# Contents

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# Introduction: Towards a Shift in Thinking about Mathematics

As teachers, we are in the midst of movement. We are continually adapting and responding to our learners as we reflect on their strengths, interests, learning preferences, and learning needs. We likely ask ourselves: *How do they learn? What excites them about learning? What personal and cultural experiences do they bring to learning?* Understanding our learners helps to shape the curriculum so that our actions as teachers are intentional, purposeful, personalized, and constantly shifting — ***moving***.

In many people's minds, the role of teacher is synonymous with mentor, facilitator, activator of knowledge, and guide. What is of utmost importance, however, is creating the conditions for students to *want* to think and do things differently. The outcome of learning, after all, constitutes a permanent shift in thinking or behavior: one of the intended outcomes of teaching (Katz and Dack 2013).

## A Way of Thinking about Cross-Curricular Ideas

In developing this resource, we seek to provide an opportunity for educators to think differently about mathematics and the way we engage our students with it. *Moving Math* was designed to support professional learning journeys, whether formal or informal, as we all strive to further deepen our understanding of the teaching and learning of mathematics. Once we, as educators, think and do things differently, we can begin to design learning experiences that will allow students to make sense of mathematical concepts and skills. The focus of the book is to support learning — both teacher learning and student learning.

Integral to teacher and student learning is the notion of teacher and student efficacy. *Teacher efficacy* is the teacher's belief that he or she can influence student learning. Teachers with high self-efficacy believe that they can positively affect student growth and learning despite any perceived challenging circumstances (Bruce 2015). Our vision is for all teachers to see themselves as confident and capable teachers of mathematics as we support numeracy development.

Parallel to teacher efficacy is *student efficacy*, which is the student's belief that he or she is able to learn (Bruce et al. 2010). Students with high self-efficacy believe in their abilities, persevere in their learning, and value their learning. Like our vision for teachers, our vision for young learners is for them to see themselves as confident and capable users of mathematics while learning to ask questions, pose problems to solve, explore ideas, and make informed decisions.

### Not Just Mathematics, But Numeracy

Note that we are not using the term *mathematics* interchangeably with *numeracy*. As with language, mathematics is the basis for thinking, understanding, and reflecting. *Numeracy*, on the other hand, refers to understanding how mathematical concepts and skills can be used to better understand the world. There, the goal is for us to lead more informed lives and, as we adopt a critical perspective, to advocate for fairness and equity.

*The unemployment rate can be defined as the number of people unemployed divided by the number of people in the labor force. Consider these figures and calculate the unemployment rate.*

1) 5,500,000 employed full time
2) 1,000,000 employed part time, want part-time work
3) 220,000 employed part time, want full-time work
4) 300,000 not employed, looked for work in last month, not on temporary layoff
5) 60,000 not employed, on temporary layoff
6) 22,000 not employed, want a job now, looked for work in the last year, stopped looking because discouraged about prospects of finding work
7) 76,000 not employed, want a job now, looked for work in last year, stopped looking for work for other reasons
8) 3,300,000 not employed, don't want a job now

Consider the task at right. If we deconstruct this task through the lens of the characteristics of numeracy, the task requires an understanding of mathematics, specifically referring to the concept of rate, but it also pertains to political struggle: How do we decide who makes up the number of people unemployed and the total labor force? It is this political struggle that requires a critical perspective and provides us with an opportunity to use mathematics to make sense of the world around us. The equivalent phrase in reference to language is *critical literacy*.

Adapted from Gutstein and Peterson (2006)

Over the past few years, we have identified support for the development of the numerate learner as a priority. To become numerate, students must learn to *make sense* of mathematical concepts and skills, *use* mathematical *skills* effectively, *communicate* mathematical thinking clearly, and *critically interpret* mathematical knowledge and skills (Fiore and Lebar 2016). When students become numerate, they see mathematics as a way of thinking about cross-curricular ideas — not, as is so often the case, as a series of isolated topics or skills without broader application.

## A Different Approach to Teaching and Learning

Thinking is fundamental to the teaching and learning of mathematics. Educational reformer John Dewey (1910, 5) states, "No words are oftener on our lips than *thinking* and *thought*. So profuse and varied, indeed, is our use of these words that it is not easy to define just what we mean by them" (emphasis added). What comes to mind when you hear the word *think*? What do you mean when you use the word *think*? How do your students interpret the word *think* when you




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"Learning is a consequence of thinking. This sentence turns topsy turvey the conventional pattern of schooling. The conventional pattern says that first students acquire knowledge. Only then do they think with and about the knowledge they have absorbed. But it's really just the opposite: Far from thinking coming after knowledge, knowledge comes on the coattails of thinking. As we think about and with the content we are learning, we truly learn it . . . Knowledge does not just sit there; it functions richly in people's lives so they can learn about and deal with the world."
— David Perkins (1992)

use it? How do you know that your students are engaging in thinking? We will address these questions as we focus on making students' thinking visible.

Dewey (1910, 6) further states, "In its loosest sense, thinking signifies everything that, as we say, is in our heads or that goes through our minds." We can describe things that go through our minds as we engage in thinking as

- inferring and interpreting
- analyzing
- evaluating
- making connections
- synthesizing
- reasoning and proving
- reflecting

These are the thinking skills to be addressed in this resource, and through them we can reach a new understanding of how to approach the teaching and learning of mathematics.

Teachers often refer to the big ideas in mathematics, but we offer an innovative variation on that. The big ideas are those statements that link mathematical concepts between various math topics or strands, for example, measurement, geometry, and probability, into a coherent whole. We are proposing that teachers consider the ***Bigger*** ideas: the mental actions or processes associated with the thinking skills noted. As we have designed them, the Bigger ideas can help teachers and students make connections between the thinking skills to support mathematical thinking and the development of numerate learners.

*Moving Math* focuses on *moving* the teaching and learning of mathematics by shifting instruction and assessment practices. The shift is achieved by looking at the mathematics curriculum through the thinking skills and the Bigger ideas. The key thinking skills are deconstructed and modeled through suggested learning experiences across various grades. The book offers authentic student solutions that demonstrate how the thinking skills support the intended mathematical learning. Evidence of student thinking and learning is described through the lens of the learning intentions and success criteria. Teaching questions to elicit further mathematical thinking are suggested. The goal of *Moving Math* is to enhance students' mathematical thinking and numeracy development.

## Learning Experiences versus Tasks

*Moving Math* has been designed for teachers, school administrators, and those with the responsibility for leading change. It will serve as a guide for planning and implementing improvements in the teaching and learning of mathematics. It is also intended to engage educators in meaningful conversations about effective practices to support student thinking and learning.

The notion of a "learning experience" implies that students will adopt an interdisciplinary approach to learning.

In this resource and as a reflection of how our thinking has shifted over time, we are inclined to use the phrase *learning experience* rather than the term *task*. By so doing, we believe that we can better emphasize two key ideas about students engaging in learning. The first idea is the importance of the learning *process*; the second idea is about capturing the *multimodal* nature of learning where students draw on their multiple intelligences, which relate to their learning preferences, to experience learning in different ways. They will use linguistic, kinesthetic, visual, and auditory modes to engage in learning. Doing so will help them to conceptualize and apply their understandings.

We believe that the tasks presented to students are meaningful only when teachers reflect on the learning intentions they serve and on how well they

incorporate practices whereby students can make their thinking visible while making sense of concepts and skills. A mathematics task is meaningful only when we have a curricular purpose and a deep understanding of our learners. Before we present students with a task to support learning, we must always ask ourselves: Why this learning, in this way, for this learner, at this time? (Ontario Ministry of Education 2016, 21). Reflecting on these questions allows us to devise learning experiences that promote sense making and place the learner at the centre of the decisions made about teaching and learning.

Why this learning, in this way, for this learner, at this time?

## Goals of the Featured Learning Experiences

You will notice that, throughout the book, the suggested learning experiences emphasize student voice with many opportunities for students to ask questions, pose problems to solve, explore ideas, and shift their thinking about mathematics and themselves as math users.

Learning experiences across three school divisions are presented in Chapters 4 to 10. Chapter by chapter, we are striving to do the following:

- to provide the Bigger ideas and the mental actions associated with each of the thinking skills
- to describe each of the thinking skills
- to unpack the thinking skills through the lenses of literacy and numeracy, so that you can better make sense of the thinking skills
- to support each thinking skill through the inclusion of thinking stems and prompting questions
- to develop success criteria for each of the thinking skills
- to suggest learning experiences focusing on each of the thinking skills, with possible learning intentions, success criteria, and effective questions to elicit further mathematical thinking
- to incorporate one student's solution, highlighting evidence of thinking and learning while recognizing that there are many approaches
- to capture math talk gleaned as a result of more probing from the teacher
- to provide reflective prompts to enhance professional learning
- to inspire educators such as yourself to engage in purposeful talk about student learning needs and related teacher learning

The focus on cross-curricular thinking skills provides teachers with opportunities to explore the mathematics curriculum through the Bigger ideas and to shift thinking about how they engage their students with mathematics. Indeed, *Moving Math* is an opportunity to shift thinking about the teaching and learning of mathematics so that *all* teachers can come to see themselves as teachers of mathematics and thereby make it more likely that *all* students will come to view themselves as capable and competent math users.

### Opportunity for Reflection

Educators gain insights and skills as they reflect on their practices through professional learning, whether formal or informal. You can engage with this book to support your professional learning. As you begin to reflect on teaching practices to support the teaching and learning of mathematics in your classroom, what do you wonder about?





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# Professional Learning as Catalyst

The world is rapidly changing and evolving into a future that we cannot even imagine. So, when we think about preparing our students for the future, what is it we are undertaking to do? If we think about teaching and learning in terms of the twenty-first century, what might this look like? sound like? feel like?

We want students to feel safe, trusted, and respected so they feel empowered to ask questions, pose problems to be solved, explore ideas, and make informed decisions. When we promote well-being within a learning environment that contributes to students' overall positive sense of self, it helps students become healthy, active, engaged, reflective, and responsible citizens in our rapidly changing world.

Although the phrase *teaching and learning* appears uncomplicated, teaching and learning is a complex endeavor. It becomes more so as society becomes more intricate. Indeed, as far back as the early 1900s, John Dewey (1916/1966) observed that as societies become more complex in structure and resources, the need for intentional teaching and learning increases. Therefore, it is important for us as educators to reflect on teaching and learning as we plan for learning experiences: experiences that are empowering and engaging and that allow for sense making, regardless of the subject area.

**Opportunity for Reflection**

What constitutes teaching and learning? How is student learning like teacher learning? How is it different?

## What Is Learning All About?

Although there are varying descriptions of what constitutes learning, the notion of experiencing lasting changes in the way we think and how we do things remains consistent. Learning involves moving beyond existing routines. It requires people to rethink ideas, practices, attitudes, and values to change what they are currently doing (Hammerness et al. 2005). The shift in thinking and behavior, regardless of whether we are talking about teacher learning or student learning, denotes a *deep* learning — a conceptual understanding of concepts and skills — that is transferrable to other contexts.

Suppose, then, that teachers have a conceptual understanding of effective practices to support student learning; suppose as well that they have a deep understanding of the curriculum, which includes knowing their learners. They are thereby able to purposefully plan for learning experiences that will allow students to make sense of concepts and skills. Their deep understanding enables them to clearly articulate what students need to know and be able to do. As students engage in meaningful learning experiences, teachers are noticing, naming, and gathering evidence of the learning; they are also providing feedback, using prompts and questioning, to further enhance the learning. In other words, the conceptual understanding of the teachers enables them to bridge assessment and instruction based on knowing their learners.

When students have a deep conceptual understanding of mathematical concepts, for example, they can see patterns and relationships. They can make connections and generalizations. What's most important is that they have made sense of the concepts. When students have made sense of mathematical concepts and skills, they begin to see mathematics as a connected whole as opposed to separate topics. They begin to view mathematics as a way of thinking, not as rules and procedures. They are able to transfer this knowledge and understanding to better appreciate the role mathematics plays in helping them understand the world around them.

### A Planned Experience

Similar learning experiences will be presented in subsequent chapters.

Consider the meaningful learning experience captured in Figure 1.1. The experience has been intentionally planned based on an understanding of the teaching and learning of mathematics and of the students. When students have made sense of multiplication, they are able to make connections between addition and multiplication, describe the meaning of multiplication of addition, and understand when to use multiplication and addition.

**PLANNING FOR MEANINGFUL LEARNING EXPERIENCES**

| **What do I want my students to know and be able to do?** | **How will I know they have learned it?** |
|---|---|
| • Students will be learning to make sense of multiplication by applying the skill of addition so they can represent their thinking and understanding in multiple ways. | Students can . . .<br>• make connections between representations<br>• describe multiplication as repeated addition |





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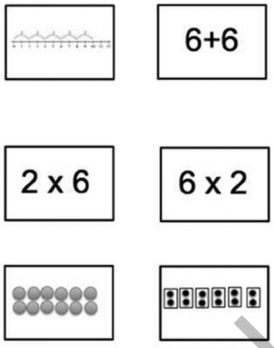
| | |
|---|---|
| | • describe multiplication as equal groups<br>• describe multiplication as an array<br>• use multiplication to represent addition<br>• describe the meaning of addition<br>• describe the meaning of multiplication<br>• understand when to use multiplication<br>• understand when to use addition |

**Learning Experience**




Write a story so that any of the cards could be used to represent it.

Share the story with a classmate and explain your thinking.

Fiore and Lebar (2016)

| Effective Questions to Support Math Talk | Effective Questions to Elicit Further Mathematical Thinking |
|---|---|
| • What card did you consider first?<br>• How did you come up with your story?<br>• How are the cards the same?<br>• How are the cards different?<br>• How are your stories the same?<br>• How are your stories different?<br>• What did this question make you think about? | • I noticed that you are struggling to come up with a story. You might want to pick just one card first.<br>• I noticed you came up with a story quickly. Which cards helped you?<br>• What might another story be?<br>• What is it we have been learning today? |

***Figure 1.1*** *Stories can be told for mathematics as well as for language arts.*