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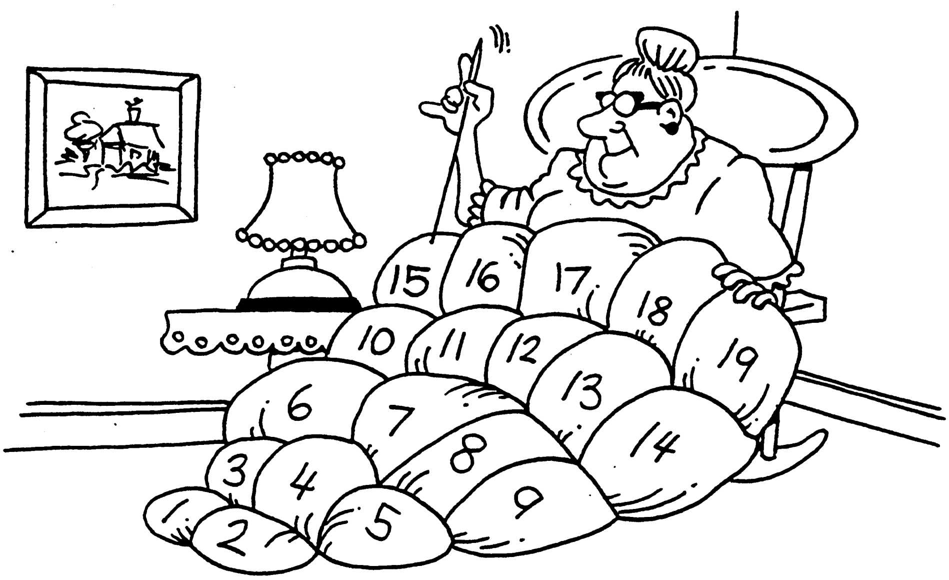
Introduction

Exploring Patterns in Mathematics provides teachers with information about—and activities that explore—various patterns that form the basis of mathematical concepts. Numerical and geometric patterns are included. Numerical concepts, such as sequences, powers, place value, operations with integers, binary numbers, modular (clock) arithmetic, expansion of binomials, and probabilities are presented through related patterns such as the Fibonacci sequence and Pascal's triangle. Geometric concepts of angle measures, transformations, tessellations, Platonic solids, and fractals are used to identify and create patterns. Students determine and develop code patterns in the section on cryptology. Patterns are presented to justify and give meaning to basic rules of mathematics, such as 'invert and multiply' and 'a negative times a negative equals a positive'.

The book includes six sections: Sequences, Number Theory Patterns, Pascal's Triangle, Encryption Patterns, Mathematical Art Designs, and Patterns in Chaos: Fractals. An answer key is provided. Although some activities complement other activities, the order of presentation is left to the discretion of the teacher.

Each section contains teacher notes that provide mathematical and historical background information for the section topic, such as definitions and general explanations. A brief description of each activity as well as comments that include explanations, mathematical proofs, and extension suggestions are provided. The blackline masters of the activities may be used to make copies for students to use individually or in small-group settings or to make a transparency for whole-class discussions.

Throughout the materials, patterns are identified, described, and created to explain and explore mathematical concepts. Students are asked to communicate verbally about mathematical patterns and to justify conclusions. Logical reasoning, critical thinking, and problem solving are encouraged and reinforced.



Teacher Notes

Sequences

A ***number sequence*** is an arrangement of numbers (shapes) in which each number (shape) follows the preceding number (shape) according to a rule. The numbers (shapes) in the sequence, called ***terms,*** form a pattern.

An ***arithmetic sequence*** is a number sequence in which each term is found by *adding a constant* called the ***common difference.***

3, 6, 9, 12, . . . is an arithmetic sequence with a common difference of 3.
27, 23, 19, . . . is an arithmetic sequence with a common difference of -4.

A ***geometric sequence*** is a number sequence in which each term is found by *multiplying by a constant* called the ***common ratio.***

3, 6, 12, 24, . . . is a geometric sequence with a common ratio of 2.
250, 50, 10, . . . is a geometric sequence with a common ratio of $\frac{1}{5}$.

A ***power sequence*** is a number sequence in which each term is found by raising consecutive counting numbers to the same power.

1, 4, 9, 16, . . . is a 2nd power sequence—the sequence of squares.
1, 8, 27, 64, . . . is a 3rd power sequence—the sequence of cubes.

The ***Fibonacci sequence*** is the number sequence 1, 1, 2, 3, 5, 8, 13, . . . The first two terms are 1 and 1, and the successive terms are found by adding the two preceding terms. The Fibonacci sequence is found throughout nature from pine cones and sunflower centers to the location of leaves on stems.

A ***Fibonacci-like sequence*** is any sequence in which terms are found by adding the two preceding terms.

3, 3, 6, 9, 15, 24, . . . is a Fibonacci-like sequence.

On and On and On

ACTIVITY Students complete various sequences and describe the pattern (rule) used.

COMMENTS Not all sequences are numerical. Geometric shapes affected by reflection, rotation, size, or shading changes can be used.

Some sequences may be continued in more than one way.

Example:
The sequence 2, 4, 8, . . . could be 2, 4, 8, 16, . . . (multiply by 2).
The sequence 2, 4, 8, . . . could be 2, 4, 8, 14, . . . (add consecutively larger evens—+2, +4, +6 . . .)

On and On and On . . . problems #6 and #7 have the same three initial terms but different 7th terms so they follow different rules.

It All Adds Up

ACTIVITY Students complete arithmetic sequences and determine the common differences. Students identify arithmetic sequences and determine if adding or multiplying terms by a constant will result in arithmetic sequences.

COMMENTS A term of an arithmetic (ar´ith met´ic) sequence is found by adding the common difference to the preceding term. The common difference can be positive or negative.

The sequence 32 29 26 23 . . . has a common difference of -3. The 5th term is 20 —23 + -3.

The sequence 2 5 8 11 . . . has a common difference of 3. The 5th term is 14—11 + 3.

Adding a constant to each term results in an arithmetic sequence
1 4 7 10 . . . (+4) becomes 5 8 11 14 . . .

Multiplying each term by a constant results in an arithmetic sequence
1 4 7 10 . . . (x 3) becomes 3 12 21 30 . . .

Teacher Notes

Pascal's Triangle

Pascal's triangle is named after Blaise Pascal (1623–1662), a French mathematician and physicist who wrote about the triangle and probability theory. The triangle was known as early as the thirteenth century to Chinese mathematicians.

Row:	
0	1
1	1 1
2	1 2 1
3	1 3 3 1
4	1 4 6 4 1
5	1 5 10 10 5 1
6	1 6 15 20 15 6 1

The triangle can be extended indefinitely. It contains many patterns. Each number is the sum of the two numbers to the left and right above it. The diagonals of the triangle contain several patterns. Each row gives the numbers of ways of getting each outcome of an event with binomial probabilities such as the number of heads or tails from tossing a coin, the number of girls or boys in a family, etc. The numbers in each row (0–n) also represent the coefficients of binomial expansions of the form $(x + y)^n$.

A Triangle of Patterns

ACTIVITY Students complete rows of Pascal's triangle and determine patterns of the various diagonals.

COMMENTS Each number is found by adding the pair of numbers above it. The diagonals contain several patterns. For instance, the first diagonal contains all 1s, the second diagonal is the set of counting numbers, the third diagonal is the set of triangular numbers. The sequence of the sums of number pairs in the third diagonal is the sequence of square numbers.