

The Four Roles of the **Numerate Learner**

Effective teaching and assessment strategies to help
students think differently about mathematics

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Developing an Understanding of the *Four Roles of the Numerate Learner*

A group of educators in a professional learning community asked a thought-provoking question: How do we teach mathematics for today's learners? We began our journey by exploring the concept of lifelong learning.

Setting the Context

It is the nature of our profession to make constant reference to the idea that we are lifelong learners. But what does this mean? To begin, it is important to define the word *learning*. Educators often engage in professional learning opportunities because we feel the need to learn something new, or perhaps to confirm that current practices are in fact impacting student achievement. However, what is our role as a participant of a professional learning community? What learning is taking place when we bring people together? Having a sense of urgency and a growth mindset are the driving forces of change. If something different is going to happen for students, then it is our moral obligation as educators to reflect on our practices and examine alternate possibilities that are responsive to our students' most urgent learning needs.

"Learning is a permanent change in thinking or behaviour."

Katz, 2013, p. vii

For real change to occur, we also need to understand our students' lives and what makes them excited about learning. How do they learn? What personal and cultural experiences do they bring to the learning? Our students help us shape the curriculum so that teaching and learning are intentional, purposeful, and personalized. Being a lifelong learner means that our thinking is always changing, and we embrace a responsive pedagogy that requires us to be reflective, adaptive, innovative, creative, and collaborative. The journey of lifelong learning begins with knowing our students—their strengths, interests, learning preferences, and learning needs—as well as understanding the elements of cultural competence so that we can implement a curriculum that values diversity and embeds cultural knowledge. Lifelong learning is about being open to trying new practices and reflecting on their impact through evidence of student learning.

Adopting an Inquiry Mindset

At a time when we are paying close attention to mathematics, many questions are emerging about instruction, assessment, differentiation, and integration of cross-curricular skills. How do we disrupt the way we have been teaching mathematics to support our twenty-first century learners? What essential skills do our students need to live in a multi-faceted world? How do we teach mathematical skills and concepts in a way that makes sense for today's learners?

After engaging in professional dialogue with a group of educators, we made the following confirmations that eventually became our guiding principles in moving forward:

- Student thinking and learning are at the forefront of teaching and learning.
- Educators and students need to be presented with opportunities to reflect on their current attitudes and beliefs about what mathematics is and the way we view ourselves as mathematicians.
- Mathematics requires a new way of thinking and doing.
- Mathematics, like language, is the basis for thinking, communicating, and reflecting.
- A cross-curricular and integrated approach is needed if the teaching and learning of mathematics is going to extend beyond the mathematics curriculum to support the development of a numerate learner.
- Differentiation and inclusion are at the heart of meaningful, intentional, and personalized instruction and assessment.

Identifying Connections between Literacy and Numeracy

"(Effective literacy instruction) is about how we communicate in society. It is about social practices and relationships, about knowledge, language, and culture."

Ontario Ministry of Education, 2004, p. ix

We soon realized that effective mathematics instruction is closely linked to effective literacy instruction. If the goal of literacy instruction is to develop a literate learner, then how does effective mathematics instruction support the development of a numerate learner? Are mathematics and numeracy different? And if so, how are they different?

To help us understand the difference between mathematics and numeracy, our conversations took us back to literacy and the fine differences between literacy, new literacies, and critical literacy (see Figure 1.1). Although the columns appear linear, they are conceptually integrated. In addition, it is important to be aware that language is embedded across all literacies.

FIGURE 1.1: COMPARING LITERACY, NEW LITERACIES, AND CRITICAL LITERACY

Literacy	New Literacies	Critical Literacy
"Literacy is defined as the ability to use language and images in rich and varied forms to read, write, listen, speak, view, represent, and think critically about ideas." (Ontario Ministry of Education, 2006)	"As technological convergence develops apace, one needs to combine the skills of critical media literacy with traditional print literacy and new forms of multiple literacies to access and master the new multimedia hypertext environments.	"Critical literacy is not something to be added to the literacy program, but a lens for learning that is an integral part of classroom practice." (Ontario Ministry of Education, 2009) Critical literacy goes one step further than

In literacy, students use all the skills they have learned as they acquire one or multiple languages to engage in thinking, talking, reading, writing, responding to, and reflecting on a wide variety of multi-modal texts.

As students reflect on their thinking, metacognition becomes essential to develop a sense of self-efficacy and self-regulation.

Literacy in this conception involves the abilities to engage effectively in socially-constructed forms of communication and representation.” (Kellner, 2000)

“The shift from *literacy* to *literacies* has created possibilities and reconsiderations of pedagogies that look at literacy in multiple ways, through a variety of media and approaches. The term *new literacies* points to multiple linguistic systems within literacy. Literacy practices, which are multiple, shift based on the context, speaker, text, and the function of the literacy event (e.g., doing a Google search).” It’s Critical! (Booth, 2008)

“Even definition of the term *text* has gone beyond the traditional acts of reading and writing using an alphabetic code or symbol system, to include digital technology, images, sounds, and oral discourse. Now we refer to a text as a medium with which we make meaning through a variety of modes—written, visual, tactile, or oral. Texts span audio books, magazines, paintings, films, computer screens, narratives, graphics, information, opinions, poetry, songs, scripts, instructions, and procedures. It’s Critical!” (Booth, 2008)

basic literacy does, and asks students to question the authority of texts and identify and examine the author’s bias or perspective, then engage in dialogue and discourse about text, relating it to their daily lives and their own point of view. It also examines writing as power.

Social justice issues are explored and questions are asked such as:

- Whose voice is missing from this text?
- What social action could I take in my school or community as a result of reading this article on environmental stewardship?
- As a global citizen, how do I view this text?

We need to use particular habits of mind to support critical literacy, such as persisting, thinking flexibly, questioning and posing problems, applying past knowledge to new situations, thinking and communicating with clarity and precision, and creating, imagining and innovating. (Costa & Kallick, 2008)

Fiore, Lebar, & Scott-Dunne, 2014

As we worked to unpack the similarities between literacy and numeracy, we examined the definitions of critical literacy as they guided our thinking with respect to numeracy development and what it might mean to be a numerate learner. At the same time, we referred to a similar chart to parallel our thinking on numeracy and critical numeracy (see Figure 1.2).

FIGURE 1.2: COMPARING MATHEMATICAL KNOWLEDGE, NUMERACY, AND CRITICAL NUMERACY

Mathematical Knowledge	Numeracy	Critical Numeracy
<p>Mathematical skills and concepts are used as tools for thinking (reasoning, proving, justifying, reflecting, and communicating).</p>	<p>Numeracy involves using mathematical knowledge to develop an understanding of the role mathematics plays in everyday life.</p> <p>Students apply mathematical skills and concepts to explore and solve practical real-life problems.</p> <p>As students reflect on their mathematical thinking, metacognition becomes essential to develop a sense of self-efficacy and the belief in themselves as successful mathematicians.</p>	<p>“Critical numeracy is a focus on the ways in which practical mathematical situations are implicated in the power relationships and face-to-face politics of everyday life. It is a focus on how numeracy in all its forms is involved in our relationships to each other and the world.” (Stoessinger, 2002)</p> <p>Critical numeracy goes one step further than basic numeracy by asking students to question the source(s) of mathematical information (e.g., the notion of using numbers to persuade).</p> <p>Students explore social and political issues and ask questions such as:</p> <ul style="list-style-type: none"> • How are numbers or shapes being used to support this viewpoint? • Whose numeracy is being advantaged through the presentation of these numbers?

Fiore, Lebar, & Scott-Dunne, 2014

We realized that we could leverage what we know about literacy development to deepen our understanding of mathematics instruction and numeracy development. In addition, we determined that *thinking* and *communicating* are the powerful connections between literacy and numeracy.

The following detailed definition of literacy includes these concepts of thinking and communicating.

Literacy is . . . “the ability to use language and images in rich and varied forms to read, write, listen, speak, view, represent, and think critically about ideas. It involves the capacity to **access, manage, and evaluate** information; to **think** imaginatively and analytically; and to **communicate** thoughts and ideas effectively. Literacy includes **critical thinking** and **reasoning** to solve problems and make decisions related to issues of **fairness, equity, and social justice**. Literacy connects individuals and communities and is an **essential** tool for personal growth and active **participation in a cohesive, democratic society.**”

Literacy for Learning: The Report of the Expert Panel on Literacy in Grades 4 to 6 in Ontario, Ontario Ministry of Education, 2004.

“That is why numeracy—like literacy—is considered so fundamental. Whether the call to action is ‘mathematical literacy,’ ‘quantitative literacy,’ ‘everyday math,’ or ‘numeracy,’ our challenge as K to 12 educators is to help prepare all our graduates to apply mathematics in the context of their everyday world.”

Literacy and Numeracy Secretariat, 2012, p. 1

We determined that the ideas presented in this definition of literacy could easily be used to define numeracy, as indicated below.

Numeracy is . . . the ability to use mathematical knowledge and skills to think critically about ideas in everyday life. It involves the capacity to **make conjectures** and **interpretations, form conclusions, evaluate** reasonableness, and **represent** and **justify** solutions; to **think** creatively and analytically; and to **communicate** thoughts, ideas, and solutions logically. Numeracy includes **critical thinking** and **reasoning** to solve problems, make sense of mathematical concepts, and question issues of **fairness, equity and social justice**. Numeracy connects individuals and communities and is an essential tool for personal growth and active **participation in a cohesive, democratic society.**

Aligning Effective Literacy and Numeracy Practices

Thinking is an integral component of literacy development. As a result, to prepare students for today’s world, making student thinking visible needs to be the underlying goal of every teaching and learning experience. Figure 1.3 outlines the instructional and assessment actions that will help make thinking visible across the curriculum. They are interdisciplinary in nature and align literacy to numeracy.

FIGURE 1.3: INTEGRATION OF INSTRUCTION AND ASSESSMENT

Begin with the students as personal and cultural resources

- Assess students’ already acquired knowledge and experiences.
- Get to know students’ interests, learning preferences, and natural curiosities.

Implement comprehensive/balanced instruction using the gradual release of responsibility

- Based on evidence gathered from observations, conversations, and products, decide which instructional approach students primarily require at any given time to support identified learning needs (e.g., *modelled, shared, guided, or independent practice*).

Use assessment for, as, of learning

- Know the intended learning outcomes and different purposes for gathering evidence during the learning process.
- Use assessment *for* learning to improve student learning and as the basis for providing descriptive feedback (emphasize developing learning goals and co-constructing success criteria to describe successful attainment of intended learning).
- Use assessment *as* learning to help students develop metacognitive practices to monitor their own learning and develop goals for improving learning.
- Use assessment *of* learning for the purpose of evaluation.

Adapted from Growing Success – Assessment, Evaluation, and Reporting in Ontario Schools, Ontario Ministry of Education, 2010.

Use accountable talk

- Use accountable talk strategies, such as turn and talk, wait time, paraphrasing (e.g., *Who can repeat what _____ just said?*), re-voicing (*I think that I am hearing you say that . . .*), adding on (*Tell me more about that. Why do you think that?*).
- Provide opportunities for students to engage in intentional, focused talk about a specific topic where they can deliberate, exchange ideas, ask questions, check understanding, defend opinions, draw conclusions, explain their thinking, make connections, and justify.

Adapted from West, Metamorphosis Teaching Learning Communities, <http://www.metamorphosistlc.com>.

Promote inquiry-based and problem-based learning

- Create classroom learning conditions and spaces that foster wonderings and curiosities.
- Provide authentic learning opportunities where students are required to use skills to ask questions, think, problem solve, resolve doubts, and find the truth.

Adapted from IQ – A Practical Guide to Inquiry-Based Learning, Watt & Colyer, 2014.

Adopt a three-part lesson mindset

- Activate student thinking.
- Develop student thinking.
- Consolidate student thinking.

Adapted from Asking Effective Questions, Literacy and Numeracy Secretariat, 2011.

Provide thinking tools and engaging texts

- Encourage students to use a range of manipulatives as thinking tools that support knowledge construction through exploration, investigation, discovery, and creation.
- Provide students with choice to select from a range of engaging texts, including literary, informational, and graphic, to develop essential thinking skills (e.g., *inferring, interpreting, extending understanding, analyzing, evaluating, synthesizing*).

Create rich tasks and authentic learning experiences

- Develop learning tasks/experiences that promote student thinking, problem solving, and decision making, and that help students adopt a critical stance about the world they live in.

Foster a responsive learning environment

- Adapt and adjust instruction “in the moment” to respond to students’ specific learning needs.