about how people around the world develop skills important to their way of life.

Howard Gardner's theory validates what many teachers find in their classrooms: students think and learn in many different ways. It also provides teachers with a conceptual framework for organising and reflecting on curriculum assessment and pedagogical practices. Using Gardner's core list of eight intelligences (including the recent addition of the naturalist intelligence), varied, creative and high interest questions can be developed. Most people are generally stronger in one or two modes than the others, but can improve their intelligence in all areas.

## Using MI theory in the mathematics classroom

A diverse set of tasks written for a given concept around Multiple Intelligences can give each individual student the opportunity to use their maths skills in a context in which they feel most comfortable. Students can also be encouraged to use a set of Multiple Intelligences tasks to develop their abilities to work in a range of contexts they don't usually experience.

Short tasks based on Multiple Intelligences can supplement or replace part of the normal skills practice. Skills practice initially coming from a traditional textbook will typically work mainly in only one or two different intelligences. Students may be able to further deepen their understanding of a concept by choosing a challenging question from a set of Multiple Intelligences tasks. They should be encouraged to use this opportunity to explore maths concepts in contexts which most assist them to make sense of the concept. Also, students should be encouraged to develop their ability in a range of intelligences.

# 3 Bloom's Taxonomy

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Bloom's taxonomy allows all students from a range of abilities to explore the same concept by offering a range of questions of varying complexity and difficulty to challenge the depth of their understanding.

## What is Bloom's taxonomy?

Bloom's taxonomy is a well-known model for using critical thinking skills in any subject area (Anderson and Krathwohl, 2001). The different levels of Bloom's taxonomy require students to use a particular skill at increasingly more complex levels. Any student can function at each level of the taxonomy, provided the content is appropriate for his or her reading ability. Bloom's Taxonomy can be used to structure sets of learning tasks, student worksheets, cooperative learning assignments and independent study units. In order to consistently design lesson plans that incorporate all six levels, teachers can use the taxonomy to structure all student objectives, information sessions, questions, assigned tasks and tests.

## Using Bloom's taxonomy in the mathematics classroom

A set of tasks written at each level of Bloom's taxonomy for a given concept can allow students from a range of abilities to be challenged while working on the same concept. Students may also use a set of Bloom's taxonomy tasks to deepen their understanding of a concept by working through several tasks at progressively more challenging levels. The higher levels of Bloom's taxonomy typically also require, and develop, a broad range of thinking skills.

Short Bloom's taxonomy tasks can add to or replace part of the normal skills practice. Skills practice initially coming from a text book will typically work mainly in the more basic levels of Bloom's taxonomy. Following sufficient skills practice, students may challenge their understanding of a concept by choosing a task from a Bloom's taxonomy set at a more challenging level than what they had been doing. Students should be encouraged to work at the level which is appropriately challenging to them, and work through the levels over a period of time where possible.

Bloom's taxonomy can also be used as a simple and effective tool for structuring questions within a broader task such as an assignment. Students are provided with a range of questions on a topic and select from the range as they challenge and develop their understanding by moving to progressively higher levels.

# 4

# Tony Ryan's Thinker's Keys

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Ryan's Thinkers Keys challenge all students to develop their lateral thinking and explore the properties of maths concepts by forcing them to think about concepts from perspectives they would otherwise not usually consider.

## What are Ryan's Thinkers Keys?

Thinker's Keys (Ryan, 1990) are a set of activities designed to engage students in a range of thinking tasks. The keys are presented as 'question starters' with the majority emphasising the development of innovative and creative thinking. Each 'key' encourage students to challenge and develop their thinking and thereby extend their learning.

The Thinker's Keys were first published in 1990 in the book, *Thinker's Keys for Kids* (Ryan, 1990). More information about the keys and how to use them can be accessed via the website at [www.tonyryan.com.au](http://www.tonyryan.com.au).

For information on the thinking keys, example activities and details of suggested revisions to the model can be found in the book, *STILL Learning to Think/Thinking to Learn* (Pohl, 2006)

## Using Ryan's Thinkers Keys in the mathematics classroom

Traditional maths work focuses on developing students' understanding of concepts in routine situations. By asking students to explore different properties, uses and advantages of mathematical concepts, ideas and physical objects, students can further enrich their understanding of mathematical concepts.

Short open-ended tasks from Ryan's Thinkers Keys can be used to supplement students' skills practice. Following the development of their understanding of maths concepts in routine contexts, students can challenge and deepen their understanding using a Thinkers Keys task at the completion of skills practice, for homework, or as an introduction to a subsequent lesson.

Students should be encouraged to choose tasks from a variety of categories over time to experience a range of creative and lateral thinking using mathematical concepts in highly non-routine contexts.

# Key Card

**Fluency**

Year Level

**seven and eight**

Task encourages thinking at this level or category of the learning theory denoted below

Write down as many representations of  $\frac{1}{2}$  as you can.

Open-ended task question

Learning theory task is written with

**Number**

Mathematical content area

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## Key for Learning Theories

**Creative and critical thinking**

**Bloom's Taxonomy**

**Multiple Intelligences**

**Tony Ryan's Thinkers Keys**