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Introduction

Math Magic: Slick Tricks with Numbers provides the teacher with magic tricks, activities, and shortcuts that are mathematically based. Concepts such as even/odd, place value, factors and multiples, fraction-decimal equivalents, casting out nines, algebraic identities, and integer trees are used to explain the tricks and develop the computational shortcuts. Activities explore and explain the Fibonacci series, geometric numbers, and Pascal's triangle. The activities may be used for class-wide discussions as well as partner and small group settings. Several activities ask students to make predictions and describe patterns. Student responses can be written or oral. Communication of mathematical concepts among students and between the teacher and students is emphasized.

The book includes six sections: Magical Mathematical Patterns; Magical Computational Tricks and Shortcuts; Magical Predictions; Magic Squares, Triangles, and Circles; Math Magic with Cards and Cups; and Seasonal Magic Activities. An Answer Key is provided. Although several activities complement one another, the order of presentation is left to the discretion of the teacher as each activity can be presented alone.

Each section contains teacher notes and blackline masters of the activities. The teacher notes contain a brief description of the activity, a list of needed materials, and comments that include an explanation of the activity, mathematical proofs, extensions, and suggestions for additional activities and class discussions. The blackline masters of the activities may be used to make a transparency for class-wide discussions or to make copies for students to use individually or in small-group settings. When two activities are printed on a page, it is recommended that the teacher make a copy of the page, make the necessary transparency and copies, and then cut the activities apart.

Throughout the materials, mathematics is used to explain the mystery of an activity and to let students be excited by, and explore, its magic.

Magical Mathematical Patterns

Teacher Notes

The activities in this section have students explore and describe mathematical patterns based on place value, the number 9, fraction/decimal equivalents, geometric numbers, and the Fibonacci series. "Switch-a-Roo" and "Are All Things Equal?" provide some surprising results that can be explained mathematically.

Switch-a-Roo

Activity Students compute products of pairs of numbers that contain the same four digits and are asked to determine the restrictions on the factors for the products to be equal.

Comments The comparison of the factors used in the multiplication problems requires considering place value.

The factors in each pair of problems use the same digits so that the products of the tens place and the ones place digits are equal.

For example, consider

$$\begin{array}{r} 39 \\ \underline{\times\ 62} \end{array} \quad \text{and} \quad \begin{array}{r} 93 \\ \underline{\times\ 26} \end{array}$$

$3 \times 6 = 18$ equals $9 \times 2 = 18$

The product of the tens digits equals the product of the ones digits.

In general, consider the pair of problems:

$$\begin{array}{r} 10A+B \\ \underline{\times\ 10C+D} \end{array} \quad \text{and} \quad \begin{array}{r} 10B+A \\ \underline{\times\ 10D+C} \end{array} \qquad \text{where } \mathbf{AC = BD}$$

For the first problem, $(10A + B) \times (10C + D) = 100\,AC + 10AD + 10BC + BD$
Since the digits were selected so that **AC = BD**,
$100AC + 10AD + 10BC + \mathbf{BD} = 100BD + 10AD + 10BC + AC$, which equals $(10B + A)(10D + C)$, the second problem.

Magical Nines

Activity Students complete three patterns based on the number 9 and are then asked to describe the patterns.

Comments The activity is appropriate for small-group work with calculators.

1. 8 x 9 = 8 x 9(1) = 72
   8 x 99 = (8 x 9)(10 + 1) = 720 + 72 = 792
   8 x 999 = (8 x 9)(100 + 10 + 1) = 7,200 + 720 + 72 = 7,992
   Largest place value is 7. Ones digit is 2. Other digits are 7 + 2 or 9.

2. 99 x 12 = (100 - 1) x 12 = 100 x 12 - 12
   In general, 99 x n = (100 - 1) x n = 100 x n - n

3. 1 x 9,109 = 9,109
   2 x 9,109 = 18,218
   In general, n x 9,109 = (n x 9,000) + (n x 100) + (n x 0) + (n x 9) = 9,000n + 100n + 9n.
   The digits in the product are n x 9 followed by n followed by n x 9.

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Could You Repeat That?

Activity Students explore the pattern of the digits in the repeating decimal equivalents for given fractions.

Comments This activity reinforces fraction and decimal equivalences. A topic for class discussion would be how to tell if a fraction has a terminating or repeating decimal equivalent—a fraction with a denominator containing prime factors of only 2s and/or 5s will terminate since the only prime factors of 10 are 2 and 5. All other fractions repeat. A repeating decimal will repeat in no more decimal places than one less than the equivalent fraction's denominator.

The most interesting pattern is perhaps the $\frac{1}{7}$s. The decimal equivalent of $\frac{1}{7}$ repeats in six digits: $\frac{1}{7}$ = 0.142857142857 . . . When the decimal 0.142857 is multiplied by the numbers 1 through 6, the products have the same order of digits as the original decimal.

Are All Things Equal?

Activity Students complete fraction problems which seem to prove that multiplication and division are the same as addition and subtraction.

Comments It must be noted that the equality statements rely on the pattern of the numbers used. The proofs offer a review of algebra concepts such as operating with rational expressions, factoring trinomials, and simplifying expressions.

Multiplication		Addition
$1\frac{1}{a} \times (a + 1)$	?	$1\frac{1}{a} + (a + 1)$
$\frac{a+1}{a} \times \frac{a+1}{1}$		$2 + \frac{1}{a} + a$
$\frac{a^2 + 2a + 1}{a}$		
$a + 2 + \frac{1}{a}$	=	$a + 2 + \frac{1}{a}$

Multiplication		Subtraction
$a \times \frac{a}{a+1}$	?	$a - \frac{a}{a+1}$
$\frac{a^2}{a+1}$		$\frac{a^2 + a - a}{a+1}$
$\frac{a^2}{a+1}$	=	$\frac{a^2}{a+1}$

Division		Addition
$\left(a + \frac{1}{a+2}\right) \div \frac{a+1}{a+2}$	?	$a + \frac{1}{a+2} + \frac{a+1}{a+2}$
$\frac{a^2 + 2a + 1}{a+2} \times \frac{a+2}{a+1}$		$\frac{a^2 + 2a + 1 + a + 1}{a+2}$
$\frac{(a+1)^2}{a+2} \times \frac{a+2}{a+1}$		$\frac{a^2 + 3a + 2}{a+2}$
$a + 1$		$\frac{(a+2)(a+1)}{a+2}$
$a + 1$	=	$a + 1$

Division		Subtraction
$\left(a + \frac{1}{a-2}\right) \div (a - 1)$	?	$a + \frac{1}{a-2} - (a - 1)$
$\frac{a^2 - 2a + 1}{a-2} \times \frac{1}{a-1}$		$\frac{a^2 - 2a + 1 - a^2 + 3a - 2}{a-2}$
$\frac{(a-1)^2}{a-2} \times \frac{1}{a-1}$		$\frac{a-1}{a-2}$
$\frac{a-1}{a-2}$	=	$\frac{a-1}{a-2}$