YEARS 2–3

Beyond the BUBBLE

How to Use Multiple-Choice Tests to Improve Maths Instruction

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EDUCATION

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Introduction

Multiple-choice testing is an educational reality for students and teachers. Rather than continue to complain about how these tests can adversely affect teaching and learning, we thought it better to turn the testing situation on its head – that is, to take full advantage of all that multiple-choice testing can offer. The purpose of *Beyond the Bubble* is to do just that: show teachers how to get more from multiple-choice tests. By asking students just a few carefully chosen questions, teachers can gain valuable insight into students' mathematical thinking.

Many schools and regions rely on multiple-choice testing to assess students' maths progress. The assumption is that if a student marks a correct answer – if they fill in the *right* bubble – that student is proficient in the corresponding skill or objective. However, a correct answer can often mask fragile knowledge or misconceptions or it may have been just a lucky guess. (There are many examples of this throughout the book.) The inverse is also true. If a student marks an incorrect answer – if they fill in the *wrong* bubble – that student is considered to be in need of remediation. But the student may have just misread the problem or made a mistake when selecting an answer. Both "correct" and "incorrect" answers reveal little about what a student truly does or doesn't understand. Consequently, instructional decisions based on this testing information may be misguided. Again, taking just a few moments to probe students' thinking can provide valuable insight leading to more effective instruction for all students.

Using typical multiple-choice questions often found on Year 2 and Year 3 assessments and in test-prep materials, we asked hundreds of students to explain their answer choices in writing and verbally. We found that both correct and incorrect multiple-choice responses often painted an inaccurate or incomplete picture of students' mathematical understanding. For example, we assumed students who answered questions correctly would consistently show strong understanding and demonstrate logical thinking, but they just as often showed partial understanding, confusion or no understanding at all. We were surprised to find the same was also true for students who marked incorrect responses. But using these test questions and probing with a few additional questions allowed us to get "beyond the bubble" – suddenly we were using the questions to our (and our students') advantage, uncovering understanding and misconceptions, which, in turn, allowed us to make more effective instructional decisions as we considered what our students needed next.

Beyond the Bubble is divided into five strands: number, measurement, algebra, geometry and probability. There are six problems per strand. Each problem begins with a brief overview of the test question's objective, followed by the sample test question, typical student strategies used to solve the problem, conversation starters, actual student work, student-teacher conversations along with teachers' insights and suggestions for instructional strategies that should help advance individual students' learning. Reassessment questions are also provided.

Each strand is followed by a brief list of additional resources to support your instruction. At the end of the book there is a more general list of teaching resources as well as a general list of questions for you to use to start conversations with your students. We've found that posting these questions on the back wall of your classroom provides a quick and easy way to use them with students when having conversations throughout the school day.

Some Dos and Don'ts

- Do take the time to ask questions and listen carefully to students' responses. They will provide you with valuable opportunities to understand and appreciate their strengths and weaknesses.
- Don't rely on a single multiple-choice response alone. It may mask true understanding or misunderstanding, making purposeful instruction difficult.
- Do discuss what you find out with colleagues. Talk about surprises, victories and methods to engage students and help them move forward with understanding.
- Don't be afraid to follow a child's lead. You may not understand their thinking initially, but by listening carefully with an open mind, you may discover brilliance in ways you've never before considered.
- Do reflect on our examples and see if you can find similar outcomes in your class. The more connections you make to your children, the more comfortable you will be in engaging students in meaningful mathematical conversations, ultimately improving your instruction and children's learning.
- Do keep asking good questions that uncover students' learning and understanding, providing you with valuable insights. Children deserve our attention and our best instructional decisions.

As educators, the more information we can gather, the better instructional decisions we can make for our students. We wrote *Beyond the Bubble* for all educators who want better, more focused mathematics instruction for their students. This includes teachers, administrators and pre-service teachers. The results of our work with students provide the basis for excellent in-service discussions or professional learning community (PLC) planning and conversations. When instructional decision makers, both teachers and administrators, examine students' written and verbal explanations, differentiation becomes quicker, easier and more targeted. It is our hope that *Beyond the Bubble* will be used as a tool for insightful, engaging mathematics instruction for you and your students.

PROBLEM ONE

Overview

This problem is typical of measurement problems found on multiple-choice tests for Years 2 and 3. To solve it, students must use the provided conversion information. Questions such as this often ask students to convert metres to centimetres, quarts to cups, hours to minutes and so on. Students can use multiplication or repeated addition to compute conversions from larger units to smaller units. It is important to note that conversion from smaller units to larger units involve division.

Sample Problem

One afternoon, Ellen set up a lemonade stand. She sold 6 quarts of lemonade. How many cups did she sell? (1 quart = 4 cups)

A. 10 cups
B. 12 cups
C. 24 cups
D. 6 cups

Show how you know.

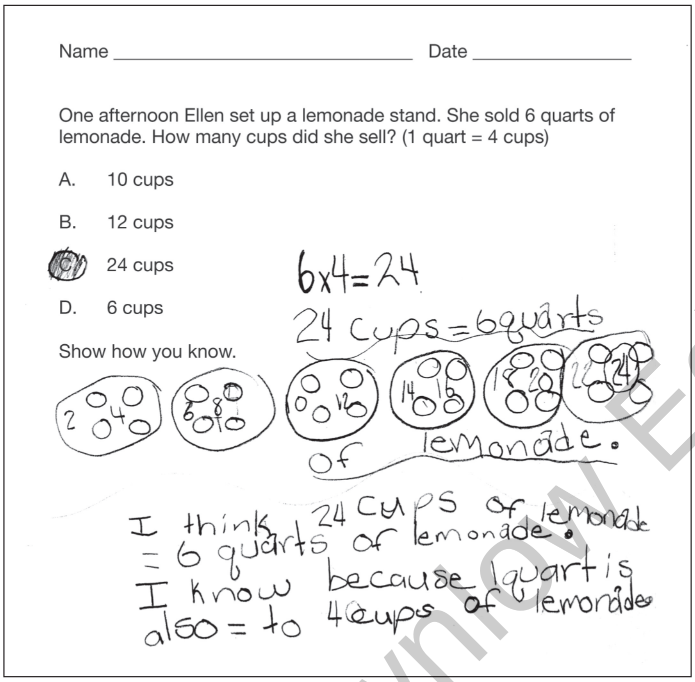
Possible Student Solution Strategies

- Students use multiplication to find the total cups of lemonade sold.
- Students find the total cups of lemonade using repeated addition.
- Students make a computational error while attempting to use a multiplication or repeated addition strategy.
- Students fail to use the information provided by the problem – 1 quart equals 4 cups – and find an incorrect total.

Conversation Starters

- What information is needed to solve this problem?
- How can multiplication help you solve this problem?
- How could drawing a picture help you answer this question?
- What do you know about quarts and cups?
- What do the numbers you used in your solution represent?
- How could you explain your solution to a younger student?

Student Work Sample: Mallory


Name ______________________________ Date ________________

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A. 10 cups

B. 12 cups

C. 24 cups

D. 6 cups

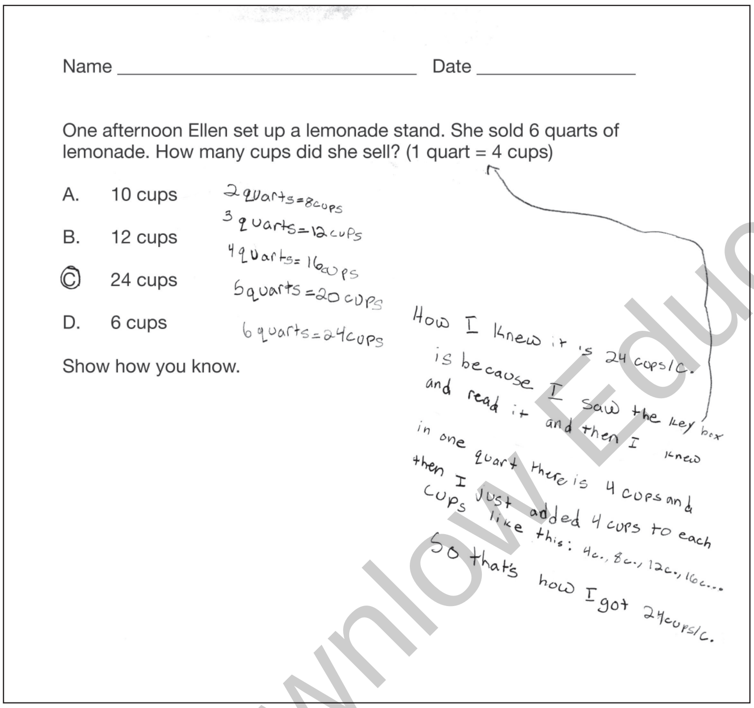
Show how you know.

A Conversation with Mallory	Teacher Insights
T: I see you used a picture to show how you solved this problem. Tell me more about how your picture represents the problem. **Mallory:** The problem says 6 quarts of lemonade were sold. I drew six circles because of the number 6. Next it says that each quart is the same amount as 4 cups. So I put 4 cups in each circle. I counted the cups by twos. That made 24 cups. I also wrote *6 × 4 = 24* because I drew six groups of 4 cups and that made 24 cups altogether. **T:** How would you explain your thinking to a younger student? **Mallory:** I'd draw a picture just like I did and explain as I was drawing what everything meant.	**T:** *Mallory used the information provided by the problem to effectively find an accurate solution. She was able to link her picture and the numbers she used directly to the problem.*

Informed Instructional Suggestions

Mallory understood what the problem was asking her to do and she had an accurate strategy to find an answer. Next steps for Mallory might be to work on measurement problems requiring other conversions involving multiplication, for example, finding the number of quarts in a given number of litres or the number of centimetres in a given number of metres. Then, to provide a challenge, we can introduce Mallory to conversions involving division.

Student Work Sample: Annika



Name ______________________________ Date ________________

One afternoon Ellen set up a lemonade stand. She sold 6 quarts of lemonade. How many cups did she sell? (1 quart = 4 cups)

A. 10 cups

B. 12 cups

(C) 24 cups

D. 6 cups

Show how you know.

2 quarts = 8 cups
3 quarts = 12 cups
4 quarts = 16 cups
5 quarts = 20 cups
6 quarts = 24 cups

How I knew it's 24 cups/c. is because I saw the key box and read it and then I knew in one quart there is 4 cups and then I just added 4 cups to each cups like this: 4c., 8c., 12c., 16c.... So that's how I got 24 cups/c.

A Conversation with Annika	Teacher Insights
T: What information did you need to solve this problem? **Annika:** I needed to know how many cups in a quart. There are 4 cups in 1 quart. **T:** Why did you need to know the number of cups in a quart? **Annika:** Because the problem told me Ellen sold 6 quarts of lemonade. But the question it asked was, How many cups is that? So somehow, I had to have some information to put into my brain to figure that out. The information was that there are 4 cups in a quart. **T:** What did you do with that information? **Annika:** I made a chart. I put down that 2 quarts is equal to 8 cups because four plus four equals eight. Then every time I added a quart, I added 4 more cups. I stopped when I got to 6 quarts and it was 24 cups. And, yippee, 24 was one of the choices! **T:** What would you have done if your answer weren't a choice? **Annika:** I would have been sad and maybe tried again or made my best guess.	**T:** *Annika was able to make sense of the problem and was able to use the information provided to accurately find the number of cups in 6 quarts. Both her written and verbal explanations were clear.*

PROBLEM ONE

Overview

Equivalence is key to this problem and an important idea in algebra. Many young children think the equal sign indicates an action when in reality it indicates a relationship. No computation is actually required to find the solution in this sample question. In this case, if 13 is on both sides of the equation and 7 is on one side and a box is on the other, the only possible number that can correctly complete the equation is 7. Students who do not understand equality often find the sum of the first two addends to fill in the box and complete the equation.

Sample Problem

What number goes in the box to make this number sentence true?

$$13 + 7 = \square + 13$$

A. 13
B. 6
C. 7
D. 20

Show how you know.

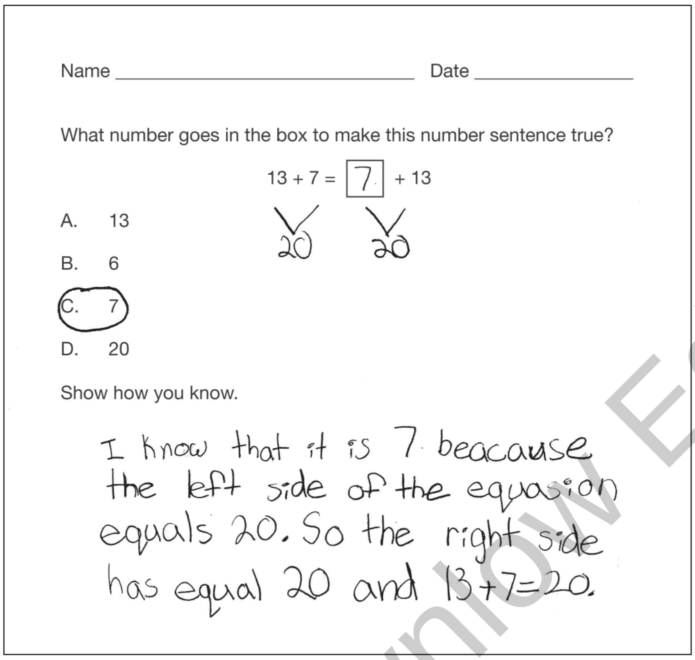
Possible Student Solution Strategies

- Students recognise that the equal sign indicates an equivalent relationship between two quantities and apply this knowledge to find the variable.
- Students recognise and apply the commutative property of addition and realise that 13 + 7 equals 7 + 13.
- Students add the addends on the left side of the equation to find the sum and then find the difference of that sum and the known addend on the right side of the equation to find the variable.
- Students add thirteen and seven to find the sum of twenty and use this sum as the variable, not recognising the meaning of the equal sign.
- Students subtract seven from thirteen to get six.

Conversation Starters

- What do you think the equal sign means?
- Why does your answer make sense?
- Why doesn't one of the other answers make sense?
- Does the equal sign mean to do something or does it show a relationship? How do you know?
- How could someone get the answer of twenty? Why doesn't it make sense?
- If you switch the order of the addends in any addition problem, will it change the sum? How do you know?

Student Work Sample: Tommy



Name ______________________ Date ____________

What number goes in the box to make this number sentence true?

13 + 7 = [7] + 13

20 20

A. 13

B. 6

C. 7

D. 20

Show how you know.

I know that it is 7 beacause the left side of the equasion equals 20. So the right side has equal 20 and 13+7=20.

A Conversation with Tommy	Teacher Insights
T: Tell me about what you think the equal sign means. **Tommy:** I know that it means whatever is on one side of the equal sign is the same amount as what is on the other side of the equal sign. **T:** How did your understanding help you with this problem? **Tommy:** I figured out the left side of the problem first because it had both numbers. I added thirteen plus seven and it is twenty. I know that the thirteen plus the missing number has to be twenty. I started with thirteen and counted up to twenty and it was seven. The missing number is seven.	**T:** *Tommy used his understanding of the meaning of the equal sign to help him find the missing addend. Rather than use the commutative property of addition to find the missing addend, he used a counting-on strategy.*

Informed Instructional Suggestions

Tommy's strategy worked well for these small numbers. Next, Tommy needs experiences that will help him see the usefulness of the commutative property of addition for quickly solving this type of problem, especially when larger numbers are involved.