

ADVANCED CURRICULUM FROM THE
CENTER FOR GIFTED EDUCATION AT WILLIAM & MARY

Challenging Australian Curriculum Maths Lessons

ACTIVITIES AND EXTENSIONS FOR GIFTED AND ADVANCED LEARNERS IN
YEAR 7

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INTRODUCTION

The Australian Curriculum: Mathematics is a framework for F–10 curriculum standards that describe the mathematics skills and concepts students need to develop for success in higher education and the 21st-century workplace. The structure of the Australian Curriculum: Mathematics consists of two parts:

- Content strands, which describe what students should be taught and be able to do in their study of mathematics. These content strands are Number and Algebra, Measurement and Geometry, and Statistics and Probability, and are made up of a number of individual sub-strands.
- Proficiency strands, which describe how the content should be explored and developed. These proficiency strands have been integrated into the content strands, and help to ensure that students' mathematical skills become more sophisticated as their learning progresses.

With the adoption of the Australian Curriculum nation-wide, gifted and advanced learners need opportunities to master year-level standards and mathematical practices with greater depth, rigour and understanding. This revised edition of *Challenging Common Core Maths Lessons* is one of a series of books developed in conjunction with the Center for Gifted Education at William & Mary intended to give gifted and advanced learners additional practice and activities to master and engage with the Australian Curriculum: Mathematics. Each book in the series is organised by the content standards in one year level.

The lessons in this book cover Year 7 mathematics content. In Year 7, the content descriptions are addressed in 11 sub-strands:

- Number and Algebra
 - Number and Place Value
 - Real Numbers
 - Money and Financial Mathematics
 - Patterns and Algebra
 - Linear and Non-Linear Relationships
- Measurement and Geometry
 - Using Units of Measurement
 - Shape
 - Location and Transformation
 - Geometric Reasoning
- Statistics and Probability
 - Chance
 - Data Representation and Interpretation

STANDARDS FOR MATHEMATICAL PRACTICE

To engage learners with the content strands, the Australian Curriculum: Mathematics provides four proficiency strands – ways in which the student can engage with the content descriptions at every year level:

1. Understanding
2. Fluency
3. Problem-Solving
4. Reasoning

Each lesson in this book identifies the proficiency strands by number. Activities and practice problems are structured to develop proficiency in learners. Teachers should be aware of the proficiency strands and look for opportunities to connect these practices to content understanding in every lesson.

PURPOSE

The lessons in this book were written with the assumption that a teacher has already introduced a mathematical content description through a primary curriculum source. Problem solving, practice problems, and activities enrich and extend current year-level mathematics content rather than accelerate students to above-year-level content. Each lesson is specific to a sub-strand, usually only focusing on one or two content descriptions, and provides additional support and enrichment for gifted and advanced learners.

LESSON STRUCTURE

Each lesson follows a predictable structure. It first begins by naming the focal content description(s) – what students should already know or to which they have been introduced. Next, the proficiency strands covered within the activities and problems are listed by number. The lesson includes an estimate for the time it might take to complete the lesson, but this will vary by teacher and classroom. Key terms are listed, and are included based on when the terms are first introduced in the Australian Curriculum or are a prerequisite for understanding the activity or problems in a lesson. Teachers should be sure their students already have a working knowledge of these terms before beginning the lesson.

Every lesson includes a list of materials needed, including handouts. It is assumed that students will have access to commonplace items such as pencils and paper, and the materials noted are those items that teachers will need to obtain/acquire in advance. The lesson objectives highlight what students will learn or be able to do as a result of completing the activities and problems.

All lessons include an opening activity to allow students to explore the concept (e.g. multiple representations, open-ended problems, observing number patterns). Each activity is followed by practice problems that challenge students (e.g. harder or less familiar numbers) and – more importantly – extend students' thinking beyond calculating an answer. The practice problems ask students to grapple with their understanding of the lesson concepts. The lessons conclude with an assessment practice that allows teachers to evaluate student learning. The practice problems were written to engage gifted and advanced learners in higher level thinking and deeper understanding of a mathematical concept. The Australian Curriculum Assessment Practice problems in this book were intentionally written for students to prepare for on-level standardised test questions similar to Australian Curriculum-based year-level assessments, given all students are required to take these types of assessments.

GROUPING OPTIONS

The lessons in this book can be used for whole-group, small-group and individual instruction.

Whole-Group Instruction

Teachers can use this book in one academic year in conjunction with the primary curriculum in a gifted or advanced mathematics class. All students would complete each lesson after being introduced to a particular content description. Teachers can integrate the lessons into the primary curriculum taught to a whole group and address higher-order thinking questions through the lesson activity and practice problems.

Small-Group Instruction

Teachers can use this book to differentiate learning in any mathematics class by creating flexible student groups and giving students who need enrichment an opportunity for deeper understanding and engagement with a concept. Students can complete activities and practise at a self-guided pace with a partner or small group and engage in peer discussion, with or without directed supervision or intervention from the teacher.

Individual Instruction

The practice problems and assessment questions in each lesson are a good way to determine individual understanding of a certain mathematics concept on a deeper level. Nearly every practice problem emphasises making sense of and communicating the process of problem solving and asks students to explain their thinking.

LESSON 2.2

Parts of a Partition

Australian Curriculum: Mathematics Content Description

- Multiply and divide fractions and decimals using efficient written strategies and digital technologies (ACMNA154)

Mathematical Practices

- 1, 2, 3 and 4

Estimated Time

- 60 minutes

Key Terms

- Partition
- Fraction model
- Relationship between percentages and fractions

Materials

- Lesson 2.2 Activity: Monthly Budget
- Lesson 2.2 Practice: Parts of a Partition
- Lesson 2.2 Australian Curriculum Assessment Practice

Objectives

In this lesson, students will

- interpret the products of fractions related to a sequence of operations
- compose a budget plan involving fractional amounts
- convert percentages to fractions.

Lesson 2.2 Activity: Monthly Budget

Inform students that they will be determining their monthly budget plan. Students can work independently or within small groups for this activity. Each student has earned \$2500 for the month. Discuss with students the difference between needs and wants. Also inform students of the relationship between percentages and fractions. Ask students what they think some needed categories would be where a percentage of this money would have to be spent. The categories for the activity are: rent/mortgage, groceries, unexpected expenses, electricity/water and entertainment. Students will answer questions to determine what percentage of money will be spent in each category. Upon completion, students will create a tape diagram to display their work.

LESSON 4.2

Finding Volume and Applying the Formula

Australian Curriculum: Mathematics Content Description

- Calculate volumes of rectangular prisms (ACMMG160)

Mathematical Practices

- 1, 2, 3 and 4

Estimated Time

- 60 minutes

Key Terms

- Volume
- Length
- Width
- Height

Materials

- Lesson 4.2 Activity: Librarian Helpers
- Lesson 4.2 Practice: Finding Volume and Applying the Formula
- Lesson 4.2 Australian Curriculum Assessment Practice

Objectives

In this lesson, students will

- apply formulas to solve for volume
- determine how many objects will fit in certain containers based on volume.

Lesson 4.2 Activity: Librarian Helpers

The students will first estimate and then calculate the volume of a bookshelf within the classroom and will be given the volume of a classroom book. Students will determine how many of the classroom books they think can fit on the shelf. Be sure to measure the volume of the shelves that you will allow students to measure to ensure their responses are accurate. A different number of books will fit on the shelf depending on how the books are laid. Be sure to give clear directions that students must have all books positioned the same and in an organised fashion, just like a librarian would have in a library. All book titles must be visible, meaning no books can be placed in front of others.

LESSON 5.1

Classifying Figures

Australian Curriculum: Mathematics Content Description

- Classify triangles according to their side and angle properties and describe quadrilaterals (ACMMG165)

Mathematical Practices

- 1, 2, 3 and 4

Estimated Time

- 60 minutes

Key Terms

- Attribute
- Hierarchy

Teacher's Note. During the practice portion of this lesson, students will work together as a class. The index cards referenced in the students' directions have been provided. Add in shapes as necessary so everyone has a card.

Materials

- Lesson 5.1 Activity: Flow Chart
- Lesson 5.1 Practice: Classifying Figures
- Lesson 5.1 Shape Index Cards
- Lesson 5.1 Australian Curriculum Assessment Practice
- Computers for flow chart research and creation of flow chart
- Tape (or something else to place index cards on students)

Objectives

In this lesson, students will

- design flow charts to classify shapes in a hierarchy
- utilise cooperative learning skills to categorise shapes.

Lesson 5.1 Activity: Flow Chart

Students will research how to create flow charts on Microsoft Word or other available computer software. Numerous free flow chart programs are also a possibility. Once students discover how to create the flow chart, they will create a hierarchy flow chart using the following terms: polygon, rhombus, rectangle, square, triangle, quadrilateral, pentagon, hexagon, cube, trapezium, circle and kite. Students will justify the placement of each shape in the flow chart and provide examples and non-examples to support their reasoning. Students will print the flow charts, which can be displayed as student work.

LESSON 6.1

Representing Measurements on Dot Plots

Australian Curriculum: Mathematics Content Description

- Construct and compare a range of data displays including stem-and-leaf plots and dot plots (ACMSP170)

Mathematical Practices

- 1, 2, 3 and 4

Estimated Time

- 60 minutes

Key Terms

- Dot plots

Materials

- Lesson 6.1 Activity: Plotted Recipe
- Lesson 6.1 Practice: Representing Measurements on Dot Plots
- Lesson 6.1 Australian Curriculum Assessment Practice
- Measuring cup (one cup)
- Ingredients (see recipe)

Objectives

In this lesson, students will

- determine the frequency of events and plot their discoveries on a dot plot
- interpret dot plots to solve fraction problems using mathematical operations.

Lesson 6.1 Activity: Plotted Recipe

Prior to the lesson, read the recipe and have enough of each ingredient to allow students to measure the correct amounts of each item. Placing students into groups will decrease the amount of materials needed. Be aware of any food allergies before beginning the activity. If food is not an option, pictures of the specific items will be sufficient. Each group will receive almonds, pretzels, popcorn, banana chips, cashews, sultanas, dried cranberries, sunflower seeds, pumpkin seeds and toasted coconut flakes. Students will also need a measuring cup (one cup).

Students will follow the recipe to create a snack. Next, students will create a dot plot that displays all of the necessary measurements in order from least to greatest. Once the snack and dot plot are created, students can enjoy it while determining the frequency of measurements and plotting the results on the dot plot. For example, students are required to use $\frac{1}{2}$ a cup of sunflower seeds, $\frac{2}{4}$ of a cup of pumpkin seeds, $\frac{4}{8}$ of a cup of sultanas and $\frac{3}{6}$ of a cup of cashews. Students will continue the lesson by completing the Lesson 6.1 Practice, which is connected to the recipe activity.