THE Xs AND WHYs OF ALGEBRA

Key Ideas and Common Misconceptons

Anne Collins and Linda Dacey

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Hawker Brownlow Education

PO Box 580, Moorabbin, Victoria 3189, Australia
Phone: (03) 8558 2444 Fax: (03) 8558 2400
Toll Free Phone: 1800 334 603 Fax: 1800 150 445
Website: www.hbe.com.au
Email: orders@hbe.com.au

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Introduction

Algebra is so integral to many of today's careers that the class is often referred to as a gatekeeper course. It is also the cornerstone on which all higher mathematics is built. The long-term advantages to successful learners are so great that Robert Moses, the founder of the Algebra Project and a noted civil rights leader, has identified the learning of algebra as a civil right. Though algebraic reasoning is introduced in the earlier grades, most students usually enter into a more formal study of algebra in grade eight.

Too often, educators pay inadequate attention to the conceptual development of algebraic ideas as they focus almost exclusively on procedural knowledge. In fact, many teachers believe that algebra is simply the manipulation of variables and symbols, and that mastery of that manipulation is the goal for a successful algebra program. Further, there is a vast discrepancy among teachers about how algebra should be taught, how it relates to arithmetic, and how it connects to real-world experiences. It is no wonder, then, that teachers are often unable to identify algebra's key ideas or address students' common misconceptions.

By the end of grade eight, all students should have a strong foundation for algebra and should be able to reason about and make sense of algebra. This reasoning and sense making is essential to students' future success in mathematics. This flipchart will focus on the following key ideas:

- using variables meaningfully
- using multiple representations for expressions
- connecting algebra with number
- connecting algebra with geometry
- manipulating symbols with understanding

The thirty modules in this flipchart are designed to engage all students in mathematical learning that develops conceptual understanding, addresses common misconceptions, and builds key ideas essential to future learning. The modules are research based and can be used to support response to intervention (RTI), a philosophy that utilizes quality interventions matched to student needs. They offer suggestions and resources for teachers seeking material for students identified as most likely to benefit from tier 1 or 2 supports as well as enrichment activities and challenges for all students.

Following the recommendations of the National Council of Teachers of Mathematics (2010) and the National Governors Association along with the Council of Chief State School Officers (2010), we have organized the modules at this level into three sections: Expressions, Equations, and Functions. Each module begins with the identification of its **Mathematical Focus** and the **Potential Challenges and Misconceptions** associated with those ideas. **In the Classroom** then suggests instructional strategies and specific activities to implement with your students. **Meeting Individual Needs** offers ideas for adjusting the activities to reach a broader range of learners. All modules are supported by one or more reproducibles (located in the appendix), and **References/Further Reading** provides resources for enriching your knowledge of the topic and gathering more ideas.

We encourage you to keep this resource on your desk or next to your plan book so that you will have these ideas at your fingertips throughout the year.

REFERENCES/FURTHER READING

Collins, Anne, and Linda Dacey. 2010. *Zeroing in on Number and Operations: Key Ideas and Common Misconceptions, Grades 7–8*. Portland, ME: Stenhouse.

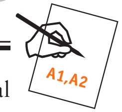
National Council of Teachers of Mathematics (NCTM). 2000. *Principles and Standards for School Mathematics*. Reston, VA: NCTM.

———. 2006. *Curriculum Focal Points for Prekindergarten Through Grade 8 Mathematics: A Quest for Coherence*. Reston, VA: NCTM.

National Council of Teachers of Mathematics (NCTM), National Governors Association (NGA) and Council of Chief State School Officers (CCSSO). 2010. *Reaching Higher: The Common Core State Standards Validation Committee: A Report from the National Governors Association Center for Best Practices and the Council of Chief State School Officers*. Washington, DC: NGA Center and CCSSO.

National Mathematics Advisory Panel. 2008. *Foundations for Success: The Final Report of the National Mathematics Advisory Panel*. Washington, DC: U.S. Department of Education.

Walking the Cartesian Coordinate Plane



Mathematical Focus

- Identify points in all four quadrants.
- Identify x- and y- intercepts.

Potential Challenges and Misconceptions

Many students confuse the sequence of the coordinates in an ordered pair, interpreting the first coordinate as the y-value and the second as the x-value. Some students mistakenly believe they must mark the x-value on the x-axis before plotting the y-value on the y-axis. These students erroneously see two points instead of one ordered pair. For example, they would mark (3, 0) and (0, 4) for the ordered pair (3, 4). Students who have multiple opportunities to physically plot points have a good chance of overcoming these misconceptions.

In the Classroom

One teacher moves all the student desks to the perimeter of the room and draws a large Cartesian coordinate plane on the classroom floor, using liquid shoe polish—a washable material custodians prefer—to mark the axes and label the intervals. If the floor is tiled, it is convenient to use the seams on the floor for the intervals. If the floor is carpeted, you might use masking tape to mark the coordinate plane.

This teacher instructs one volunteer to stand on the origin on the floor grid, a second volunteer to go to the board, and a third volunteer to go to the overhead. The teacher dictates an ordered pair. The student at the origin on the floor must graph the ordered pair by moving to the appropriate point on the grid; the student at the board records the ordered pair and indicates in which quadrant it will be located. The ordered pairs and the quadrants in which they are located remain on the board to allow all students to examine the growing list to identify any patterns they notice about the signs of the coordinates and their corresponding quadrants. The third student plots the point on a grid which is projected for all students to examine. Students at their seats also plot the point and label the quadrant in which the point lies either on individual whiteboards or in their notebooks. After ensuring all students have correctly labeled the quadrant and correctly plotted the point, the teacher has students rotate through the roles. Each student has a turn to "walk the Cartesian coordinate plane."

This teacher makes sure to include ordered pairs that fall in each of the four quadrants as well as points that lie on the x- and y-axes. She also reviews the meanings of the terms that the students will use when playing the *Match It* game (see the *Match It Cards* reproducible on pages A1 and A2 in the appendix). The class reviews the terms *quadrant I*, *quadrant II*, *quadrant III*, *quadrant IV*, x-*intercept*, y-*intercept*, *origin*, *abscissa*, *ordinate*, x-*axis*, and *y-axis*. Then students pair up to play *Match It*, a game similar to concentration. Cards are dealt facedown in an array and students take turns trying to match a term with its mathematical representation. If a player makes a match, he or she takes another turn. If the player doesn't make a match, the other player takes a turn. The game ends when all the cards have been matched.

Meeting Individual Needs

Some students may benefit from taping a template that shows the direction in which the positive and negative integers move to their desks. For instance:

$+x$ goes → $+y$ goes ↑ $-x$ goes ← $-y$ goes ↓

Other students benefit from playing *Paper Battleship*. They see the relevance of using ordered pairs when set in either a game format or a real-world context. Directions are available at http://en.wikipedia.org/wiki/Battleship_game.

REFERENCE/FURTHER READING

Bay, Jennifer, and Deanna Wasman. 2000. "Sharing Teaching Ideas: Making the Coordinate Grid Come to Life with Human Graphing." *Mathematics Teacher* 93 (7): 553–554.

Simplifying Expressions

Mathematical Focus

- Simplify expressions with variables.
- Model operations with integers.

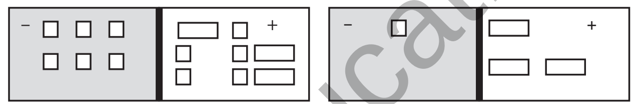
Potential Challenges and Misconceptions

Many students struggle to understand the difference between operators and positive and negative signs. For instance, $+4b$ or $+ (+4b)$ means *add a positive* $4b$, while $+ (-4b)$ means *add a negative* $4b$. Also, $+ (-4b)$ means *add a negative* $4b$, while $- (+4b)$ means *subtract a positive* $4b$. Notice the operator is the first sign and the integer sign is the second sign. The greatest challenge for students is to interpret which operation to perform if there is only one sign given, such as $2b - 3b$. Students must recognize that the operation is subtraction and the lack of a second sign indicates a positive integer. So the difference in this case is $-1b$ or just $-b$.

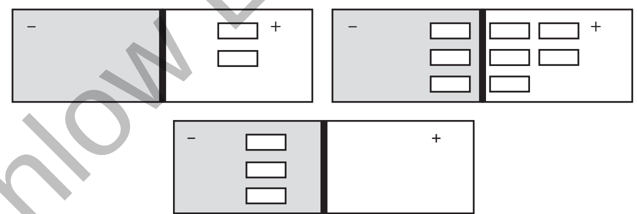
In the Classroom

One teacher introduces algebra tiles (see the *Algebra Tiles Template* reproducible on page A3 in the appendix) to engage students in developing an understanding of simplifying expressions. The class agrees to call the rectangular tile b. They agree that the square tiles are b^2 since the length and width are both b units long. She asks her students what the sum of 5 plus its opposite, -5, would equal. All agree it is 0. Next she queries her students to tell her what the sum of b plus its opposite, $-b$ equals. Again all students answer 0. She introduces the term *zero pairs* and reminds the students that any term plus its opposite is zero.

The teacher distributes copies of the *Chip Board* reproducible on page A4 to the students and challenges them to model each of the following expressions using the tiles (from page A3 in the appendix) on their chip boards: $+3$, -2, $b + 4$, $b - 8$. She also has them record a pictorial model of each expression in their notebooks. She follows this with simple addition computations, emphasizing the need to simplify. For instance, when demonstrating $b + 5 + 2b + (-6)$, students should simplify their mat to show $3b + (-1)$, as shown in the following figures. Students identify zero pairs and remove them from their mats.



Modeling subtraction is helpful, as is thinking about the subtraction symbol as "do the opposite of." This teacher provides her students with the expression $2b - 5b$ and has a volunteer show how that would look using algebra tiles (see figures below).



Next, the class begins a round-robin activity, which is described on the *Equivalent Expressions Round-Robin* reproducible on page A5 in the appendix, to reinforce students' understanding of simplifying expressions.

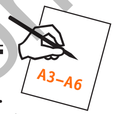
Meeting Individual Needs

Provide students with many experiences to use the chip boards until they show proficiency. As students become more proficient, you may increase the number of terms on the chip board or challenge the students to work in pairs creating and simplifying their own expressions.

REFERENCE/FURTHER READING

Leitze, Annette Ricks, and Nancy A. Kitt. 2000. "Algebra for All: Using Homemade Algebra Tiles to Develop Algebra and Prealgebra Concepts." *Mathematics Teacher* 93 (6): 462–466.

Generalizing Patterns

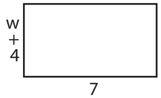


Mathematical Focus

- Model the distributive property.
- Model an expression with algebra tiles.
- Convert an expression in factored form into expanded form.

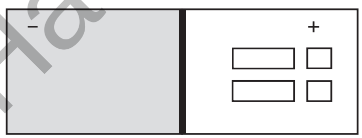
Potential Challenges and Misconceptions

Many students struggle with understanding how to simplify expressions that include variables and operation signs. Some ignore the variable altogether; for example, they assume the sum of $3x + 4$ is 7. Many students do not believe an operator can be contained within an equivalent answer, so they think that $r + s = rs$ or that $3s + 5 = 8s$. Other students neglect to understand the notation or importance of parentheses. If they are given the following rectangle, with a length of 7 and a width of $w + 4$, they indicate the area as $7 \times w + 4$ or $w + 4 \times 7$.



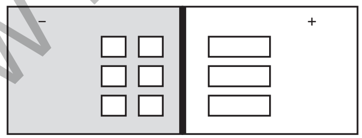
In the Classroom

Research indicates that student learning occurs in three phases, from the concrete to the pictorial to the abstract. Students move through the stages at different rates. One teacher requires her students to model, draw a picture in their notebooks, then write an expression in expanded form for each factored expression she provides. She uses the chip board in the *Chip Board* reproducible on page A4 in the appendix to engage students in modeling the distributive property. She begins by instructing her students to use algebra tiles (see page A3 in the appendix) to model $2(a + 1)$. She calls upon a student to go to the overhead to model and explain what he did. He says he placed the quantity of $(a + 1)$ twice to get a product of $2a + 2$, as shown in the following figure.



The students record the expanded expression $2a + 2$. They discuss whether the expression is equivalent or if they could represent it as $4a$. This engenders a conversation about whether the students can combine the a tile with a unit tile. Students agree it is not possible.

The teacher provides a second expression: $3(a - 2)$. She calls upon a student to share her solution at the overhead. The student explains that she multiplied both the a and the -2 by 3 by using the distributive property to arrive at a product of $3a - 6$, as shown in the following figure.



The class moves on to distributing a negative factor in the expression $-2(b + 1)$. As the teacher walks around, she listens for *accountable talk* (which includes using correct mathematical vocabulary and restating what someone else has said).

The teacher brings the class together to share the students' strategies and assigns the *Working with Expressions* reproducible on page A6 in the appendix.

Meeting Individual Needs

The amount of time students need to rely on the manipulatives varies. Allow students unrestricted use of the manipulatives until they decide they are ready to work pictorially or abstractly. Some students benefit from drawing a picture of the algebra tiles to ensure they have correctly simplified the symbolic expressions.

REFERENCE/FURTHER READING

Chappell, Michaele F., and Marilyn E. Strutchens. 2001. "Creating Connections: Promoting Algebraic Thinking with Concrete Models." *Mathematics Teaching in the Middle School* 7 (1): 20–25.

Geometric Patterns

Mathematical Focus

- Create an expression to generalize a geometric pattern.
- Identify equivalent expressions.
- Connect expressions to visual representations.

Potential Challenges and Misconceptions

Teachers often use visual representations of patterns as a way to concretize algebraic relationships. Unfortunately, rather than building strong figural reasoning, they often direct students to make a table to find the pattern. This oversimplification of the task means that students merely count to complete a table and then ignore the visual model. Generalizations then result from iterative thinking (*I just need to add five each time*), guessing and checking, or the application of rote procedures, rather than an understanding of the structural relationships within the model. Wallace concludes that as a result, "Students who can provide a formula to solve a problem are usually unable to explain why it works" (2007, 511).

In the Classroom

Teachers can show geometric patterns on interactive whiteboards or projectors, as available. One teacher presents the following pattern to his students:

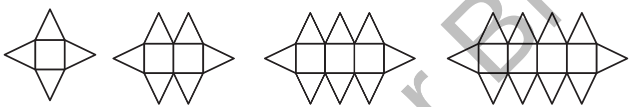


Figure 1 Figure 2 Figure 3 Figure 4

He then distributes a written list of tasks for the students to consider in groups:

1. Describe Figure 5.
2. Draw a sketch of Figure 10.
3. If there were 20 squares, how many triangles would there be? How do you know?
4. If there were 20 squares, how many total pieces (triangles and squares) would there be? What if there were n squares? How do you know?

The teacher walks from table to table as the students work. He notes whether students sketch freely or appear hesitant. He notices who makes a table and who does not. He is particularly interested in the explanations students give to one another. He then brings the groups back together for a discussion of their findings.

Students identify two expressions for the total number of pieces for a figure with n squares: $3n + 2$ and $2n + n + 2$. The teacher records these and the students decide that the expressions are equivalent. Then the teacher asks the students where they can *see* these expressions in the figures. Students who made tables to form a generalization refer to seeing a difference of three in the total. Reid explains, "It keeps going up by three each time."

Squares	1	2	3	4	5	10	20
Triangles	4	6	8	10	12	22	42
Total pieces	5	8	11	14	17	32	62

The teacher again asks about the figures. Julie explains, "Each time there is a row of squares, there are two rows of triangles. The numbers are the same. So our group came up with n plus $2n$ to show the one row of squares and the two rows of triangles. Then we added 2 for the two triangles at the ends."

Jacob added, "Our group just saw the three rows as the same, so we wrote *3n plus 2*." To help them practice this thinking, he has students to complete the *See the Pattern* reproducible on page A7 in the appendix.

Meeting Individual Needs

Some students will almost always prefer to make a table and rely on numerical methods, while others gravitate toward the visual images. Allow your students to begin with their preferred technique, but then encourage them to make connections. For example, ask a student who finds a generalization using a table to then look for those features in the figures.

REFERENCE/FURTHER READING

Wallace, Ann H. 2007. "Anticipating Student Responses to Improve Problem Solving." *Mathematics Teaching in the Middle School* 12 (9): 504–511.

Equivalence of Expressions

Mathematical Focus

- Determine equivalence among representations.
- Simplify expressions to determine equivalence.

Potential Challenges and Misconceptions

Changing the symbolic form of an expression or equation, while maintaining equivalence, is considered a transformational algebraic activity (Driscoll 2010, 13). It is crucial that students become proficient in recognizing and collecting like terms, factoring, expanding, substituting, simplifying, and solving expressions and equations. These skills help students become proficient at taking an expression and either expanding it or simplifying it into an equivalent expression. Many students struggle with simplifying the general form of $(n + 1)^2 - n^2$ by expanding it to $n^2 + 2n + 1 - n^2$ and finding it is equivalent to $2n + 1$, with n being any integer. Yet if they were presented with an arithmetic expression such as $(5 + 1)^2 - 5^2$, most would easily recognize it as $36 - 25$, which is equivalent to 11.

In the Classroom

One teacher provides every student with multiple opportunities to work with expressions. She begins one lesson with a *range question* designed to let her gauge the students' prior knowledge. She posts the following task on the projection device:

> List everything you can about the expression $8 - 2(a + 3) + 2$.

She records their observations and notices that many students suggest they have to subtract before distributing. She tailors her lesson to address that misconception.

She continues the lesson by presenting the following expression: $4 + 3(n + 5)$. She instructs them to simplify it on their individual whiteboards and when finished to hold their whiteboards up for her to see. She examines the whiteboards for common misconceptions such as adding $4 + 3$ before distributing the 3 over the quantity of $n + 5$. After discussing the solution, she groups the students into threes and plays *Equivalent Expressions Round-Robin* (see page A5 in the appendix for directions). This allows her to intervene as soon as she perceives a student is struggling. She conducts the following activities on other days.

- *Do You Have a_____?*: The game *Do You Have a _____?* is similar to go fish. Students play in groups of two or three. One student deals four cards to each participant. The player to the dealer's left begins the game by asking one of the other players, for example, "Do you have a $3n - 15$?" If the second player has an equivalent expression, he or she reads, for example, "I have $3(n - 5)$." If the first player receives the card requested and makes a match, he or she takes another turn. If the second player does not have a matching expression, he or she states, "No match; draw an expression." The first player draws a card and his or her turn ends. Play alternates among the players until they've made all the matches. The *Do You Have a _____?* reproducible on pages A8–A10 in the appendix contains a set of sample expression cards to use for this game.
- *I Have ________; Who Has ________?*: In this game, each student receives one or two cards with a question and a statement. The statement tells the rest of the class what expression the student has, and the question asks the rests of the students if they have an equivalent expression. The game begins with one student asking, "Who has ________?" Whichever student has a match responds, "I have ________." The *I Have ________; Who Has ________?* reproducible on pages A11–A13 in the appendix has sample cards to use for this game.

Meeting Individual Needs

When students demonstrate proficiency, the teacher challenges them to pose a problem that each expression on the game cards might represent. She instructs those students who need more time to master the concept to model each expression with algebra tiles. She also encourages them to sketch each expression if they need a visual representation.

REFERENCE/FURTHER READING

Driscoll, Mark. 2010. "Learning and Teaching Algebra in Secondary School Classrooms." In *Teaching and Learning Mathematics: Translating Research for Secondary School Teachers*, ed. Frank Lester, 13–20. Reston, VA: National Council of Teachers of Mathematics.

Input/Output

Mathematical Focus

- Create tables and graphs to represent real-world situations.
- Translate between natural language and algebraic expressions.

Potential Challenges and Misconceptions

Diana Steele states, "Algebraic thinking includes the ability to analyze and recognize patterns, to represent the quantitative relationships between the patterns, and to generalize these quantitative relationships" (2005, 142). Beginning in the elementary grades, teachers often introduce *Guess My Rule* games to motivate such thinking. Students suggest values for x and the game leader discloses the corresponding values for y. This process continues until students can guess the rule, that is, identify the expression that generalizes the relationship between the x- and y-values. Playing such games familiarizes students with the notion of inputs and outputs and with creating generalizations to connect them. Unfortunately, teachers pay far less attention to helping students connect tables of input and output values to contextual situations, leading students to form incomplete concepts.

When students translate natural language to an algebraic statement, they often translate in the order that the information is given. For example, they are likely to misrepresent a relationship such as *there are three times as many crayons as markers* as $3c = m$. This misconception is known as a syntactic error and accounts for many of the difficulties students experience in the solution of word problems. Though identified many years ago, the misconception persists.

In the Classroom

Connecting situations to tables and graphs first, instead of moving immediately to finding expressions that generalize the relationships, can help students recognize errors in their thinking. Describe a situational relationship to students such as *There are four times as many children as adults at the picnic*. Ask: *Are there more children or adults at the picnic? What is one possible answer for how many adults and children there could be at the picnic? If there are six adults, how many children are there?* Ask for additional possibilities and record these responses in a table with the x-column labeled Adults and the y-column labeled Children. Challenge students to work in groups to write an expression that will allow them to identify the *output* (number of children) if they know the *input* (number of adults). Do not be surprised if some of the students reverse the relationship and state that four times the number of children tells the number of adults. Encourage them to check their ideas by seeing if they work for the ordered pairs in the table. Finally, point to a particular ordered pair in the table and ask, *What does this pair tell us about this situation?*

Talk explicitly with the students about this common error and help them develop ways to check their thinking.

1. Find one ordered pair that works.
2. Pay attention to which variable is greater.
3. Think about how to balance the relationship when you write your expression.
4. Check your idea with your ordered pair.

Continue to explore examples (both linear and nonlinear), sometimes asking the students to make a table, sometimes asking them to make a graph. After several examples, ask, *How can you predict whether the graph will be a straight line or not?* Students can practice these ideas by completing the *It's a Relationship* reproducible on page A14 in the appendix.

Meeting Individual Needs

Working in pairs or small groups can be helpful to students because they can talk about their thinking. You may want to have students record a procedure similar to the four-step process listed in the previous section and keep the list in view.

For students ready for a greater challenge, have them identify the differences between the values in the y-column of the tables when the x-values are listed from least to greatest and the intervals between them are equal. What do they notice when the relationships are linear? What about when they are nonlinear? Why do they think this is so?

REFERENCES/FURTHER READING

Clement, John. 1982. "Algebra Word Problem Solutions: Thought Processes Underlying a Common Misconception." *Journal for Research in Mathematics Education* 13 (1): 16–30.

Steele, Diana. 2005. "Using Writing to Access Students' Schemata Knowledge for Algebraic Thinking." *School Science and Mathematics* 105 (3): 142–154.

Problem Solving with Patterns

Mathematical Focus

- Identify and select appropriate problem-solving strategies.
- Identify and generalize mathematical patterns in given situations.

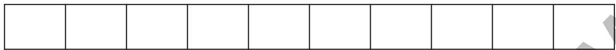
Potential Challenges and Misconceptions

There is a pervasive misconception that one has to have a good memory to be successful in mathematics. Contrary to that belief, Booth and Koedinger (2008) found that conceptual understanding is key to solving algebraic problems. It is essential to *not* encourage students to rely on cute sayings or tricks to work through procedures. Strategies are tools or representations that can be applied to any problem situation, thereby eliminating the need for memory and tricks.

In the Classroom

One teacher poses problems such as the following:

Altogether, how many rectangles are there in this figure?



"And," she adds, "what if the figure has *n* rectangles?"

Some of the students call out, "There is only one; the rest are squares." This response initiates a short conversation about the attributes of squares and rectangles.

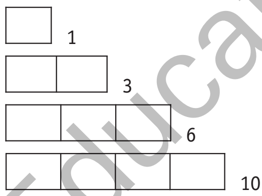
After giving students some time to think about the problem, the teacher asks them to tell her how many rectangles they found. She lists the numbers. She then asks groups to report out the strategies they used.

Jeremy's group made an organized list for identifying all the rectangles. (See following figure.)

1	2	3	4	5	6	7	8	9	10

1-2, 2-3, 3-4, 4-5, 5-6, 6-7, 7-8, 8-9, 9-10 — 9
1-2-3, 2-3-4, 3-4-5, 4-5-6, 5-6-7, 6-7-8, 7-8-9, 8-9-10 — 8
1-2-3-4, 2-3-4-5, 3-4-5-6, 4-5-6-7, 5-6-7-8, 6-7-8-9, 7-8-9-10 — 7
And so on for 6 + 5 + 4 + 3 + 2 + 1

Henry's group solved a simpler problem, drew a picture, and found a pattern, as shown in the following figure:



# of Rectangles	1	2	3	4	5	6	7	8	9	10	*n*
# of Total Ways	1	3	6	10	15	21	28	36	45	55	

The class then discusses the strategies and solutions, identifying similarities and differences in how the students were thinking about the problem.

To give students additional practice with similar problems, hand out copies of the *Express It* reproducible on page A15 in the appendix.

Meeting Individual Needs

When one teacher assigns a problem, she asks her students to think individually about which problem-solving strategy would be appropriate to use. After a minute of individual think time, she instructs her students to share their strategies with their partners. After a brief discussion, the students report their strategies and the teacher records them on the board. This allows all students to access the problem by choosing one of the reported strategies if they could not decide on one by themselves.

REFERENCE/FURTHER READING

Booth, Julie, and Kenneth R. Koedinger. 2008. "Key Misconceptions in Algebraic Problem Solving." In *Proceedings of the 30th Annual Conference of the Cognitive Science Society*, ed. B. C. Love, K. McRae, and V. M. Sloutsky, 571–576. Austin, TX: Cognitive Science Society.

Posing Problems

Mathematical Focus

- Analyze representations.
- Make connections among representations.
- Write story problems.

Potential Challenges and Misconceptions

Rarely are students asked to actually pose problems; rather, they are given word problems that always provide the question to be answered. "When students begin posing their own original mathematical questions and see these questions become the focus of discussion, their perception of the subject is profoundly altered. When they get to spend time working on these questions, their ownership of the experience produces excitement and motivation" (Abrams 2003, 1). Posing problems builds on curiosity, a rare commodity in too many mathematics classes. Knowing what question to ask is a key component in the inquiry and problem-solving process.

In the Classroom

It is important for teachers to model problem posing before assigning such a task to students. When students hear questions such as *I wonder what would happen if* . . . and *What if* . . . as part of regular conversations, they often begin to raise questions themselves. One teacher begins her discussion of problem posing by presenting her students with the following values: n, 3, and 5. She challenges her students to use all the values in a problem situation.

Rylie's group writes the following problem:

> Write an expression for the following situation. Charlie has five more than three times the number of video games his cousin Walter has.

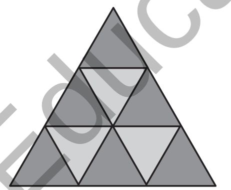
Amanda's group poses this problem:

> Deesha sold three less than five times the number of chocolate chip cookies Marty sold at the school fair. Write an expression for this situation.

Next, the teacher presents a table, as shown in the following figure.

Input	2	4	6	8	n
Output	40	50	60	70	$5n + 30$

She instructs her students to write a story problem for which the table would fit. Students share their problems during a class discussion. Before assigning the situations in the *So What's the Problem?* reproducible on page A16 in the appendix, the teacher instructs her students to pose a story problem for the following stack of triangles.



She is pleased with the assorted questions, which include the following:

- How many triangles would be in row 7?
- If there were n rows, how many triangles would row n have?
- What is the surface area of the triangle?
- How might you find the volume of the triangle?
- What patterns do you notice?

Meeting Individual Needs

To help students who struggle with posing problems, instruct them to examine a familiar problem and change the numerical values. Any time the values change, a new problem is posed. A second strategy is to examine a familiar problem but ask a different question. For example, if the original question is *How long did it take?* students can change the problem by asking, *What was the rate of change?* or even *How far did they go?* Still another strategy is to change the representation that is given. For example, if an original problem gives a table and asks for a graph, students can provide a graph and ask for the data to be represented in a table or give an equation and ask for a graph or a table.

REFERENCES/FURTHER READING

Abrams, Joshua. 2003. "Problem Posing." *Teacher Handbook*. Education Development Center. http://www2.edc.org/makingmath/mathproj.asp#rsskil.

Collier, C. Patrick. 2000. *Menu Collection: Problems Adapted from "Mathematics Teaching in the Middle School,"* 73–74. Reston, VA: National Council of Teachers of Mathematics.