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Introduction

Critical thinking skills are the mental skills and processes involved in the act of learning, such as remembering and understanding *facts, ideas* and/or *concepts.* In recent years cognitive psychologists have compiled a great deal of new information about thinking and learning, much of it done in conjunction with neurobiologists interested in how the human brain thinks, learns and remembers.

Thinking processes describe what goes on in the learner's brain during learning – how knowledge is acquired, organised, stored in memory and used in further learning and problem solving. It is often helpful to classify knowledge as either declarative – knowledge about something – or procedural – knowledge of how to do something. Some theorists suggest that knowledge begins as declarative and becomes procedural as it is used in solving problems.

For development of conceptual mathematics and mathematical problem solving, building thinking skills and processes is essential. The National Council of Teachers of Mathematics stresses the importance of reading, writing and discussion for the development of students' mathematical thinking:

> The development of a student's power to use mathematics involves learning the signs, symbols and terms of mathematics. This is best accomplished in problem situations in which students have an opportunity to read, write and discuss ideas in which the use of the language of mathematics becomes natural. As students communicate their ideas, they learn to clarify, refine and consolidate their thinking. (NCTM 1989)

This position (consolidated from the NCTM's previously published standards) has been reiterated in the new Principles and Standards for School Mathematics (2000) and continues to remain key for the success of *all* students (pre-K–12) in our 21st century technologically advanced society.

This booklet will offer ways to promote critical thinking and make students more conscious of their thinking in three contexts:

- cooperative problem solving,
- reflective discussion and writing about mathematical ideas and
- steps and strategies for problem solving.

Also offered to teachers is a hierarchy of instructional tasks to guide their own choices as they move students to more challenging and rewarding mathematical tasks.

Promoting Critical Thinking in Cooperative Groups

> Students' understanding of mathematical ideas can be built throughout school years if they actively engage in tasks and experiences designed to deepen and connect their knowledge. Learning with understanding can be further enhanced by classroom interactions, as students propose mathematical ideas and conjectures, learn to evaluate their own thinking and that of others, and develop mathematical reasoning skills. Classroom discourse and social interaction can be used to promote the recognition of connections among ideas and the reorganisation of knowledge . . . Moreover, in such settings, procedural fluency and conceptual understanding can be developed through problem solving, reasoning and argumentation.
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> —*Principles and Standards for School Mathematics* (2000, NCTM, p. 21)

For students to understand mathematics conceptually, they need to interact with each other as well as the teacher. Students also need to discuss their own ideas about mathematics. For small group work to be successful, the team members must learn to function as a unit. The teacher needs to lay the groundwork for effective group relationships by promoting the cooperative and collaborative social skills associated with positive group relationships, including the following:

- Praise good ideas
- Describe feelings
- Express support for one another
- Listen to each other
- Be positive
- Give encouragement

Once the climate for positive group work is established, the teacher's next step is to make the expectation clear that when solving a problem, all students must be able to explain their thinking, to justify their answer and to explain why an answer is reasonable. When students defend their solutions to others in small groups, they develop a better understanding of the mathematics involved and become more confident about their own ability to solve difficult problems.

The following three expectations need to be discussed with students for the establishment of an environment conducive to small group problem-solving work:

1. Students are expected to work together on their assigned problem and make sure that each member of the group participates.
2. Students are expected to listen to each other carefully and to then build on each other's ideas.
3. Each individual team member is expected to be able to explain and justify the team solution.