

Contents

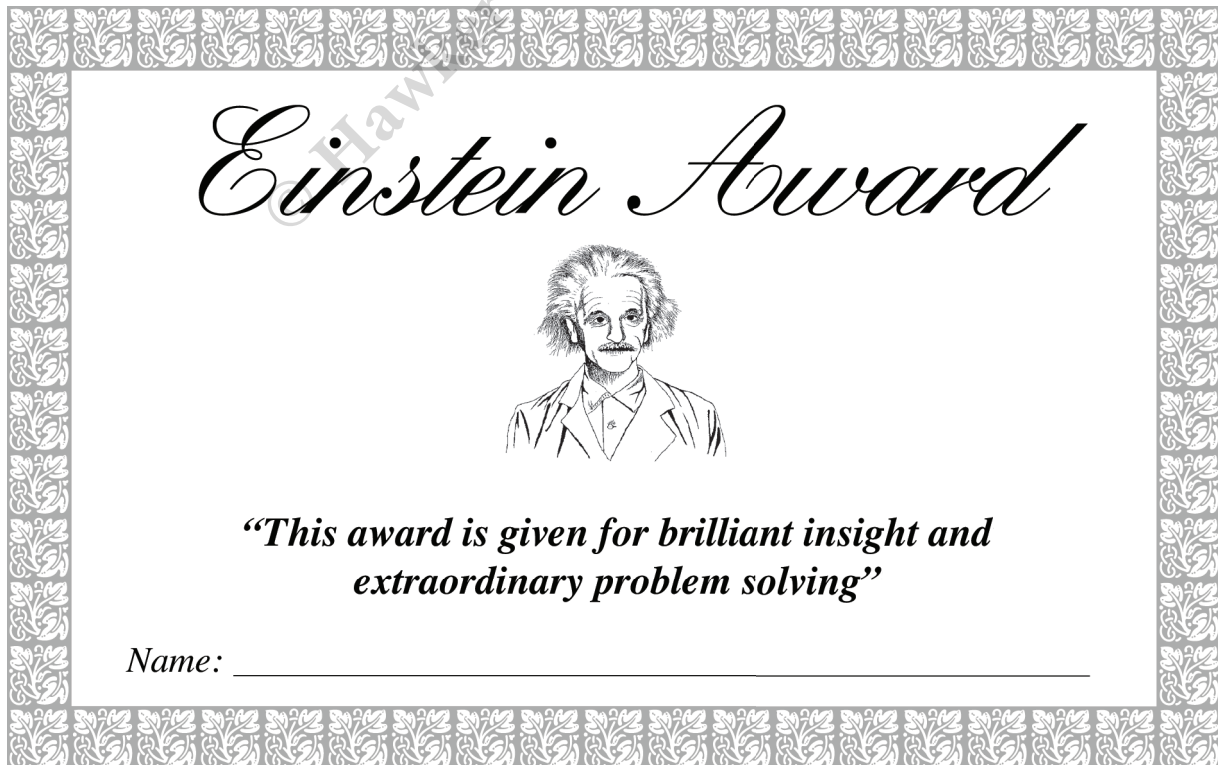
Chapter 1: Language of Algebra	1–8
Chapter 2: Solving Equations.....	9–24
Chapter 3: Using Algebra to Solve Problems	25–34
Chapter 4: Negative Numbers	35–46
Chapter 5: Distributive Property	47–54
Chapter 6: Algebra and Proportions.....	55–69
Chapter 7: Algebra and Percentages	71–90
Chapter 8: Exponents, Radicals and Scientific Notation	91–104
Chapter 9: Pythagorean Theorem.....	105–114
Chapter 10: Geometry and Algebra.....	115–124
Chapter 11: Algebra and Money	125–136
Chapter 12: Algebra and Physics	137–148
Chapter 13: Different Kind of Average Speed	149–156
Chapter 14: Distance = Rate \times Time (advanced).....	157–166
Chapter 15: Algebra and Work.....	167–172
Chapter 16: Simultaneous Equations	173–188
Chapter 17: Fun With Variables	189–194
Chapter 18: Order of Operations	195–200
Chapter 19: Function Machines	201–210
Chapter 20: Revision Activities.....	211–218
Solutions	219–278

Recognising and Honouring Academic Brilliance

Can you imagine what it feels like for an athlete to have hundreds of parents and classmates cheering for him or her? Add to that the newspaper articles, trophies, medals and other awards. This kind of reinforcement pushes athletes to excel. It is unlikely that this kind of motivating environment will ever become routine for those students who excel in maths and science. Because there are precious few opportunities for gifted children to be formally recognised and honoured, it is important that teachers make students feel that their gifts are something to be treasured.

For several years I have been recognising and honouring my students by handing out “Einstein Awards” for problem solving brilliance. I’d like to share an experience that shows the impact this kind of recognition can have on children. During a workshop I was conducting, one of the children solved a difficult problem with a very clever and insightful solution, for which she was given an Einstein Award. The next day, her teacher said that the child’s parents had called and mentioned that their child felt that the Einstein Award was “the best thing that ever happened to her”.

Some might say that this was a sad commentary on this child’s life, but at that moment, the power and importance of recognising and honouring academic brilliance in children became apparent. Children who have a special capacity for maths must learn to treasure and value their gift.



Introduction

Disaster struck Apollo 13 when it was 320,000 kilometres from earth. A routine journey to the moon suddenly became perilous when an oxygen tank aboard the service module exploded. As precious oxygen and power began draining away from the spacecraft, NASA and the astronauts of Apollo 13 struggled to find a way to bring their crippled vehicle safely back to earth. Throughout the four-day drama, as problem after problem arose, the astronauts depended on mathematicians and engineers to save their lives. The intellectual brilliance of these professionals helped avert a tragedy.

While the story of Apollo 13 is fairly well known, the important role of algebra in the rescue is not. The problem-solving power of algebra was used throughout the mission as NASA struggled with oxygen levels, carbon dioxide poisoning, engine thrust and re-entry angles.

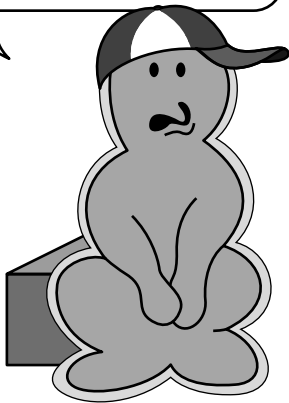
Algebra is the most powerful problem-solving tool ever invented. It was used during the Apollo 13 rescue, and it has also been relied on during the development of much of our civilisation. Unfortunately, many people are frightened and confused by algebra symbols and letters because they don't understand that algebra is simply a different language—a maths language.

Real World Algebra begins by explaining how to turn words into the maths language of algebra. Once students are comfortable working with the language of algebra, they easily step into working with equations and then real problem-solving. Soon they will not only be “translating” complex problems into algebra and solving them, but they will appreciate the many uses of algebra.



help of Grandma's machine. Now I am using algebra to help build buildings, design cars, and help the space program be successful. Have fun using the maths language of algebra. It will make you a great problem solver!!

I've heard that algebra is so amazing that it is almost like magic. What is algebra, and can it really allow me to solve very difficult problems?



Algebra is just like a foreign language, but it is a maths language. Watch me turn a maths problem into the maths language called algebra. Don't get scared by all the fancy letters though, I'll teach you how to "speak" algebra later. All I want to do now is show you how powerful algebra really is.



Maths Problem

Luke is twice as old as Dan, who is 5 years younger than Rachel. Sara is 9 years older than twice Luke's age and all their ages add up to 94. How old is Dan?

Maths Language of Algebra

$$n + 2n + n + 5 + 4n + 9 = 94$$

$$8n + 14 = 94$$

$$n = 10$$

Dan is 10 years old

That was unbelievable! I would have spent all day doing guess and check. You have really confused me, but I really want to learn algebra now.



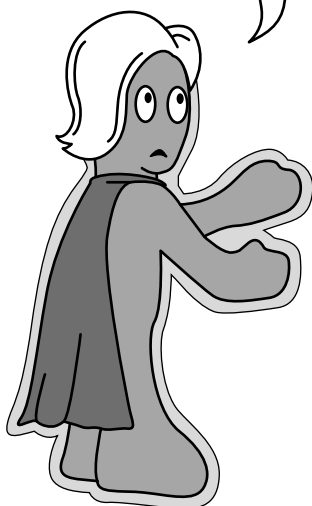
Before you can learn algebra, you have to learn how to change words into algebra. We call this turning words into algebraic expressions. Look at the sentence I wrote and watch how easy it is to change it into an algebraic expression. Whenever you don't know something, you use a letter to take its place. We will be using the same letter so it isn't too confusing. We will use the letter n .

Sentence

Kristin had \$17 and then she found a bag of money.

Algebraic Expression

$$17 + n$$



That's easy, you just put in an n for the amount of money in the bag because you don't know how much it is. I think I'll be able to learn the language of algebra pretty quickly!

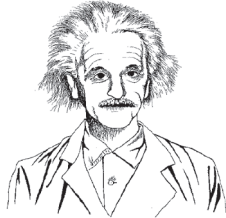


That was pretty easy. Try this one: If I said Jordan was twice as old as Steve, what expressions would you write for each of the children?



I see how to do that. I would call Steve n because I don't know his age. I would then call Jordan $2n$ because he is twice as old as Steve. If Steve had a brother three times his age, I would call him $3n$.





Always call the smallest unknown n . If we didn't do that in the last example we would have called Jordan n , and Steve would have been $\frac{1}{2}n$. There's nothing wrong with having fractions in our expressions, but it makes them a little bit messy.

Look at the sentences below and the translations into the language of algebra. See if you can understand why the algebraic expressions are written the way they are.

Sentences	Algebraic Expressions
Jill is three years older than Nancy	Expression for Nancy: n Expression for Jill: $n + 3$
Lee weighs 25 kilograms more than Ed	Expression for Ed: n Expression for Lee: $n + 25$
Three consecutive numbers	Expression for the smallest: n Expression for the next number: $n + 1$ Expression for the largest: $n + 2$
Three consecutive even numbers	Expression for the smallest: n Expression for the next number: $n + 2$ Expression for the largest: $n + 4$

These were easy. The three consecutive even numbers almost fooled me, but then I realised that even numbers are two away from each other.



Now you can tell people that you know another language. I think it's fun turning sentences into the maths language of algebra.



Match the words on the left with the expressions on the right.

Steve had an unknown amount of money in his pocket. He then lost \$23. What is the expression that shows how much money he has now? _____

$n \div 23$

$n - 23$

Adam found a bag of money that he split with 22 friends. What is the expression that shows the amount of money that each person has? (Don't forget to include Adam.) _____

$n + 23$

$n - 7$

Rachel found a box with money in it. What is the expression for this money? _____

n

πn

Steve cashed a cheque and then found \$23. What is the expression that shows how much money Steve has now? _____

$4n$

A dog lost 7 kilograms. What is the expression that shows the dog's current weight? _____

$6n$

$10n$

Ryan weighs 6 times as much as his dog. What is an expression for Ryan's weight if you call his dog's weight n ? _____

$50n$

What is an expression for the value of an unknown number of 10-cent coins? _____

$2n + 6$

$n + 7$

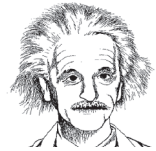
Jamie is 7 years older than Nancy. What is an expression for Jamie's age if Nancy is called n ? _____

Fred is 6 years older than twice his brother's age. What is an expression for Fred's age if his brother's age is called n ? _____

What is an expression for the circumference of a circle with a diameter of n centimetres? _____

What is an expression for the value of an unknown number of 50-cent coins? _____

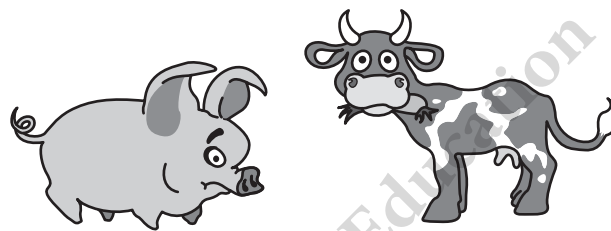
If there are 4 times as many dollar coins in a pile of coins as there are 2-dollar coins, what is the expression for the number of dollar coins if you call the number of 2-dollar coins n ? _____



The Language of Algebra

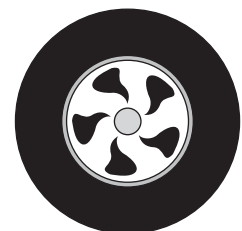
Einstein Level

- 1) Write an expression for the perimeter of a rectangle that has a length which is three times its width.
- 2) A farmer has 50 pigs and an unknown number of cows on his farm. Write an expression for the number of legs on the farm where n is the number of cows. (Make sure you include the farmer.)



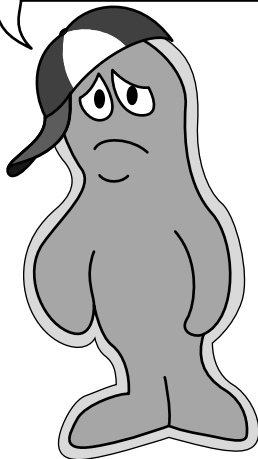
- 3) Dan receives \$9 per hour plus a bonus of \$55 each week. Write an expression for the amount of money Dan makes in a year (52 weeks) if he works the same number of hours each week.
- 4) Write an expression for the average of 4 consecutive even numbers where n is the smallest number.
- 5) A garden's length is five times its width. Write an expression for the number of sections of 10 metre fence that are needed for this garden.
- 6) A farmer has pigs and chickens. If the farmer has 75 total animals, write an expression for the number of chickens. Pigs: n
Chickens:

- 7) Write an expression for the radius of a circle with a circumference of n .
- 8) Write an expression for the number of revolutions a tyre with a circumference of n centimetres makes when it travels one kilometre.



Adding and Subtracting Negative numbers

I always get confused when I use negative numbers. I am starting to feel like I'll never understand them. Are there any rules I can use to help me?



There are a few rules for when you use negative numbers, but most of the time plain old common sense will tell you the correct answer.

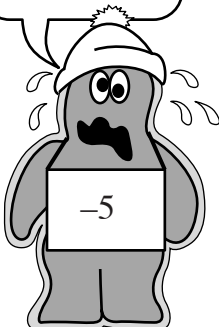


If I give you a number like -5, that is like saying someone owes \$5. Look at the problems below and see how easy it is to find the answers by using my amoeba friends who owe money. Don't let the parentheses frighten you, they are only there to separate the numbers we are adding.

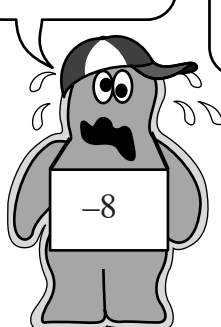


$$(-5) + (-8) = ?$$

I owe \$5



I owe \$8



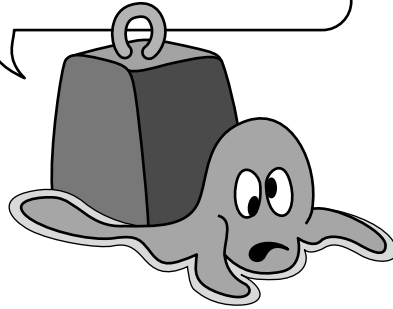
+

= -13

The answer is -13 because together they owe \$13.



The parentheses are really bothering me. You wrote the problem $(-5) + (-8)$. Is it okay if I write it as $-5 - 8$?



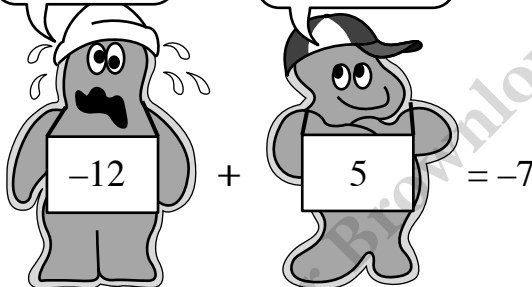
You can write it that way if you want. $-5 - 8$ also equals -13 . Look at the next two problems.



$$(-12) + (5) = ?$$

I owe \$12

I have \$5



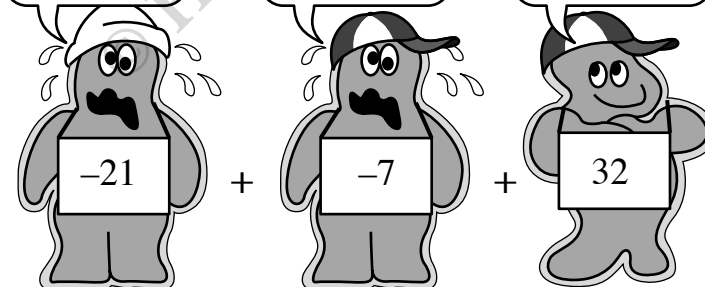
If they combine the money they owe with the money they have, my amoeba friends end up owing \$7. The answer is therefore -7 .

$$(-21) + (-7) + (32) = ?$$

I owe \$21

I owe \$7

I have \$32



If you combine the amounts of owed money with money they have, you would get \$4. This is because when the \$28 debt is paid from the \$32, the amoebas end up with \$4.

