IT'S ALL RELATIVE

Key Ideas and Common Misconceptions About Ratio and Proportion, Grades 6–7

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Introduction

Understanding ratios and proportional relationships and acquiring the accompanying skills associated with their conceptual development are essential. These ideas permeate our daily lives and underpin further study in mathematics and science (Common Core Standards Writing Team 2011). The Common Core State Standards for Mathematics (CCSS-M) identify this area of study as critical at both grades six and seven (NGA and CCSSO 2010).

The thirty modules in this flipchart are designed to engage all students in mathematical learning that develops conceptual understanding, addresses common misconceptions, and builds key ideas essential to future learning. The modules are research based and can be used to support response to intervention (RTI) as well as offer enrichment activities and challenges for all students. The modules are organized in three sections: Representing Ratios; Unit and Scale Factors; and Percents. While building on students' understanding of multiplication and division, the activities in this flipchart will focus on these key ideas:

- Understanding the language of ratios
- Understanding the multiplicative relationships of ratios
- Using tables, tape diagrams, double number line diagrams, and graphs to represent ratios
- Using unit rates and scale factors to solve problems
- Solving multistep ratio and percent problems

The modules increase in complexity by section, though we do not assume that you will focus on only one section at a time nor that you will necessarily complete each component of an activity or section. You can return to many of these activities as students build their mathematical expertise. Each activity begins with the identification of its **Mathematical Focus**, through identification of specific CCSS-M standards. (Either complete standards or portions thereof are provided.) The **Potential Challenges and Misconceptions** associated with those ideas follow. **In the Classroom** then suggests instructional strategies and specific activities to implement with your students. **Meeting Individual Needs** offers ideas for adjusting the activities to reach a broader range of learners. Opportunities to assess student thinking are often embedded within one section or another. Each activity is supported by one or more reproducibles (located in the appendix), and **References/Further Reading** provides resources for enriching your knowledge of the topic and gathering more ideas.

We encourage you to keep this chart on your desk or next to your plan book so that you will have these ideas at your fingertips throughout the year.

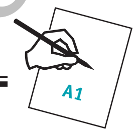
REFERENCES/FURTHER READING

Collins, Anne, and Linda Dacey. 2010. *Zeroing in on Number and Operations: Key Ideas and Common Misconceptions, Grades 7–8*. Portland, ME: Stenhouse.

Common Core Standards Writing Team. 2011. *Progressions for the Common Core State Standards in Mathematics: 6–7, Ratios and Proportional Relationships. Draft.* http://commoncoretools.files.wordpress.com /2012/02/ccss_progression_rp_67_2011_11_12_corrected.pdf.

National Governors Association (NGA) and Council of Chief State School Officers (CCSSO). 2010. *Reaching Higher: The Common Core State Standards Validation Committee—A Report from the National Governors Association Center for Best Practices and the Council of Chief State School Officers*. Washington, DC: NGA Center and CCSSO.

Squatters



Mathematical Focus

- (6.RP.1) Understand the concept of a ratio; describe a ratio relationship between two quantities.

Potential Challenges and Misconceptions

"Ratios arise in situations in which two (or more) quantities are related" (Common Core Standards Writing Team 2011, 2). The greatest challenge for many students is to think multiplicatively, which requires that students think about relationships among numbers of equal groups, rather than countable objects, and pay attention to two quantities at the same time. Teachers must explicitly model an emphasis on the multiplicative nature of ratios. Many students benefit from acting out or modeling ratios using multiple representations.

In the Classroom

It is exciting for students to act out a situation such as doing squats, jumping rope, or bouncing a ball as many times as possible in a specified period of time as they explore ratios. Divide your class into groups of four and tell them to record the number of squats (or other activity) each student in the group can complete in thirty seconds. After everyone has had a turn, have them repeat the exploration for another activity, such as hopping. Students in each group can take turns acting as the squatter (hopper), the record keeper, the counter, and the timer. Students can use the *What's My Ratio? Recording Sheet* reproducible on page A1 of the appendix to record their data.

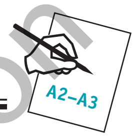
Invite student volunteers to share their ratios. If students do not use labels, omit them as you record what they say, and then ask, "Is this ratio about squats or jumping jacks? How do you know?" Then encourage students to use labels as they report. Once you have recorded several examples, facilitate a discussion about the ratios. For example, you might ask, *What does this ratio tell us about the number of squats completed in thirty seconds? Who can state this relationship starting with the phrase "For every . . ."?* or *What can you tell me about this ratio using the word* per*?* Next ask students to predict, at this rate, how many squats they would complete in one minute.

Meeting Individual Needs

Some students may have physical limitations that prevent them from squatting or hopping. Choose an activity in which all students can participate or provide options such as snapping fingers, typing a name, counting by twelve, drawing stars, or repeating a tongue twister.

REFERENCE/FURTHER READING

Common Core Standards Writing Team. 2011. *Progressions for the Common Core State Standards in Mathematics: 6–7, Ratios and Proportional Relationships*. Draft. http://commoncoretools.files.wordpress.com/2012/02/ccss_progression_rp_67_2011_11_12_corrected.pdf.



Ratio Drama

Mathematical Focus

- (6.RP.1) Understand the concept of a ratio; describe a ratio relationship between two quantities.

Potential Challenges and Misconceptions

Although most students recognize a ratio when it is stated numerically, they often lack recognition of ratio relationships in their daily lives.

In the Classroom

Begin by reading the script provided in the *Script* reproducible on page A2 of the appendix, inviting a student to read one of the roles. After students listen to the script, ask them, “What was different about the ways in which the two people talked? What ratio do you think might represent the number of words Chris said as compared to the number of words Jamie used?” After students estimate, tell them that Chris said 250 words and Jamie said 10. Ask, “So what ratio tells how many words Chris said for every one word Jamie said? How does this ratio compare with your estimate?” In one classroom a student said, “I thought Chris said about one hundred times as many words as Jamie, but it was really only twenty-five.” The teacher was able to emphasize the *times as many as* language to highlight the multiplicative relationship. She also asked, “How could we state this relationship using the words *for every*? What about *per*?”

Challenge students to list other aspects of a conversation someone might count, for instance, the number of times someone said “um” compared to the number of words the person spoke. Assign students to groups of four and have them brainstorm their own scenarios. Note that the scenarios may, but do not need to, be related to conversations; however, they should include people in a real-world situation. For example, students might consider the number of ice-cream cones sold in December compared to the number sold in July.

Have students use the *Planning Your Script* reproducible on page A3 of the appendix to help them plan the dramatizations of their scenarios. They will not write scripts, but instead they’ll make plans that will allow them to create dramatic examples of ratios. When the groups are ready, have each group present its dramatization while the rest of the students identify the relationship dramatized and estimate the ratio that states the comparison.

Meeting Individual Needs

Presenting a video, rather than reading a script, provides students with visual as well as audio input of an “unbalanced” conversation. Students can view the three-minute animation “Bad Date,” one of the “math snacks” found at http://www.mathsnacks.com/.

REFERENCE/FURTHER READING

Van de Walle, John A., Karen Karp, and Jennifer M. Bay-Williams. 2013. *Elementary and Middle School Mathematics: Teaching Developmentally*. 8th ed. New York: Pearson Education.

Equal Values

Mathematical Focus

- (6.RP.3a) Make tables of equivalent ratios; use tables to compare ratios.

Potential Challenges and Misconceptions

There are multiple representations available for determining when ratios are equivalent. However, when using tables or lists, many students think additively and add the same number to both quantities in a ratio to create an equivalent ratio. It is extremely important for students to understand and use the multiplicative relationship.

In the Classroom

Begin this lesson with an assessment question to inform your sense of which students might need more support during this work: *Tell me what you think the term* equivalent *means in mathematics*. Record your students' responses on the class conjecture board. Tell them they are going to investigate equivalent ratios. Then pose the following problem while projecting, or copying on the board, the ratio table shown below.

> Jamal is having pizzas at his birthday party. He knows that he needs 2 pizzas to feed 6 people. How many pizzas should he order to feed 24 people?

Challenge students to complete the following ratio table, illustrating equivalent ratios that might be used to determine the number of pizzas Jamal needs.

Pizzas	2	4	10		
People	6			16	24

As the students work, walk around and observe if any students are using addition to complete the table. For example, some students may suggest 8 pizzas for 4 people, as 4 is two more than 2, and 8 is two more than 6. Ask, "How many people can you feed for every 2 pizzas? Can you show me what that means for the 4 pizzas?"

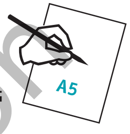
When appropriate, ask student volunteers to share their solutions. Ask questions to stress the importance of using the multiplicative identity. For example: *What was the 2 multiplied by to get a product of 6? How could you use that factor to find the number of people those 12 pizzas can feed?* Then ask why multiplying by the same factor results in equivalent fractions. If no one refers to the multiplicative identity (one), introduce the language to the conversation and make sure students recognize how they multiplied by one. Finally, have students reflect back on their discussion of the meaning of *equivalent* to see if there is anything they want to add, revise, or delete from the conjecture list. Then assign the problem set in the *Equivalent Ratios* reproducible on page A4 of the appendix.

Meeting Individual Needs

Students may model the pizzas-to-people ratio using differently colored tiles. As they model the multiplication through duplication, encourage them to use the language of two pizzas *for every* six people.

REFERENCE/FURTHER READING

Lobato, Joanne, Amy B. Ellis, Randall I. Charles, and Rose Mary Zbiek. 2010. *Developing Essential Understanding of Ratios, Proportions, and Proportional Reasoning for Teaching Mathematics: Grades 6–8*. Reston, VA: National Council of Teachers of Mathematics.



Tape Diagrams

Mathematical Focus

- (6.RP.3) Use ratio reasoning to solve real-world and mathematical problems, e.g., by reasoning about tape diagrams.

Potential Challenges and Misconceptions

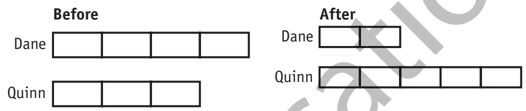
Many students have used tables to represent ratios, but few have worked with tape (or bar) diagrams to show ratio relationships. Often students are unsure about what data to use or how to use given data when solving word problems. The use of tape diagrams is an effective strategy for visualizing the data given in real-world situations as well as for determining an unknown quantity. Tape diagrams effectively model the relationship among the parts being compared to the whole. The whole is divided into equivalent cells, which are labeled to illustrate the various parts. So when comparing boys to girls, for example, some of the cells would be labeled "girls," the remaining "boys," and the total diagram would represent total students. Tape diagrams clearly identify the ratio of those parts so if the ratio of 5 girls to 7 boys is being explored, 5 cells would be labeled "girls," 7 labeled "boys," and the 12 cells would represent the total number of students.

In the Classroom

Present this problem:

> Dane and Quinn collect sports cards. Dane has 4 cards for every 3 cards that Quinn has. If Dane gives Quinn $\frac{1}{2}$ of his cards, what will be the new ratio of Dane's cards to Quinn's?

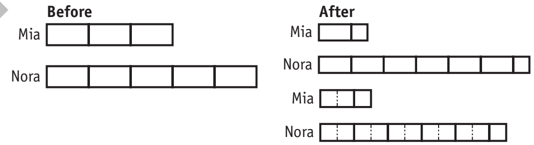
Draw and explain a tape diagram of the "before" ratio before challenging students to model the "after" ratio to solve the problem. This before-and-after model (shown in the figure below) allows students to visualize the changes that occur in the problem. Notice the new ratio is 2:5. Ask if this means that Dane has two cards and Quinn has five cards.



Before assigning the problems in the *How Might Ratios Look?* reproducible on page A5, invite the students to model and solve the following problem:

> Mia and Nora each have a collection of mystery books. For every 3 mystery books Mia has, Nora has 5. Mia decides to give half of her books to Nora. What will be the new ratio of Mia's mystery books to Nora's?

In this situation, students should be able to represent the problem as shown in the following figure. To determine the ratio, however, all parts need to be the same, which requires that the model be partitioned.



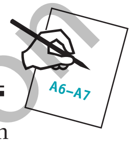
The new ratio is 3:11. Ask if Mia has three books and Nora eleven books.

Meeting Individual Needs

Some students may need more explicit directions to understand that each component in the tape diagrams must be the same size and shape. To enable students who need this support, provide them with Cuisenaire rods that they can use to model the situations. After building the models, they should draw sketches of how the models look.

REFERENCE/FURTHER READING

Murata, Aki. 2008. "Mathematics Teaching and Learning as a Mediating Process: The Case of Tape Diagrams." *Mathematical Thinking and Learning* 10 (4): 374–406.



Double Number Lines

Mathematical Focus

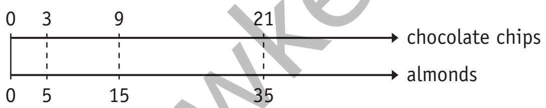
- (6.RP.1) Understand the concept of a ratio; describe a ratio relationship between two quantities.
- (6.RP.3) Reason about double number line diagrams.

Potential Challenges and Misconceptions

Few students have had experience with double number lines and may not recognize that the connections between the lines illustrate that there is a relationship between the two. It is helpful to engage students in exploring double number lines and their multiplicative relationship before expecting students to apply them in problem-solving situations.

In the Classroom

One teacher draws two number lines on her tiled floor with liquid shoe polish. (Custodians find the shoe polish easy to wash off.) Then she asks student volunteers to model the ratio 3 parts chocolate chips to 5 parts almonds. She provides them with sticky notes with the numerals 3, 5, 9, 15, 21, and 35 and masking tape to mark approximate intervals. The students place the sticky notes where they belong on the double number line. The rest of the class observes, noting the students' conversations and decisions. Finally, the students label the number lines as shown in the figure below.



The teacher asks the observing students to comment on how their peers made their decisions and whether or not they agree with those decisions. The teacher then asks all the students to describe what relationship is shown. Sarah states, "The difference is plus 3, then plus 6, then plus 12 on the first line and plus 5, then plus 10, then plus 20 on the bottom."

The teacher asks if anyone noticed anything else. Scott explains, "If you multiply 3 by 3 you get 9, and 3 by 7 you get 21 for the first line, and on the second line if you multiply 5 by 3 you get 15, and 5 by 7 gives 35. That keeps the ratio of 3 to 5."

This teacher reinforces the multiplicative relationship by saying, "So you think multiplication by the same number is important. What property do we use when we multiply each value on each of the number lines by the same factor?" She asks her students to each make a table to check their thinking. To make sure that students understand the ratios are all equivalent, she asks, "Is the ratio twenty-one to thirty-five greater than, equal to, or less than the ratio three to five?" Once students have confirmed the ratios are equivalent, she assigns the *Double Up* reproducible from page A6 of the appendix.

Meeting Individual Needs

For students who need more scaffolding, it might be helpful to actually mark intervals on the number lines until the students gain more experience with the multiplicative reasoning. It may also be appropriate to make "slide rules" for the students to manipulate as they develop their reasoning. A *Ratio Slide Rules* template can be found on page A7 of the appendix.

REFERENCE/FURTHER READING

Lamon, Susan. 2012. *Teaching Fractions and Ratios for Understanding: Essential Content Knowledge and Instructional Strategies for Teachers*. New York: Routledge.

Comparing Ratios

A8

Mathematical Focus

- (6.RP.3) Use ratio reasoning to solve real-world and mathematical problems.

Potential Challenges and Misconceptions

Too often students apply algorithms or formulas erroneously. Helping students develop the ability to estimate and compare ratios informally before introducing such techniques provides opportunities for students to reason quantitatively while developing conceptual foundations for later work.

In the Classroom

Present the following information to students:

> There were 20 problems on the quiz.
>
> Student A answered 4 problems correctly for every 1 problem answered incorrectly.
>
> Student B answered 7 problems correctly for every 3 problems answered incorrectly.

Have the students work individually for about four minutes, writing down everything this information tells them. Circulate with a clipboard as they write, noting those students who have several ideas and those that have fewer. Then have students turn to their partners to exchange ideas. Again circulate, this time paying attention to the words the students use to describe and compare the ratios.

Have pairs share one idea at a time with the whole group for as many times as it is possible to do so without repeating. Record each of the comments for all to see. With each suggestion, ask other students if they agree or disagree and discuss as necessary. Consider asking the following questions if no one brings up these ideas:

- *How many problems on the quiz did Student A solve correctly? How do you know?*
- *Who solved more problems correctly on the quiz, Student A or Student B? How do you know? Does anyone else have another way to find this answer?*

Next display the questions below and encourage students to share their thinking with the class.

> Which of the following ratios would you rather have describe how your correct answers compared to your incorrect answers?

5:6 or 6:5
10:3 or 30:9
7:3 or 14:6

Assign the *Which Ratio Do You Want*? reproducible on page A8 in the appendix for more practice.

Meeting Individual Needs

Encourage some students to create tables, tape diagrams, or double number lines of equivalent ratios to help them compare ratios.

REFERENCE/FURTHER READING

Sharp, Janet M., and Barbara Adams. 2003. "Using a Pattern Table to Solve Contextualized Proportion Problems." *Mathematics Teaching in the Middle School* 8 (8): 432–39.

Graphical Representations

A9–A16

Mathematical Focus

- (6.RP.3a) Plot the pairs of values on the coordinate plane. Use tables to compare ratios.
- (7.RP.2a) Test for equivalent ratios on a coordinate plane.

Potential Challenges and Misconceptions

A ratio is, by definition, a comparison between two quantities, and a ratio can be represented on a Cartesian coordinate plane. Most students do not realize this. They do not have experience with finding a ratio from a graph. Further, they think of coordinate pairs only as points on a grid, not as relationships between two quantities. It is important that students recognize this relationship as it sets the foundational understanding for thinking about ratio as slope.

In the Classroom

Project the graph *Which Is Saltier?* from page A9 of the appendix. Challenge your students to work in small groups to interpret the graph and talk about what they notice about the rays. Give the students chart paper and ask them to list any similarities or differences in the rays. When they've completed that task, direct the students to hang their papers around the room and conduct a gallery walk. Instruct each group to post one positive comment and one question on each paper. In one classroom the lists included these observations:

> The steeper the graph the more water there is in the mixture. We can take the points and put them in a table. One ray has more water for the same amount of salt.

Discuss their observations. Begin by focusing on particular points, for example when there are 4 units of salt, and ask students what they can conclude. If the students do not identify which of the rays represents the saltier mixture, ask them which they think would taste saltier. Be sure they understand why the ray that is closest to the salt axis represents the saltier mixture. To ensure your students understand this concept, challenge them to graph the salt to water ratios for 2:3 and 3:4 on the same set of axes. Tell them to list all the observations they can make about the two graphs. Ask for volunteers to share their ideas. Look for responses like these:

> Both graphs have a lattice point at $x = 12$.
> They both have a lattice point at $y = 6$.
> The graph of 3:4 is steeper than the graph of 2:3.
> The graph of 2:3 represents the saltier mixture.

Pair your students and have them play the *Match It* game from the appendix (pages A10–A14). Students shuffle the cards and deal them faceup in an array. The object of the game is to match each graph with its table. The first player selects a card and tries to match it to its corresponding representation. That student must convince his or her opponent that the cards are equivalent. If they *are* equivalent, the student keeps the two cards and the next player takes a turn. If the cards are *not* equivalent, they are returned to the array and the next player takes a turn. If there is a disagreement, the teacher may play the role of referee and tell the students whether they are correct. Play alternates between the two students until all the cards are matched. The student with the most cards wins the game. After students play the game, assign the *Graph It* reproducible, found on pages A15–A16 in the appendix.

Meeting Individual Needs

Some students will benefit from making a table of the data shown in the graph representing the amount of salt in two solutions of water. When discussing the idea of the steepness of a ray, it is important for many students to explicitly use language referring to the axes, such as *The ray that approaches the axis labeled "Salt" is saltier*, instead of just referring to the steepness of the ray.

To offer some students a challenge, have them play a different version of the *Match It* game. Students should arrange the cards facedown and play like Concentration. If a student turns over two matching cards, he or she keeps them; otherwise, the student turns them facedown again and play continues.

REFERENCE/FURTHER READING

Ercole, Leslie K., Marny Frantz, and George Ashline. 2011. "Multiple Ways to Solve Proportions." *Mathematics Teaching in the Middle School* 16 (8): 483–90.

Mathematical Focus

- (6.RP.3) Use ratio reasoning to solve real-world and mathematical problems.
- (7.RP.2a) Test for equivalent ratios.

Potential Challenges and Misconceptions

Too often students are shown a procedure for converting ratios to decimals without using any quantitative reasoning. This lack of understanding is compounded when students use calculators to make conversions before they understand in which order they should enter the values. When asked if a conversion makes sense, students often reply, "Well, that is what the calculator got." Students need multiple opportunities to learn how to determine the reasonableness of their answers.

In the Classroom

One teacher believes strongly that students who are adept at working with ratios in fraction and decimal forms have an easier time building on this knowledge when performing problem-solving tasks. She instructs her students to work in small groups to complete the *Conversion Tables* sheet on page A17 and to record any and all patterns they notice. As they work, she asks questions *such as Is there an answer in the table that you know? How can you use that information to find another decimal?* After an allotted period of time, she invites student volunteers to share their reasoning for the first table. While a few students know that they can change a ratio to a decimal using division, others suggest alternative strategies. As this thinking supports students' sense making, the teacher wants to emphasize it.

Manny: Our group started with the commonest ones, like $\frac{1}{2}$ and $\frac{1}{4}$. Like, 4:8 is $\frac{1}{2}$, which is 0.50, so we filled that in the chart. We also knew that 2:8 is the same as 1:4, which is a quarter, so we put in 0.25.

Sam: We also figured 2:8 was the same as 1:4, which we know is a quarter, or 0.25 . . . it's like 25 cents. Then we added the 0.25 to the 0.50 and got 0.75. So, like we said, 1:4 plus 2:4 must be 3:4 and be 0.75, like 75 cents. Since 3:4 is the same as 6:8, we wrote that on the chart. We figured that 3:8 is halfway between 2:8 and 4:8, so we divided 0.25 by 2 and got 0.125. So then we knew we could just keep adding that to finish the table.

Lucas: We know you can change a ratio to a decimal by dividing. So, we divided each numerator by each denominator. We got the same answers as Manny and Sam.

The class discusses each strategy the students have shared. At the end of the lesson this teacher assigns an exit card and asks her students to find an equivalent decimal for 3:8.

Meeting Individual Needs

Students who struggle with division can use calculators to do the conversions if needed. However, it is often necessary to work with those students to help them understand the order in which to enter the numbers and how to determine if their answers are reasonable.

REFERENCE/FURTHER READING

Lappan, Glenda, James Fey, William Fitzgerald, Susan Friel, and Elisabeth Phillips. 2012. *Comparing and Scaling: Ratio, Proportion, and Percent.* Upper Saddle River, NJ: Pearson Prentice Hall.