

INTENTIONAL TALK

HOW TO
STRUCTURE
AND LEAD
PRODUCTIVE
MATHEMATICAL
DISCUSSIONS

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Foreword by Megan Franke

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CHAPTER I

INTRODUCTION

Leading mathematical discussions can be both invigorating and challenging. It's easy to start a discussion by asking children to share their thinking. And nothing beats those moments when children proudly share something they figured out. But then what? Maths discussions aren't just about show-and-tell: stand up, sit down, clap, clap, clap. Knowing what to do with students' ideas and teaching children how to meaningfully participate in discussions can be a lot more daunting.

Open Strategy Sharing: The Case of Mental Maths

You might already have some experience leading discussions that fall into our open strategy sharing category. Discussing mental maths strategies is a good example of open strategy sharing. It is a routine practice in primary mathematics classrooms and is designed to build children's ability to flexibly, efficiently and accurately compute. The teacher starts by posing a computational problem, such as $5 + 2$, $12 - 7$, 21×4 or $96 \div 6$ and invites children to share the different ways they figured out the answer.

Ms Lind picks a multiplication problem for her Year 4 students to solve as a warm-up to her main lesson. She's expecting to have the students spend about ten minutes sharing a few different ways of solving the problem. After writing 25×18 on the board, she steps to the side and provides time for her students to solve this problem mentally. As she sees that children have arrived at their solutions, she whispers to them to write their strategies in their maths journals (she hopes this will help students remember the steps of their strategies). She circulates through the room, noticing the ways that students have approached the problem. When it looks like everyone has at least one solution, she asks the students to call out together what they got for the product. She records their ideas on the board to help make sure she doesn't put any one child on the spot to be correct or incorrect and to give herself the chance to see if there are multiple ideas in the room. She hears two different answers, 498 and 450. With all the ideas out, she begins by calling on a child who she could see has used a strategy that's fairly common in the class. Ms Lind knows that asking Faduma to use her notebook will help her feel more comfortable sharing.

Ms Lind: Okay, Faduma, tell us about what you wrote as you figured out this solution. I want everyone else to think about whether you are understanding what Faduma did and if you used a similar or different strategy.

Ms Lind's opening words help students know what to share and invite listeners into the discussion. She tries to help them know what to listen for.

Faduma: Since I can multiply numbers by 10, I broke up the 18 to a 10 and an 8. I multiplied 25 times 10 and 25 times 8. I got 250 plus 200, which is 450.

Ms Lind: Thank you.

As Ms Lind records Faduma's solution on the board, she notices that many students signal that they used the same strategy with the sign for "me too", inspired by the sign from American Sign Language. Children make the sign with one hand near their chest (or even close to their head), folding over their three middle fingers and rocking their hand back and forth (Parrish 2010; see Figure 1.2).

Figure 1.2 Students show the sign for "me too."



Ms Lind: I've written on the board what I heard Faduma say. And many of you are showing me that you did the same thing. Who can add on to help us explain why we would split the eighteen the way Faduma did?

This question reinforces Ms Lind's cue for listeners to see if they understand Faduma's ideas, which she gave at the beginning of the discussion.

Jordan: Well, it's like Faduma said, multiplying by 10 can be easier to do. So since one way of thinking about 25 times 18 is that you have 25 18 times, you can first do 25 10 times and then you have 8 more 25s.

Students signal agreement with Jordan. And Ms Lind adds some words to what she has recorded to help make this explanation visible in the class display (see Figure 1.3).

Figure 1.3 Faduma's Strategy

$$25 \times 18 = 450 \text{ (this means 25 18 times)}$$

$$25 \times 10 = 250 \text{ (this means 25 10 times)}$$

$$25 \times 8 = 200 \text{ (this means 25 8 times)}$$

Mr William: How many did you get? Let me hear your answers. Shout it out!

Mr William has chosen to invite students to shout out their answers because this first problem was very accessible to all students. As he looked at the sketches on their whiteboards and listened to their whispers, he could see and hear that three groups of seven was an easy entry point for the students.

Class: (In unison.) 21!

Mr William: Does anyone have a different answer?

There are no responses. Mr William asks this question even when he is not expecting to hear a different answer. If teachers don't ask it, they are reinforcing quick correct answers again and students don't have opportunities to learn from mistakes. If he had heard different numbers among the voices, he would have acknowledged that and said, "I am hearing a couple different numbers, which is just fine. As we work through this, I'll want us to figure out together which one is accurate and why. We often have to revise our thinking in mathematics."

Mr William: Okay, I'm going to ask you to share different ways you saw the dots. Mark, what did you see?

Mark: (Speaking softly.) It's a pattern. I can see 7 three times, or 7 plus 7 plus 7, and that's 21.

Mr William: Mark, I want you to say that again and speak loudly so everyone can hear your idea. (Turning to the whole class.) Our job as listeners is to follow what Mark is saying.

As Mr William asks Mark to speak loudly he is supporting students in knowing how to share so their ideas are understandable to others. As he reminds students of their job as listeners he is also supporting students in knowing how to listen.

Mr William: (After Mark repeats his strategy.) So you saw 7 dots in a group. How did you see the 7 dots?

Mark: I saw a chunk of 6 and then 1 above it. (He moves his finger in the air as if he is showing the 6 and 1.)

Mr William: (Circling the dots on the image under the document camera as Mark has described. [See Figure 2.5.]) Okay, so you saw 6 and 1, which makes 7 and you saw that group of 7 three times. We see just what you did. This is one good way. Who saw the dots in a different way? Neeyah?

Neeyah: Kind of like Mark – I saw that the same group happened 3 times. But I wasn't sure how many were in each group. So I counted 3 and 3 and then the 1.

Mr William: Can you come up and point to show us where you saw the 3 and 3 and 1?

Neeyah: (Pointing to the dots and circling them with her finger.) Here they are!

Mr William: Ah, yes. (Turning to the class as he recircles Neeyah's groups. [See Figure 2.6.]) Here is the 3 and 3 and 1 that Neeyah saw. And, Neeyah, how did you know how many there were all together?

Neeyah: I counted all the threes, like 3, 6, 9, 12, 15, 18 and then I counted the ones: 19, 20, 21.

Mr William: Interesting. You counted by threes and then you counted on by ones. So you saw the dots quickly in 6 groups of 3. Okay, let's find out one more way.

Mr William continues by inviting one more student to share and then wraps up the discussion by repeating the three different ways students saw the 21 dots and by emphasising grouping as a foundational idea for multiplication.

Figure 2.5 Mr William marks up the Quick Image to show how Mark decomposed it.

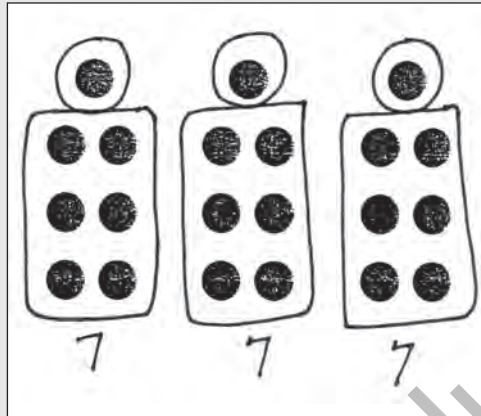
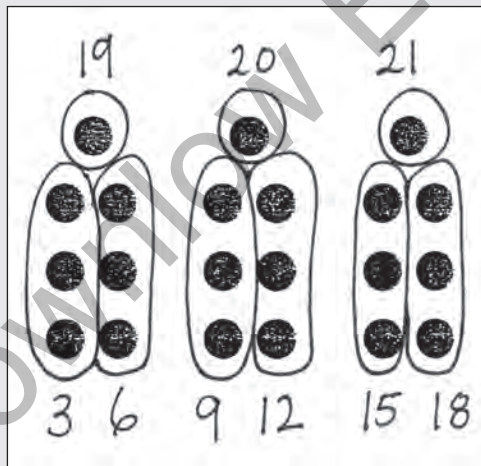


Figure 2.6 Mr William marks up the Quick Image to show how Neeyah counted all the dots.



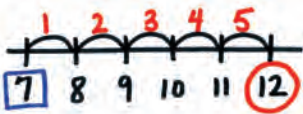
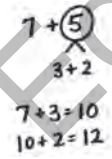
Mr William supports students in several ways. When he calls on Mark and Neeyah, he follows up on their initial responses by asking them to clarify and show the class how they counted. He uses talk moves to support them to share and clarify what they did for the rest of the class. This is one way of getting discussion going in classrooms; it is clearly directed by the teacher and generally goes back and forth between the teacher and one student at a time. This type

next day's discussion. As he prepares, he targets particular solutions to help students think about how you can make decisions about making bigger jumps.

Mr Delgado is using a previous class discussion as fodder for the Compare and Connect discussion (see Figure 3.2 for his planning notes); this helps us see how open strategy sharing can lead to targeted strategy sharing. Throughout this book, for each of our targeted discussion structures, we offer a planning

Figure 3.2

Mr Delgado's Planning Template for a Compare and Connect Discussion: Counting On Strategies

Compare and Connect	
Strategy 1	Strategy 2
$7 + 5$ Counting on by ones on fingers or a number line 	$7 + 5 = 7 + 3 + 2$ Split up the 5 into 3 and 2 and combine 7 with 3 to make 10 and then add 2 
What connections are important for students to notice? You can start with 7 and count on the five one by one. Or we can break up the five into chunks that allow us to easily make tens.	
Supporting students' thinking	
What students might notice	How I might respond to support their thinking
Both started at 7.	Why does starting at 7 make sense? How did the strategies use the second number?
Both got 12.	How did the strategy help get to 12?
One broke up 5 and added three numbers: 7, 3 and 2.	Where did the 3 and the 2 come from? Could you break up the 5 into a 4 and a 1? Why was it useful to break the 5 up that way?
What is the key mathematical idea I want to highlight? Breaking up the second number into chunks that easily make tens makes counting the total efficient.	

template that can help you anticipate and think through key ideas that will help you orchestrate the discussions. Your own experiences and lesson planning strategies will help you adapt these, but in our templates we've included the most important ideas that you will need to have ready before you begin each kind of discussion.

Mr Delgado: Yesterday we were talking about how you might add 7 plus 5 and there were many different ways we thought about this problem.

Referring to the chart from the previous day's discussion, Mr Delgado briefly recaps each strategy. In doing this he is orienting students to each other and the mathematics. He looks to the students who shared the solutions in order to thank them for their contribution and to ensure he has captured their ideas accurately. He looks to the other students to make sure they understand the solutions so they can delve further during today's discussion.

Mr Delgado: There is a lot we can learn from thinking more about each of these strategies. Today, I would like us to focus on two particular solutions. Keisha, you solved this problem by counting on by ones. *(Pointing to her solution on the chart.)* Can you model this for us again? *(Looking to the other students.)* Your job right now is to listen to Keisha explain her idea and make sure you understand her thinking.

Mr Delgado starts by inviting Keisha to share her strategy because he is working towards the mathematical goal of counting on by numbers larger than one. By starting with an example of counting on by ones he hopes to come to a collective understanding of this strategy, name the strategy as "counting on by ones" and use it as a launching point to move to counting on in larger jumps.

Keisha: *(Carrying her number line to the front of the room.)* First I pulled out my number line.

Mr Delgado: Can we watch you do that and listen to how you counted using the number line?

Keisha: Okay. *(Putting her finger at 7.)* 1, 2, 3, 4, 5.

Mr Delgado: Go on, what did you do next?

As Mr Delgado asks Keisha questions such as "Can we watch you do that and listen to how you counted?" and "What did you do next?" he is supporting Keisha and her classmates in knowing what and how to share.

Do I Really Have Time for These Types of Discussions?

You may be wondering, “Do I really have time for these types of targeted discussions?” And you’ve probably already guessed our answer to this question! Most mathematics texts direct teachers to have discussions with their students. The teacher’s guide might list questions to ask students but may not provide much explicit direction about what the goal of the discussion should be. For example, the text might say, “Have students compare their strategies” or “Discuss how to use an array”. And according to the US Common Core Standards for Mathematical Practice, students are expected to make sense of each other’s ideas and to reason through the viability of an argument. We hope we have provided some guidance on how to structure discussions to attain particular goals. We think that when teachers take the time to facilitate sustained discussions, students will learn mathematics in ways that will stick.

When Discussions Are Student Centred, What Is My Role as the Teacher?

We can see in the vignettes that discussions thrive when students take on active roles and their ideas are at the heart of the talk. But when discussions are student centred, teachers often wonder about their role. While the teacher in student-centred discussions may appear more like a “guide on the side” than a “sage on the stage”, the teacher’s role in leading productive mathematical discussions cannot be minimised. For example, remember in Chapter 7 when Mr Barber and his students were engaged in a Troubleshoot and Revise discussion in order to make sense of how students were dividing up the 24 cookies? In this vignette, students were actively making sense of and revising a common strategy that contained an error. During this discussion, Mr Barber made many important moves and decisions in order to reveal the logic in the misconception and make sense of fractions of a set. If you glance back through the vignette, you may notice some of his actions: he decided when students needed think time, he listened in during a turn-and-talk in order to select and sequence which ideas were shared in the whole group, he revoiced important ideas that emerged and invited students to repeat, he oriented students back to the problem context and narrowed upon key mathematical ideas, he provided cubes to help act out the story and he pointed out the importance of persisting through confusions and the value in examining errors.