

MAKING NUMBER TALKS MATTER

**Developing
Mathematical
Practices and
Deepening
Understanding,
GRADES 4-10**

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FOREWORD BY JO BOALER

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Introduction

Most upper-grade teachers lament their students' lack of mathematical understanding. They often secretly—or not so secretly—wonder what students did in their previous classes. Why don't they know their multiplication facts in fourth, fifth, or even tenth grade? Why do some students still hide their fingers under the table as they count? And why do they have so much trouble with fractions? The fact that this happens in so many classrooms across the country tells us that the failure lies not with the students or even their teachers, but in how mathematics has been taught, year after year, with the best of intentions.

Understanding quantities and numerical relationships is within the grasp of all students, yet many of them don't realize it. Students come to classrooms fearing and avoiding math and, worse, thinking they are no good at it. Believing that mathematics is mostly about using procedures correctly, they have learned to focus on getting the right answer, whether or not the process makes sense to them. Many students don't expect math to make sense at all. The result is that students learn to disengage their reasoning—and even distrust it.

Making Number Talks Matter is about helping students take back the authority of their own reasoning through a short, fifteen-minute daily routine called Number Talks, in which they reason mentally with numbers. This book will help you, their teacher, learn how to facilitate this routine so that, over time, students develop a strong sense of the meaning of quantities and operations while gaining proficiency with mathematical practices.

The good news is that Number Talks have a predictable structure that will support you in this rewarding path, no matter what grade you teach. The bad news is that there is not one set route to follow. We can't tell you exactly what to do, and we wouldn't want to even if we could! The process of engaging students in reasoning with numbers is one we hope you will consider as a problem-solving venture—an investigation that will help you to learn to listen to your students and learn along with them as you build your lessons around their thinking.

■ A Word About Us—Cathy and Ruth

Number Talks represented a big change in our own practice. We were both taught to make sure that our students were not confused and to explain clearly and structure our teaching so that students would know just what to do. Coming to know that cognitive dissonance is a valuable and even necessary part of the process of learning caused us to examine our practice on many levels. And the payoff has been huge. As our students came to know that they had mathematical ideas worth listening to, as they learned to defend their ways of knowing with sound mathematical arguments, and as they learned the value of listening to one another and building on the ideas of their peers, the culture of our mathematics classes was richly enhanced.

Yes, we focus on Number Talks in *Making Number Talks Matter*, but we also take time to explore principles of teaching and learning. We would love to see every classroom every day be guided by the principles we've outlined here. And we know that providing a space where for fifteen minutes a day children can be the "havers" of mathematical ideas—just this one small innovation—can profoundly change students' relationship with mathematics and their belief about themselves as learners. Ultimately, if enough students start believing in themselves, it just might change the world.

What follows is a quick overview of each chapter. While we hope that you will find the entire book to be both motivating and helpful, we are aware that readers bring a wide range of experience when it comes to Number Talks. For those of you who are new to Number Talks, we have tried to provide enough detail so that you will feel prepared to launch Number Talks in your classrooms. For those of you with a great deal of experience, we hope that we have addressed issues in ways that will help you make Number Talks an even more powerful and empowering part of your school day. The following chapter overviews are intended to help you make decisions about where to begin.

■ Chapter 1: What Are Number Talks? Why Are They So Important?

Number Talks help bring interest, excitement, and joy back into our mathematics classrooms. Before we show you how to make the short, daily Number Talks a regular part of your classes, we first examine why Number Talks are sorely needed.

■ Chapter 2: Getting Number Talks Started

Although Number Talks, sometimes called "math talks," are gaining prominence in today's literature and in many classrooms throughout the country, the term means different things to different people. In this chapter, we describe what we mean by Number Talks and explain how to set them up as a class routine. We also suggest teaching strategies and ideas to help your Number Talks be successful.

■ **Chapter 3: Guiding Principles for Enacting Number Talks in the Classroom**
Several core beliefs guide all of our teaching decisions during Number Talks. We hope these “Guiding Principles for Number Talks” will become part of your teaching, too.

■ **Prelude to the Operations**

Before looking in depth at a particular operation, as we will do in Chapters 4 through 7, we explain how these four chapters are organized. We also introduce three ideas that we consider to be the “new basics” for Number Talks and address issues that are common across the operations of addition, subtraction, multiplication, and division.

■ **Chapter 4: Subtraction Across the Grades**

In this chapter, we discuss why subtraction might be a good place to start incorporating Number Talks. We describe the main strategies that work efficiently for subtraction as it deepens in complexity across the grades, and we help you consider why you might choose one problem instead of another.

■ **Chapter 5: Multiplication Across the Grades**

This chapter takes us from early multiplication to multidigit multiplication of whole numbers and rational numbers and multiplication of algebraic expressions. We also pay close attention to geometric representations, which are particularly powerful in the transition to algebra.

■ **Chapter 6: Addition Across the Grades**

In this chapter we describe and develop strategies that work efficiently for addition as it deepens in complexity across the grades. As with the previous chapters, we also explore the arithmetic properties as they arise in students’ methods of solving problems.

■ **Chapter 7: Division Across the Grades**

Number Talks can offer a refreshing alternative to teaching the standard long-division algorithm that has long confounded many students while consuming unreasonable amounts of mathematics instruction time. In this chapter, we discuss ways that students can make sense of division, divide integers and rational numbers efficiently, and make reasonable estimates in division.

■ **Chapter 8: Making Sense of Fractions (and Decimals and Percent)**

Many middle and high school students want nothing to do with fractions, but their lack of understanding is an obstacle to success in higher mathematics courses. In this chapter, we offer tasks and ideas to help students make sense of fractions, decimals, and percent through Number Talks.

Chapter 9: Number Talks Can Spark Investigations

While it is important to limit daily Number Talks to about fifteen minutes, sometimes a mathematical idea worthy of further investigation arises during a Number Talk. In this chapter we illustrate how ideas posed during Number Talks can lead to important—albeit longer—mathematics lessons.

Chapter 10: Managing Bumps in the Road

As carefully as you might plan, and as much as you believe that Number Talks are important for your students, things won't always go smoothly. You and your students are learning together, so there are bound to be rough patches along the way. But as you will see, these inevitable rough spots offer great opportunities for learning. In this chapter, we identify thorny questions we've been asked, and we share our thoughts on what teachers can do to get beyond some of the common challenges we've all faced with Number Talks.

Chapter 11: Moving Forward

Here we return to our vision in the hopes of inspiring you to continue with what we believe is a transformative classroom practice.

CHAPTER

1

What Are Number Talks? Why Are They So Important?

Many teachers have embraced Number Talks, a brief daily practice where students mentally solve computation problems and talk about their strategies, as a way to dramatically transform teaching and learning in their mathematics classrooms. Something wonderful happens when students learn they can make sense of mathematics in their own ways, make mathematically convincing arguments, and critique and build on the ideas of their peers. As students sit on the edge of their seats, eager to share their ideas, digging deep into why mathematical procedures work, they come to like mathematics and know that they can understand it. And while middle and high school students may not demonstrate such overt enthusiasm, they also come to see Number Talks as both meaningful and fun. Teachers soon find out that if they don't make time for a Number Talk, students will remind them. Students don't want to miss Number Talks!

This is a far cry from the mathematics classrooms that most of us experienced, and even quite different from the mathematics classrooms that many of us taught in for years. We have all seen students with their heads on their desks, rubbing their foreheads, or avoiding eye contact to avoid being called on—or even acting out in other ways when they can't access the mathematical ideas. We have also seen the common mistakes that recur year after year as students try, but often fail, to remember the arithmetic rules they have been taught.

Over the past two decades, many of us who have been teaching a long time have felt the very real pressure to put aside what we know makes sense for students and instead teach the procedures students will need for the tests. In the process, too many teachers have lost their

love of teaching. It's time to bring joy back into our teaching and into our mathematics classrooms. Number Talks can be a wonderful vehicle for helping us do this.

This book is about helping teachers learn to make Number Talks matter for their students. It is about helping students learn to work flexibly with numbers and arithmetic properties; and helping them build a solid foundation and confident dispositions for future mathematics learning. And it is about empowering both teachers and students as mathematical thinkers.

Even those of us who have been doing Number Talks for some time have a lot to learn about how to make them add up to something important—beyond the power in knowing that there are many ways to solve any problem. How do we help students develop flexibility and confidence working with numbers? How can we help each student build a solid foundation for future mathematics learning? What questions do we ask that help students understand important mathematical ideas? What decisions do we make to establish an optimal learning culture in the classroom? How do we best promote a spirit of inquiry and a thirst for knowledge? These questions, and more, are ones that we will grapple with together in this book.

We have not, however, provided step-by-step recipes. Number Talks, if they are to be meaningful, will be organic in nature. While we can plan for each Number Talk by choosing the problem carefully, the discussion about the problem—and where the discussion goes—depends on how our students are thinking.

Before we begin to look at ways to enact Number Talks in classrooms, we examine why this change is needed so you will be prepared for conversations with your colleagues, administrators, parents, and students.

Why Are Number Talks Needed?

Many who have taught middle school or high school mathematics have bemoaned their students' lack of facility with arithmetic. This is nothing new. Decades of research have shown that the traditional curriculum and instructional methods in the United States have left our students with fragile skills and shallow understanding (Hiebert 1999). And every teacher routinely sees students dependent on rote procedures that they apply mindlessly. Unfortunately, examples like this are common:

$$\begin{array}{r} 0x'7 \\ - 9 \\ \hline 8 \end{array}$$

The work here isn't wrong; the algorithm is done correctly, and the answer is right. Yet, it is a bit disturbing that this student went immediately to the algorithm without first thinking

about the problem. Had she done so, she might have realized that following the algorithm would get her nowhere.

Here is another example that we have seen commonly in the middle grades:

$$\frac{1}{3} + \frac{1}{3} = \frac{2}{6} = \frac{1}{3}$$

This student has mixed up rules for fractions. But more disturbing—why didn't this answer raise a red flag that something was amiss? $\frac{1}{3} + \frac{1}{3} = \frac{1}{3}$ makes no sense at all. This brings to mind a memorable observation:

The depressing thing about arithmetic badly taught is that it destroys a child's intellect, and, to some extent, his integrity. Before they are taught arithmetic, children will not give their assent to utter nonsense; afterwards, they will. (Sawyer 1961)

Teaching arithmetic as a set of rules and procedures to be remembered is the major culprit here. This is not, however, to deny the importance of algorithms. As Hyman Bass points out, arithmetic algorithms are remarkable tools; they are reliable and efficient and they work with all numbers. The trouble is that their very compactness “hides the meaning and complexity of the steps involved” (2003, 323).

Consider, for example, a student's explanation for $63 - 27$:

You can't take 7 from 3, so borrow
1 from the 6, and make it a 5.
Put the 1 by the 3, making it a 13.
Now subtract 7 from 13 to get 6.
Then subtract 2 from 5 to get 3.

$$\begin{array}{r} 5\cancel{6}3 \\ - 27 \\ \hline 36 \end{array}$$

The subtraction algorithm conceals the concept of place value in the service of efficiency. Students can get the right answer by treating numbers as columns of *place value-neutral* digits. And, with the value of the digits so far in the background, the relationship between the quantities is lost. The numeral 6 represents 60, but students don't need to know this to get the correct answer. And we can't simply “make the 6 into a 5.” It is misleading to let students think that they can “change” numbers. Meanwhile, the idea that $50 + 13$ is equal to 63 gets lost in the shuffle.

And another misconception is born here. To help students know when to “borrow,” we often say, “You can't take 7 from 3.” Actually, you *can* take 7 from 3, and the answer is, of course, -4 .