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Preface

This book provides a comprehensive approach to the teaching of early numeracy in the classroom. The book complements the authors' other books:

Early Numeracy: Assessment for Teaching and Intervention (2000)

Teaching Number: Advancing Children's Skills and Strategies (2002)

Early Numeracy: Assessment for Teaching and Intervention, 2nd Edition (2006)

Part I focusses on approaching, organizing and designing instruction and helping teachers to understand mathematics learning. Part I also includes a detailed overview of traditional and emerging approaches to teaching early numeracy.

Part II consists of eight chapters, each of which focusses on an important topic of early numeracy learning. These topics range over learning about number words and numerals; early and advanced counting strategies; structuring numbers in the ranges 1 to 10 and 1 to 20; strategies for adding and subtracting two 2-digit numbers; and early multiplication and division. Each of the chapters in Part II is organized into three sections: topic overview; assessment tasks and instructional activities. The emphasis on assessment tasks is consistent with the fundamental principle of our approach, that is, that comprehensive assessment precedes instruction. Instruction must be based on a very sound understanding of children's current knowledge and strategies.

Part III focusses on issues related to teachers' professional learning in early numeracy, including teachers' use of videotaped interviews of assessments to support their learning about children's conceptual development.

The book sets out a constructivist, inquiry-based approach to instruction. The approach builds on clear understandings of children's current levels of knowledge, where their learning should progress to, and the means by which children learn early numeracy topics.

The methods presented in this book have been used extensively by practitioners in the United States, the United Kingdom and Australia, as well as in several other countries. The book is of interest to all who are concerned with finding new ways to teach early numeracy and with raising standards of learning in schools. Teachers, advisers, numeracy consultants, mathematics supervisors and learning support personnel, as well as teacher educators and researchers whose work relates to this field, will find much of interest and practical help to develop confidence and skill in the teaching of early numeracy.

Introduction

The authors of this book have worked extensively on research and development projects in early numeracy for at least the last 15 years. Much of our research in the early 1990s focused on the development of the Mathematics Recovery Programme – a programme of intensive instruction for children encountering significant difficulties in early number learning. From the mid-1990s onward, this programme has been used extensively in the United States, the United Kingdom, Australia and elsewhere. An additional important point is that, in many of the school districts where Mathematics Recovery has been implemented, the programme has provided the basis for major transformations in approaches to teaching early numeracy in the classroom, that is, in teaching early numeracy to all children.

The authors and contributors to this book have spent countless hours on the provision of school-based and system-based professional development and support focussing on early numeracy assessment and instruction. This work has involved many thousands of teachers in a range of countries. The work provides teachers with ways to observe and document children's mathematical activity and thinking, and approaches to instruction that take account of detailed information about children's current levels of knowledge.

The theory and approaches presented in this book have their origins in constructivist-based research into early number learning. This includes Leslie Steffe's constructivist teaching experiment research and Paul Cobb's classroom teaching experiment research, as well as research into early number learning by the first author of this book. This book also draws significantly on approaches to teaching number developed by the Freudenthal Institute.

PURPOSE OF THIS BOOK

This book has the purpose of providing a detailed and comprehensive guide to the classroom teaching of early numeracy, that is, the teaching of early numeracy to all children. This book complements our earlier two books – *Early Numeracy: Assessment for Teaching and Intervention* and *Teaching Number: Advancing Children's Skills and Strategies* which focus mainly on the provision of specialist, intervention teaching of low-attaining children.

THE STRUCTURE OF THE BOOK

The book is organized into three Parts. Part I focusses on our general approach to teaching early numeracy, Part II focusses on the teaching of specific topics of early numeracy, and Part III focusses on issues related to teacher professional development in early numeracy.

Part I

Part I consists of two chapters: Chapter 1 focusses on approaching, organizing and designing instruction and Chapter 2 provides a general introduction to Part II.

Chapter 1 consists of three sections. The first presents nine principles of classroom teaching which have been used extensively by the authors and others as a guide to classroom teaching. Also included in this section is the Classroom Instructional Framework for Early Number (CIFEN) which provides detailed guidance for the development of instructional sequences, that is, sequences of interrelated instructional topics that progressively build an important aspect of children's early number knowledge.

In the second section, scenarios are presented which demonstrate different approaches to classroom organization for instruction. Block scheduling is also discussed. The third section provides a description and illustration of the use of the Teaching and Learning Cycle for designing instruction.

Chapter 2 consists of three sections and provides a detailed overview of Part II. Section 1 sets out the common structure that applies to Chapters 4 to 11. Sections 2 and 3 provide a broad overview of the approaches to teaching number to 4–8-year-olds that are the focus of Chapters 3 to 10. Section 2 focusses on aspects of the teaching of number typical of the earlier years in this range, say 4–6-year-olds, and Section 3 focusses on aspects of the teaching of number typical of the later years in this range, say 6–8-year-olds.

Part II

Part II of the book is organized into eight chapters, each of which presents an important topic of early numeracy. There is a common structure for these chapters, consisting of three sections – topic overview, assessment task groups, and instructional activities. In each chapter the first section provides a comprehensive overview of the topic of early number learning that is the focus of the chapter.

The second section of each chapter sets out in detail up to nine assessment task groups relevant to the topic of the chapter, which can be used by the teacher to assess comprehensively the extent of children's knowledge of the topic. Assessment in this form provides a crucial basis for instruction. An assessment task group is a group of assessment tasks in which all the tasks are very similar to each other. The tasks in each assessment task group focus on a particular aspect of the number topic in the chapter. Each task group includes details of how to present the assessment tasks and notes on the purpose of the task, children's responses, and so on. Across Chapters 3 to 10 there is a total of 59 assessment task groups.

The third section of each chapter sets out in detail up to ten instructional activities relevant to the topic of the chapter. Each instructional activity has the following five-part format: title, intended learning, description, notes and materials. These activities are designed so that they are easily incorporated into lessons. Across Chapters 3 to 10 there is a total of 75 instructional activities.

Topics in Chapters 3 to 10

Chapter 3 focusses on children's early learning about number words and numerals. Developing facility with number word sequences and learning to name numerals constitute important aspects of early numeracy, and the view taken in this book is that these aspects are deserving of a renewed emphasis. This chapter presents new information about how children learn these aspects and approaches to instruction that take account of children's learning.

Chapter 4 focusses on the early development of counting, where counting is regarded as an activity oriented to solving an arithmetical task such as figuring out how many items in a collection or how many items in all when two small collections are put together. The chapter provides a detailed description of a progression of counting types of increasing sophistication – emergent, perceptual and figurative counting, and how these types of counting provide the basis for children's early addition strategies.

Chapter 5 focusses on a new and important topic in early numeracy, that is, structuring numbers in the range 1 to 10. This topic relates to children's facility to combine and partition numbers without using counting-by-ones. Instead the child uses an emerging knowledge of doubles, and the five and ten structure of numbers, that is, using five and ten as reference points. Learning this topic provides an important basis for children's early addition and subtraction strategies, and an important basis for moving beyond a reliance on counting-by-ones.

Chapter 6 continues on from Chapter 4 with a focus on the development of more advanced counting-by-ones strategies and the use of these strategies to solve addition and subtraction tasks. These are the strategies of counting-on, which includes counting-up-from and counting-up-to, and counting-back which includes counting-back-from and counting-back-to. The chapter explains clearly the place of these strategies in the overall development of early numeracy.

Chapter 7 extends the focus of Chapter 5 to the topic of structuring numbers in the range 1 to 20. This topic relates to the development and use of non-counting strategies in the range 1 to 20, and provides a crucial basis for the development of mental strategies for addition and subtraction involving 2-digit numbers. This chapter presents new approaches to teaching involving use of materials such as the arithmetic rack or the double ten frame.

Chapters 8 and 9 focus on the development of mental strategies for addition and subtraction involving two 2-digit numbers. Chapter 8 focusses on the development of a range of strategies which are referred to as jump strategies – in the case of addition, the student begins from one addend and goes forward in jumps of tens and ones according to the second addend. Chapter 9 focusses on the development of a range of strategies which are referred to as split strategies – in the case of addition, the student splits each of the two addends into tens and ones and then separately combines, tens with tens and ones with ones. The chapters include detailed descriptions of approaches to the development of these two kinds of strategies for addition and subtraction.

Chapter 10 focusses on the early development of multiplication and division knowledge. This includes the emergent notions of repeated equal groups and sharing, the development of skip counting and the use of arrays in teaching multiplication and division. Also explained are the ideas of numerical composite and abstract composite unit – important milestones in the development of numerical thinking, the idea of commutativity, and the inverse relationship between multiplication and division.

Part III

Chapter 11 focusses on issues related to teacher professional development in early numeracy. This includes discussion of four examples of children's conceptual difficulties and four examples showing how teachers can use their intuition and creativity to support children's learning. Also explained are teachers' use of videotaped interviews of assessments to support their learning about children's conceptual development and, related to their use of videotaped interviews, the three stages of growth in teachers' reflective conversations with children. Chapter 11 includes a set of six teacher activities that can be the basis of a programme of school-based professional learning focussing on children's early numeracy knowledge. Chapter 11 concludes with a detailed discussion of the role of repeatability and differentiation in early number instruction.

CONCLUSION

The Glossary lists the main terms used in this book. Each of these terms is printed in bold type in the text at its first occurrence. The Bibliography concludes the book.