Cooperative Learning & Geometry

High School Activities

by Becky Bride

Hawker Brownlow EDUCATION

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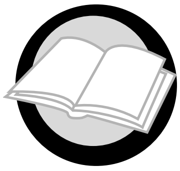

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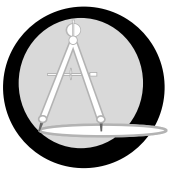

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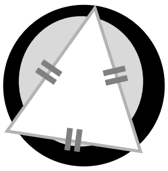

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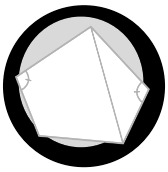

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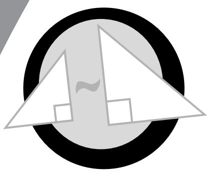

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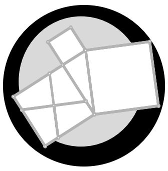

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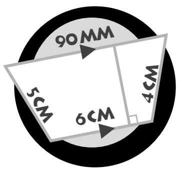

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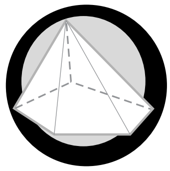

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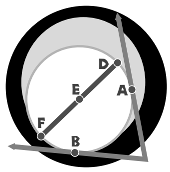

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INTRODUCTION

Have you ever asked your students if they had any questions on homework? When you saw hands go up did you ask, "What problems do you want to see?" Did the students ever reply "All of them?" This was a common occurrence in my mathematics classroom. It made me question why I bothered to come to school each day since my teaching efforts from the day before seemed fruitless. I felt like I was teaching mathematics to concrete blocks instead of students. Since I couldn't change the students, I began to re-evaluate what I did as a teacher. I began to search for a way to structure my classroom so that students would be more successful. In my search, I found Kagan Cooperative Learning. I had put students together before to work on projects and it was disaster. What I learned was that I had done group work, NOT cooperative learning. I never knew there was a difference.

Spencer Kagan has developed a way to structure student-to-student interaction so that there are no "hogs" (children that do it all) and no "logs" (children who contribute nothing). Inherent in each structure is a means to ensure individual accountability, equal participation, and positive interdependence, and have a minimum of 25 percent of the class overtly active at once. I was very intrigued. I went to every workshop I could to learn more about the structural approach to cooperative learning.

Then, five years ago, I began to use Dr. Kagan's structural approach to Cooperative Learning in my classroom. The results were incredible! The prior year I would spend on average 5 hours a week after school in help sessions for any student(s) who did not understand the material I had taught. There were on average anywhere from 7-20 students per day who came for after school help. The year I began implementing the structural approach to cooperative learning, my after school help sessions for students present in class were no longer needed. Only students who were absent from class and missed my instruction and the cooperative structures to process my curriculum came after school for help. I was amazed. That gave me time to lesson plan and mark papers. Before cooperative learning, I would spend anywhere from 20-40 minutes a class period (sometimes the entire period) to answer questions on the homework. With cooperative learning we spend 2-10 minutes to check homework and answer questions. Amazingly, the mistakes students make now on the homework are careless errors rather than conceptual mistakes. This has given me precious teaching time.

Academic achievement has also been phenomenal. In a typical geometry class, I have students in years 9-12. Year 9's are very sharp. Students in years 11 and 12 struggle at best. Before using cooperative learning, the year 12's six weeks grades were D's or F's. Using the Kagan Cooperative Learning, the same students now make B's and C's. The academic achievement of the other students has also improved with many more A's and B's. Because the students are more successful, they feel much better about themselves, their attitude toward maths has improved, and they take more mathematics classes! Students of average ability, whom I had three years ago in geometry class, are now in my calculus class. Ten years ago if I'd had these students, most would have never considered taking trigonometry, let alone calculus.

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INTRODUCTION CONTINUED...

The students enjoy coming to class because of the interaction they have with their peers. It is amazing how much bonding takes place among team members. When I change the seating chart, most students are sad to say goodbye. We do a short activity that brings closure to the group, then they meet their new teammates and begin the bonding process again.

Besides incredible advances for my students, using Kagan Cooperative Structures has had a major impact on me. When I leave the school everyday, I now feel that the students and I have accomplished incredible feats. I feel that my teaching is now productive and the classroom is certainly a whole lot more fun. Before cooperative learning, I would count the days until the summer holidays. Now the school year ends and I can't believe it went so fast. My transformation and that of my students is awesome!

My belief in Kagan Cooperative Learning is so great that I am now a national trainer for Kagan Professional Development. As I was doing workshops, I would hear maths teachers say that cooperative learning must not be very successful in high school, because they didn't see any maths curriculum for sale that was above year 8. What better way to demonstrate to teachers that Kagan Cooperative Learning is as valuable in a high school mathematics classroom as it is in a primary school classroom than to write a book that shows how versatile and encompassing cooperative learning can be? It was a labour of love. I chose to dedicate the entire book to geometry because it is such an incredible course and one that can be taught concretely. Included in the book are exploratory activities so students can concretely develop the concepts.

The book is divided into 10 chapters. The first chapter deals with vocabulary development. This lays a large foundation on which to build. A chapter on angles and lines (parallel and perpendicular) follows. Chapter 3 is devoted to constructions, compass and straightedge and tracing paper. These are included in Chapter 3 because many of the exploratory exercises in subsequent chapters require them. Chapter 4 explores triangles and the properties that guarantee that two triangles are congruent. The chapter on polygons and quadrilaterals follows and precedes the chapter on similarity because students need to compute missing angles in polygons to determine if two polygons are similar. Chapter 7 works with Pythagoras's Theorem and the special right-angled triangles because activities in the area, volume and circle chapters require students to use these concepts. Chapter 8 is a chapter on area of two-dimensional and three-dimensional figures. Chapter 9 works with volume. The final chapter of the book deals with properties of arcs, chords and angles in circles. The properties included in this chapter are the concepts needed to develop trigonometry concepts when the students take trigonometry.

Each chapter consists of several lessons. Each lesson consists of one or more exploratory activities with each exploration followed by one or more activities designed to help students process the concepts just investigated. Each activity used to process concepts just learned is based on one of Dr. Kagan's many structures. The use of these structures ensures that cooperative learning is taking place rather than group work.

INTRODUCTION CONTINUED...

The lessons begin with a brief synopsis of the activities, followed by teacher notes and directions for each structure used in the activities. The teacher notes for any activity name the structure it was designed for, materials needed and step-by-step directions. The chapters, lessons and activities are numbered sequentially. This system makes navigating the book very easy.

The activities are designed for use as a whole class. All work is done on a sheet of paper supplied by a partner or team member. Answers are included on the activities when work is necessary to solve the problems. Otherwise, answers are included in the teacher's directions. Students love the instant feedback they get from having the answers to check their work.

This book was written for teachers. I wanted to share with teachers how all geometry concepts could be developed with hands-on activities and processed using a variety of Kagan structures. I used a variety of structures in this book because it is fun to use different ones, to expose teachers to structures they may have not used, and to demonstrate how the structures can be used with many different concepts. My hope is that this book opens a whole new world for you and your students. How I wish I had a resource like this when I began teaching.

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LESSON 6
PROCESSING CONGRUENT TRIANGLES

This lesson slowly builds into proofs involving congruent triangles. It begins with identifying whether two marked triangles are congruent; progresses to marking the diagrams with given information to determine if two triangles are congruent; then there are flow chart proofs; blind sequencing with statements and reasons together, blind sequencing with statements and reasons separate; and finally proofs from scratch involving the concept, "corresponding parts of congruent triangles are congruent". The speed at which your class progresses through the next activities and the point you may choose to stop in this progression depends on the level of student you are teaching.

ACTIVITY

1 ARE WE CONGRUENT? #1

1. Student A supplies one sheet of paper, folded in half lengthwise with his/her name in one column and a partner's name in the other column.

2. Student B is the first boss and Student A is the first secretary.

3. As the boss, Student B tells Student A how to do the problem. Student A, the secretary, records what Student B says in Student B's column of the paper.

4. If the boss makes a mistake, then the secretary coaches and praises once the boss does it correctly. Otherwise, the secretary praises the boss.

5. Reverse roles for each problem and repeat steps 3 and 4.

Structure
- Boss/Secretary

Materials
- 1 Activity 1 (p. 178) per pair of students
- 1 sheet of paper and pencil per pair of students

Answers:

1. $\triangle EDF$; SAS≅
2. $\triangle AER$; SSS≅
3. $\triangle TAC$; SAS≅
4. Can't be determined
5. Can't be determined
6. $\triangle AKR$; SAS≅
7. $\triangle CDB$; HL≅
8. $\triangle KEI$; SAS≅

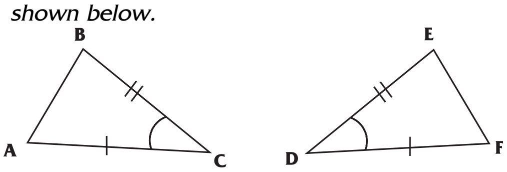
ACTIVITY

2 ARE WE CONGRUENT #2?

Structure
- Pairs Check

Materials
- 1 Activity 2 (p. 178) per pair of students
- 1 sheet of paper and pencil per pair of students

Note:
The only difference between Activity 2 and Activity 1 is that the students have to take given information and mark the diagrams first, then determine if the triangles are congruent and write the postulate or theorem that justifies the congruence. For the "why" part of the problem, have students write the theorem or postulate vertically, then write a pair of segments or angles to justify each part of the theorem or postulate. Problem 1 is shown below.



$\triangle BCA \cong \triangle EDF$
$S \mapsto \overline{BC} \cong \overline{ED}$
$A \mapsto \angle C \cong \angle D$
$S \mapsto \overline{AC} \cong \overline{FD}$

1. Student B supplies one sheet of paper, folded in half lengthwise and each member of the pair writes his/her name at the top of one column.

2. Student B does problem 1, recording his/her work on his/her side of the paper while Student A watches and coaches if necessary.

3. Student A checks Student B's work, coaches and/or praises. Then Student A does problem 2 on his/her side of the paper.

4. Student B checks Student A's work, coaches and/or praises.

5. After every two problems, the pair checks their answers with the other pair in their team, coaching and/or praising each others' work.

6. Repeat steps 2-5, reversing roles until all the problems are done.

Answers:
1. S $\overline{AB} \cong \overline{ED}$ given
A $\angle B \cong \angle D$ given
S $\overline{BC} \cong \overline{DC}$ given
$\triangle EDC$
2. S $\overline{MO} \cong \overline{TO}$ given
A $\angle NOM \cong \angle POT$ vertical angles are equal
S $\overline{NO} \cong \overline{PO}$ given
$\triangle POT$
3. A $\angle W \cong \angle C$ given
S $\overline{WA} \cong \overline{CA}$ given
A $\angle PAW \cong \angle MAC$ vertical angles are congruent
$\triangle MAC$
4. A $\angle A \cong \angle C$ given
A $\angle ADB \cong \angle CBD$ given
S $\overline{BD} \cong \overline{DB}$ reflexive
$\triangle CDB$
5. A $\angle G \cong \angle M$ given
A $\angle GHE \cong \angle MEH$ If lines parallel alternate interior angles congruent
S $\overline{HE} \cong \overline{EH}$ reflexive
$\triangle HME$
6. Can't be determined

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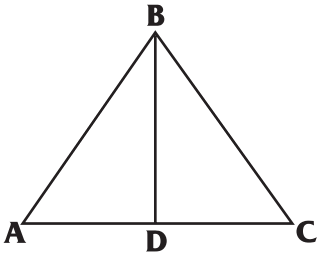
ACTIVITY

3 ARE WE CONGRUENT #3?

Note:
For this activity, the students must do all that was required in Activity 2 but now must justify each segment or angle they state is congruent. See the example below.

Given $\overline{AB} \cong \overline{CB}$

D is the midpoint of $\overline{AC}$
Show $\triangle ABD \cong \triangle CBD$



S ↦ $\overline{AB} \cong \overline{CB}$ Given congruent

S ↦ $\overline{AD} \cong \overline{CD}$ A midpoint separates a segment into two congruent parts

S ↦ $\overline{BD} \cong \overline{BD}$ It is the same segment in each triangle (Reflexive Property)

1. Student A folds a paper in half lengthwise, putting his/her name at the top of one column and his/her partners name at the top of the other column.

2. Student A marks the diagram with the given information and marks the diagram for reflexive parts or vertical angles if appropriate.

3. Student B checks to see if he/she agrees with what has been marked on the diagram, discusses disagreements with his/her partner. The pair celebrate when they agree on the markings. Student B then determines which postulate or theorem will prove the two triangles congruent and writes that postulate or theorem vertically on the paper.

4. Student A checks the postulate or theorem that Student B chose, coaches if necessary then praises a job well done. Student A then writes one set of segments or angles with the justification that corresponds to one of the parts of the theorem or postulate chosen in step 3.

5. Student B coaches and/or praises his/her partner for the work in step 4 and then writes one set of segments or angles with the justification that corresponds to another one of the parts of the theorem or postulate chosen in step 3.

6. Student A coaches and/or praises his/her partner for the work in step 5 and then writes the last set of segments or angles with the justification that corresponds to the last part of the theorem or postulate chosen in step 3.

7. Student B coaches and/or praises his/her partner for a job well done.

8. Reverse roles and repeat steps 2-7 for each remaining problem.

Structure
- RallyTable

Materials
- 1 Activity 3 (p. 180) per pair of students
- 1 sheet of paper and pencil per pair of students

Answers:

1. A $\angle A \cong \angle D$ given
S $\overline{AC} \cong \overline{DC}$ def. of midpoint
A $\angle 1 \cong \angle 2$ given
$\triangle DBC$

2. Possible solution
A $\angle 4 \cong \angle 3$ ∥ lines ↦ Alt. int. ∠s ≅
A $\angle 1 \cong \angle 2$ ∥ lines ↦ Alt. int. ∠s ≅
S $\overline{NO} \cong \overline{TO}$ given
$\triangle PTO$

3. S $\overline{AB} \cong \overline{CB}$ given
A $\angle ABD \cong \angle CBD$ def. ∠ bisector
S $\overline{BD} \cong \overline{BD}$ reflexive
$\triangle CBD$

4. A $\angle A \cong \angle D$ given
A $\angle B \cong \angle C$ given
S $\overline{AE} \cong \overline{DE}$ def. of midpoint
$\triangle DCE$

5. S $\overline{MS} \cong \overline{PS}$ def. of segment bisector
A $\angle 1 \cong \angle 2$ given
S $\overline{AS} \cong \overline{AS}$ reflexive
$\triangle PAS$

6. A $\angle ABD \cong \angle CBD$ def. of ∠ bisector
S $\overline{BD} \cong \overline{BD}$ reflexive
A $\angle ADB \cong \angle CDB$ def. of ∠ bisector
$\triangle CBD$

ACTIVITY

4 CHART IT!

Structure

- Boss/Secretary

Materials

- 1 Activity 4 (p. 181) per pair of students
- 1 sheet of paper and pencil per pair of students

Note:
This activity moves the students closer to proof with a flow chart sequence.

1. Student A supplies one sheet of paper, folded in half lengthwise with his/her name in one column and his/her partner's name in the other column.

2. Student B is the first boss and Student A is the first secretary.

3. As the boss, Student B tells Student A how to do the problem. Student A, the secretary, records what Student B says in Student B's column of the paper.

4. If the boss makes a mistake, then the secretary coaches and praises once the boss does it correctly. Otherwise, the secretary praises the boss.

5. Reverse roles for each problem and repeat steps 3 and 4.

Note:
This activity can be repeated using a tracing paper construction. To perform a "happy cyclops" tracing paper construction refer to page 123.

ACTIVITY

5 SEQUENCE IT! #1

Structure

- Blind Sequencing

Materials

- 1 set of blind sequencing cards per team – Activity 5 (pp. 182-185)

Note:
Make a copy of the blind sequencing cards before you cut them up. These will be the answer cards that students can use to check their work. Emphasise that this is one possible answer. It is sequenced so that, as the given information is put in the proof, it is immediately used in the next step to help students see the logical progression and how the given helps to develop the proof.

1. Give each team one set of the blind sequence cards on pages 182 and 185. Have the team place the diagram face up for everyone to see.

2. Ask students to take the cards, mix them up and place them face down on the table. Each student, taking turns, picks up one card until the cards run out. Some students may have only one card while others may have two.

3. Each student is responsible for his/her own card(s). Each student can only read and touch his/her own card(s).

4. Taking turns, each student reads his/her card(s) to the team.

5. Each student places his/her card(s), (face up or face down, your choice), in a logical sequence that justifies the triangles being congruent. Cards may be rearranged as the team discusses, but the original "owner" of each card is the only one who can move that card.

6. Teams check the answer cards for correctness.

7. Repeat steps 1-6 for each remaining set of blind sequence cards on pages 183 and 184.

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