YEARS
5–9

# MATHS EXPLORATIONS

## *Numbers & Operations*



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# Introduction

This introduction contains general information about the structure of the books and implementation of the activities in the *Maths Explorations* series.

## AUDIENCE

*Maths Explorations* is designed to support students, teachers and other learners as they work to deepen their understanding of middle years maths concepts. The activities have been written primarily with upper primary and middle years students and teachers in mind. But older students or those who have already studied more advanced content can also enjoy and benefit from them. The explorations can be used in classrooms, as professional development activities for mathematics teachers, in university maths content and methods courses, and by anyone who would like to extend their understanding of middle years mathematics concepts by solving challenging problems.

These explorations are designed to stretch students beyond their initial level of comfort. They are built around the belief that most of us underestimate the mathematics we are capable of learning. Although the activities are challenging, they are also meant to be accessible. Although they are targeted to the special needs of gifted and talented students, I hope that teachers will make them available to any student who would like to pursue the challenge. Most students are capable of making progress and learning something meaningful, even if they work just on the first question or two of an activity.

## PURPOSE

The investigations in this series were developed through years of work with talented middle years maths students. They are designed to

- engage students in the excitement of mathematical discovery
- deepen students' understanding of a wide range of middle years maths concepts
- encourage the use of multiple strategies for solving problems
- help students become flexible, creative, yet disciplined mathematical thinkers
- improve mathematical communication skills
- highlight connections between diverse mathematical concepts
- develop perseverance, patience and stamina in solving mathematical problems

- provide levels of depth and challenge to meet a variety of needs and interests
- enable students to work both collaboratively and independently
- offer opportunities for further exploration.

## STRUCTURE OF THE BOOKS

The Maths Explorations series contains ready-to-use explorations focused on one mathematical content area. These explorations emphasise challenge and depth, and so there is a stronger focus on concepts than on procedural skills, but most activities provide plenty of opportunities to practise computational skills as well.

When selecting activities, use your own knowledge of your students' backgrounds and abilities, rather than only focusing on the content of the exploration and how it corresponds to the student's year level at school. Information about the prior knowledge needed for each exploration is also included as a guide.

## FEATURES OF THE EXPLORATIONS

Each activity includes three stages. Stage 1 (and sometimes part of Stage 2) may be challenging enough to meet the needs of many students. The second and third stages are usually appropriate for older students, or for those who finish early, need more challenge or are highly motivated and curious to learn more. They may also be useful for teachers or other adults who have more mathematical experience and want to extend their own knowledge further. I have separated the explorations into stages in order to provide a tool for setting goals, to help measure and celebrate students' progress, and to create additional options for those who need them.

Each exploration also contains features carefully designed to support teachers in the implementation process: an introduction, the student handout, a set of questions and notes to guide conversation, detailed solutions and suggestions for a closing discussion.

## IMPLEMENTING THE EXPLORATIONS

Implementing each exploration involves five steps on the part of the teacher: prepare, introduce, follow up, summarise and assess.

### *Prepare*

The best way to prepare to teach an activity is to try it yourself. Although this involves an initial time investment on your part, it pays great dividends later. Doing the activity, ideally with a partner or two, will help you become familiar with the mathematics, anticipate potential trouble spots for students and plan ways to prepare students for success. After you have used the activity once or twice with students, very little preparation will be needed.




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## *Introduce*

The Introduction section at the beginning of each exploration provides support to help you get your students started: materials and prior knowledge needed, learning goals, motivational background and suggestions for launching the activity.

Read the Motivation and Purpose selection to students, and then follow the suggestions for leading a discussion to help them understand the problem. Often, one of the suggestions involves looking through the entire activity with them (or as much of it as they will be doing) to help them see the big picture before they begin. Let students know what kind of a time frame you have in mind for the exploration. An activity may take anywhere from a few days to two or three weeks depending on how challenging it is for students, how much of it they will complete and how much class time will be devoted to it.

The explorations are designed to allow students to spend much of their time working without direct assistance. But it's usually best if you stay with them for a few minutes just after introducing an activity to ensure that they get started successfully. This way, you can catch potential trouble spots early and prevent unnecessary discouragement.

This is also a good time to remind students about the importance of giving clear, thorough written explanations of their thinking.

## *Follow Up*

The level of challenge in these explorations makes it impractical for most students to complete them entirely on their own at school or for homework. Students' most meaningful (and enjoyable!) experiences are often the opportunities you give them to have mathematical conversations with you and with each other while the activity is in progress. If you are implementing an activity with a small group of students in a mainstream classroom, it may be sufficient to plan to meet with them a couple of times per week, for 15 or 20 minutes each time. If circumstances allow more time than this, then the conversations and learning can be still better.

The Teacher's Guide for each exploration reprints each problem and contains two main elements: (a) Questions and Conversations and (b) Solution. The Questions and Conversations feature is designed to help you facilitate these conversations with and among students. For the most part, it lists questions that students may ask or that you may pose to them. Ideas for responding to the questions are included. It isn't necessary to ask or answer all of the questions. Instead, let students' ideas and your experience and professional judgement determine the flow of the conversation. The Solution section offers ideas for follow-up discussions with students as they work. Although the answers in the Questions and Conversations sections are often intentionally incomplete or suggestive of ideas to consider, you'll find detailed answers, often with samples of multiple approaches that students pursue in the Solution section.

### *Summarise*

After students have finished an exploration, plan a brief discussion (20 minutes is usually enough) to give them a chance to share and critique one another's ideas and strategies. This is also a good time to answer any remaining questions they have. The Wrap-Up section at the end of each exploration offers ideas for this discussion, along with suggestions for further exploration.

### *Assess*

One of the most valuable things you can do for your students is to comment on their work. You don't have to write a lot, but your comments should show that you have read and thought about what they have written. Whether you give praise or offer suggestions for growth, make your comments specific and sincere. Ideally, some of your comments will relate to the detail of the mathematical content.

If you would like to give students a numerical score, consider using a rubric such as the one in *Extending the Challenge in Mathematics: Developing Mathematical Promise in K–8 Students* (Sheffield, 2003). Whatever system you use, the emphasis should be on process goals such as problem-solving, reasoning, communication and making connections – not just correct answers. You may also build in general criteria such as effort, perseverance, correct spelling and grammar, organisation, legibility, etc. But remember that the central goal is to develop students' mathematical capacity. Any scoring system should reflect this.

## GETTING STARTED

Here are some tips for getting started. First, a few "DON'Ts" to help you avoid some common pitfalls:

- *Don't feel that you have to finish the activities.* Students will learn more from thinking deeply about one or two questions than from rushing to finish an activity. Each exploration is designed to contain problems that will challenge virtually any student. Most students will not be able to answer every question.
- *Don't feel that you have to explain everything to students.* Your most important job is to help them learn to develop and test their own ideas. They will learn more if they do most of the thinking.
- *Don't be afraid to allow students to struggle.* Talented students need to know that meaningful learning takes time and hard work. Many of them need to experience some frustration – and learn to manage it.
- *Don't feel that you have to know all of the answers.* In order to challenge our students mathematically, we have to do the same for ourselves. You'll never know all of the answers, but if you're like me, you'll learn more about the maths every time you teach an exploration! Do what you can during the time you've allotted to prepare, and then allow yourself to learn from the mathematical conversations – right along with your students.




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And now some important "DOs":

- *Take your time.* Allow the students plenty of time to think about the problems. Take the time to explore the ideas in depth rather than rushing to get to the next question.
- *Play with the mathematics!* To many people's surprise, maths is very much about creative play. Of course, there are learning goals, and it takes effort, but also be sure to enjoy playing with the patterns, numbers, shapes and ideas!
- *Listen closely to students' ideas and expect them to listen closely to each other.* Meaningful mathematical conversation may be the single most important key to students' learning. It is also your key to assessing their learning.
- *Help students feel comfortable taking risks.* When you place less emphasis on the answers and show more interest in the quality of students' engagement, ideas, creativity and questions, they will feel freer to make mistakes and grow from them.
- *Believe that the students – and you – can do it!* Middle years students have great success with these activities, but it may take some time to adjust to the level of challenge.
- *Use the explorations flexibly.* You don't always have to use them exactly "as is". Feel free to insert, delete or modify questions to meet your students' needs. Adjust due dates or completion goals as necessary based on your observations of students.

Teachers who use the activities in a mainstream classroom often find it helpful to make a solid but realistic commitment at the beginning of the school year to implement the explorations. Put together a general plan for selecting students, forming groups, creating time for students to work (including time for you to meet with them), assessing the activities and communicating with parents. Stick with your basic plan, making adjustments as needed as the school year progresses.

## THE AUSTRALIAN CURRICULUM

The Maths Explorations series provides teachers with the perfect resource to extend students' learning with the Australian Curriculum: Mathematics. Each exploration in this book, and the series, is tailored to engage students in the mathematics content, as well as promote the use of practical skills. Each of the Australian Curriculum: Mathematics key ideas – understanding, fluency, problem-solving, and reasoning – are not only present in these explorations, but form an integral part of the learning, ensuring that students are actively engaged in applying the skills introduced and emphasised by the curriculum.

While these activities have not been designed to correlate directly with Australian Curriculum: Mathematics content descriptions, the knowledge and skills that students develop as they learn curriculum content form the basis of their understanding in approaching the explorations here. The prior knowledge infor-

mation included at the start of each exploration will inform you of the Australian Curriculum: Mathematics content that will need to be established before students attempt the activity.

The activities in this book are the ideal extension for gifted students using the Australian Curriculum: Mathematics, and will ensure that advanced learners remain engaged with their work in the classroom while practising essential skills




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# Exploration 1

## Triangle Sums

## INTRODUCTION

### *Prior Knowledge*

» Add, subtract, multiply and divide one- and two-digit numbers.
  • Select and apply efficient mental and written strategies and appropriate digital technologies to solve problems involving all four operations with whole numbers (ACMNA123)

### *Learning Goals*

» Experience a mathematical investigation process.
» Develop and flexibly apply problem-solving strategies.
» Analyse and extend patterns.
» Make and test mathematical conjectures.
» Justify conclusions using deductive reasoning.
» Formulate questions and generalise solutions.
» Communicate complex mathematical ideas clearly.
» Persist in solving challenging problems.

### *Launching the Exploration*

**Motivation and purpose.** To students: this exploration is an extended version of an entertaining puzzle that has been around for years. Its main purpose is to introduce you to the process that mathematicians use to discover and create new mathematical knowledge. You will gather information, organise it, analyse it, make predictions, test them, try to prove (or disprove!) your conclusions and then create new questions to explore. In the process, you will strengthen your computational, problem-solving and mathematical reasoning skills.

**Understanding the problem.** Read through the first question to ensure that students understand it. Emphasise the fact that each number must be used exactly once – none of them will be left out or repeated.

Encourage students to begin the exploration using "thinking paper". This is a place where they record their ideas, calculations, conjectures and observations – essentially everything they do – as they work. They should save this paper and use it to help them write their final copy.

Some students may enjoy cutting out nine circles, numbering them and moving them around as they try to solve the puzzle. But remind them that they should still keep track of their work on the thinking paper.

# Exploration 2

## Torran Maths

## INTRODUCTION

### *Prior Knowledge*

- » Understand how place value is used to represent numbers.
  - Apply place value to partition, rearrange and regroup numbers to at least tens of thousands to assist calculations and solve problems (ACMNA073)
- » Understand the role of place value in procedures for whole number addition, subtraction, multiplication and division.
  - Select and apply efficient mental and written strategies and appropriate digital technologies to solve problems involving all four operations with whole numbers (ACMNA123)

### *Learning Goals*

- » Deepen understanding of place value by exploring a system that groups by a number other than 10.
- » Develop, describe and justify procedures for translating between two place value systems.
- » Analyse and extend counting patterns in a new place value system.
- » Understand the difference between numbers (ideas) and numerals (symbols).
- » Communicate complex mathematical ideas clearly.
- » Persist in solving challenging problems.

### *Launching the Exploration*

**Motivation and purpose.** To students: one of the best ways to better understand your own language is to learn someone else's. In the same way, if you want to gain a deeper understanding of your own numeration system, it helps to study a different system! To do this, you will visit the imaginary planet, Torr, where they group everything by fours instead of tens. As you investigate the Torran way of writing numerals, pay close attention to the similarities and differences between our systems.

**Understanding the problem.** Ask students to read the first page of the activity very carefully. Have them discuss what they've learned and what they are still trying to understand. (They should not feel that they have to understand everything right now.) Encourage students to refer to the first page whenever they feel confused or get stuck.

Discuss the distinction between *numbers* and *numerals*. Tell students to stay focused on this distinction throughout the exploration.

Don't explain anything beyond the content of the introductory page. Especially do not teach explicit procedures for translating between the Earth and Torran numeration systems. Students will develop their own strategies based upon their own understandings.