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WHAT IS PROBABILITY?

TEACHER'S GUIDE

MATERIALS: *Student pages 29 - 32, calculators.*

This lesson introduces students to the notation and terminology used by mathematicians who study probability. The lesson focuses on these ideas:

- A definition of *probability*: “the mathematical study of uncertain or chance events.”
- The potential values for the probability of an event: a value between 0 and 1 expressed as a fraction, decimal, or percent.
- The mathematical definition for the theoretical probability of an event: a fraction expressing the number of ways the event can occur out of the total number of outcomes.
- A mathematical definition of a fair game: each player has an equal chance of winning.

OVERVIEW OF PROBLEMS FOUND ON STUDENT PAGES 29 - 32:

- Problem 1:** Students list places in which they recall hearing references to probability. See the answer key for suggested responses.
- Problem 2:** Students estimate the probability for three statements about the next Prime Minister. Remind them that only events which are *impossible* have a probability of 0, and only events which are *certain* to happen have a probability of 1.
- Problem 3:** Students choose from a list of values to assign probabilities for events in their lives. It might be useful to have students order these decimal values from smallest to largest and discuss their relative magnitude before assigning approximate values.
- Problem 4:** Students list events in their own lives which have these approximate probabilities. Encourage students to include the time frame for each event. For example, “I will go to the mall *today*” may have a probability of 50% while “I will go to the mall *this year*” would have a much higher probability. See the answer key for suggested responses.
- Problem 5:** Students determine the theoretical probability of several events as a fraction, decimal, and percent based on a description of a collection of marbles from which one marble is drawn.
- Problem 6:** Students complete a similar table for a different collection of objects. In this exercise they also have to name events with a given probability.
- Problem 7:** Students determine if three versions of a game are fair based on the theoretical probability of each player winning.

The answer key for this lesson can be found on page 80.

AN EXPERIMENT IN PROBABILITY

A CARD GAME

TEACHER'S GUIDE

MATERIALS: *Student pages 33 - 36, one set of cards per group (see pages 84 and 85 for blackline masters), calculators. Collections of corresponding standard playing cards may be used instead, if desired.*

This experiment involves a card game in which two cards are drawn at random from a set of seven cards. The seven cards include four aces, a jack, a queen, and a king. Students will see whether two cards drawn from this set of cards are more likely to produce a pair of aces or two unmatched cards.

LIKELY STUDENT PREDICTIONS

Some students are likely to predict that Player A has the advantage in this game since more than half of the cards to choose from are aces. Other students are likely to notice that even if just one of the cards is not an ace, Player A cannot score a point, and for this reason will predict that Player B is likely to win. Some may predict that the game is fair since once an ace is drawn, there are three cards left which might produce a match and three cards which will not.

LIKELY CLASS RESULTS

Allow 10–15 minutes for students to play the game. Students are instructed to draw at least 30 times according to the directions on their sheets. Groups which work quickly can be encouraged to draw more than 30 times. Additional data will enhance the analysis of the game. Students should NOT take on the role of Player A or Player B; they are simply recording the results for these players. Each draw must be recorded on the chart regardless of who drew the cards. Students may tally incorrectly if they think they only record for “their” player.

While there is no guarantee, Player B should win more games than Player A. If class results do not reflect this, check to see that students understand the game and are playing it correctly. Make certain they are shuffling or mixing the cards between draws and that the cards are not marked in any way. Check to see that each group is assigning tallies accurately. It is not uncommon for middle school students to try to manipulate the experiment to make their results match their predictions. Explain to students that to make a mathematical analysis of the game, they will need unbiased data.

The most efficient way to collect class data is to have each group post its results on the chalkboard or overhead projector. Students should copy this data onto their own sheets and compile the results. Be certain to remind students that they need to find the total number of trials for the whole class to compute the percentages.

Class percentages for Player A should be near 29% and for Player B, near 71%.

Students should conclude that this game favors Player B. This is based on the fact that Player B won most of the games played and that Player B has more than 50% of the total points in his or her favor.

Explaining the Probability in the Activity

There are 21 possible outcomes when two cards are drawn from a deck of seven cards. This can be seen by looking at the complete list below.

NOTE: Students may find it difficult to draw the card suit symbols. If this is the case, have them use letters (H for heart, D for diamond, S for spade, C for club).

A♦ - A♥	A♥ - A♣	A♣ - A♠	A♠ - J♥	J♥ - Q♥	Q♥ - K♥
A♦ - A♣	A♥ - A♠	A♣ - J♥	A♠ - Q♥	J♥ - K♥	
A♦ - A♠	A♥ - J♥	A♣ - Q♥	A♠ - K♥		
A♦ - J♥	A♥ - Q♥	A♣ - K♥			
A♦ - Q♥	A♥ - K♥				
A♦ - K♥					

Notice that a combination like A♦ - A♥ is the same as A♥ - A♦. For this reason the second pair is not included in the list. The shaded portions indicate draws which would be points for Player A. The rest of the pairs are the draws which would be points for Player B.

$$P(\text{Player A}) = \frac{2}{21} \text{ or } 29\%$$

$$P(\text{Player B}) = \frac{19}{21} \text{ or } 71\%$$

Some examples of fair games follow:

- No one scores a point if two black cards (♣ or ♠) are drawn. Player A gets draws that include two red cards (♦ and ♥). Player B gets draws that include a red (♦ or ♥) and a black (♣ or ♠) card.
- No one scores a point if the king is drawn with the queen. Player A gets draws that include the king or the queen. Player B gets draws that do not have the king or the queen.
- No one scores a point if A♦ is drawn with A♥. Player A gets draws that include the ace of diamonds or ace of hearts (but not A♦ with A♥). Player B gets draws that do not include the ace of diamonds or ace of hearts.
- Keep the directions as listed, but Player A scores 2.5 points every time a pair of aces is drawn while Player B scores 1 point each time a mismatched pair is drawn.

Students should be directed to consult the list of all possible outcomes to determine fair games. They can look for ways to assign the draws so that each player claims 10 of the 21 possible outcomes, and one outcome is a “do over” or a point for both players. Another approach is to compensate for the inequity in the game as presented by assigning points for each player in a way which accounts for the difference in the number of outcomes assigned to each.