Table of Contents

Preface to the Third Edition — xi

Chapter 1: Mathematics for All — 1

What is equity, and how is it evident in mathematics classrooms? — 2
How do ability grouping and tracking affect student learning? — 4
How can schools facilitate students' opportunity to learn mathematics? — 6
How can teachers best address different learning styles? — 8
How can mathematics teachers support English language learners? — 10
What instructional strategies help students with disabilities? — 12
How does response to intervention impact mathematics teaching? — 14

Chapter 2: Teaching Mathematics — 17

What instructional methods support mathematical reasoning and problem solving? — 18
How is mathematical thinking addressed in the mathematics classroom? — 20
What role does teacher questioning play in learning mathematics? — 22
How can teachers motivate students to enjoy and want to learn mathematics? — 24
What instructional strategies make mathematics more challenging and interesting to students? — 26
What role does vocabulary instruction play in the mathematics classroom? — 28
How does linking instruction and classroom assessment impact student learning? — 30
What impact does teacher content knowledge have on instruction? — 32
What impact does teacher pedagogical knowledge have on instruction? — 34
How do teacher attitudes about mathematics learning impact student achievement? — 36
What are the characteristics of effective professional development for mathematics? — 38

Chapter 3: Assessment in Mathematics — 41

What roles can assessment play in mathematics teaching and learning? — 42
How can the use of varied assessments provide important evidence of learning? — 44
How can mathematical thinking be assessed in the classroom? — 46
What do national and international assessments tell us about teaching and learning mathematics? — 48

Chapter 4: Mathematics Curriculum — 51

What is the importance of standards-based curricula in mathematics? — 52
How do we determine what students should know and be able to do in mathematics? — 54
What are curriculum coherence and articulation? — 56
What is the importance of reading and writing in the mathematics curriculum? — 58
What considerations are the most important in selecting textbooks and other materials? — 60
In what ways can integrating curriculum enhance learning in mathematics? — 62
How does integrated mathematics instruction affect teaching and learning? — 64
How does classroom curriculum connect to the outside world? — 66

Chapter 5: Instructional Technology in Mathematics — 69

How can using instructional technology affect mathematics reasoning and problem solving? — 70
What effect do calculators have on student learning? — 72
How does technology affect the mathematics classroom's learning environment? — 74

How can students best use information and data from the Internet and Web 2.0 applications? 76
How has technology changed the mathematics that is important for students to learn? 78

Chapter 6: Learning Mathematics 81

How can we promote the importance of learning mathematics to the public? 82
What do we know about how students learn mathematics? 84
What does learning theory show teachers about how students learn mathematics? 86
What roles do basic skills play in mathematics instruction? 88
What is the role of algorithms in mathematics instruction? 90
What factors contribute most strongly to student success in learning mathematics? 92
How do students' attitudes affect their performance and future opportunities? 94
How can teachers help students reflect on and communicate their own learning? 96
What role does active, hands-on learning play in mathematics instruction? 98
How does using contextual or applied activities improve student learning in mathematics? 100
What can parents do to support student learning in mathematics? 102
What are characteristics of effective homework in mathematics? 104
How does teacher learning affect student learning? 106

References and Resources 109

Index 127

SOT0046 • 9781743300046 • © 2012 Hawker Brownlow Education

Preface to the Third Edition

Many good intentions have gone into ambitious education reform goals, including those ratified by the U.S. Congress in 1985 and embodied in the No Child Left Behind (NCLB) Act of 2002. Despite these noble efforts, as of 2009 the United States has not made substantial progress toward the goal of becoming first in the world in mathematics and science education. In international comparisons, U.S. students' overall mathematics and science achievement ranks behind nearly a dozen other nations. The Trends in International Mathematics and Science Study (TIMSS) from 1995 showed our third- and fourth-graders scoring above the international average but our twelfth-graders scoring well below. The TIMSS-repeat results released in 2007 for fourth- and eighth-graders show significant improvement. However, countries other than the United States have statistically higher rates of growth. In 2003, the Program for International Student Assessment (PISA) reported that fifteen-year-olds' performance in mathematics literacy and problem solving ranked lower than the average performance for most industrialized countries. Although the United States appears to be making some small gains, the question still remains: what more can be done to make a difference?

What We Know About Mathematics Teaching and Learning provides a starting place by summarizing educational research and surveys of best classroom practices and offering implications for improved teaching and learning. This resource is not intended to be a complete look at all educational research; it aims only to touch on each focus area and guide the user to resources for additional study. Classroom teachers and preK–12 administrators will find this resource useful for their own professional development; educators can use it to inform and inspire their students, and parents and the public can read about the intended and achieved results of educational practices. Effective reforms in mathematics education practice and policy will require the collaboration of all these stakeholder groups. They will need a common understanding of the current status of mathematics education and of the direction that the "Research and Ideas to Know About" sections indicate for improvement, and they will need to understand how they can help accomplish reform. We hope that this resource provides a foundation for greater understanding and reflection.

As the way we live, work, and learn changes, methods of doing and communicating mathematics continue to emerge and evolve. The level of mathematics needed for thoughtful citizenship is increasing along with the need for greater mathematical problem solving. Today's students must develop skills to manage and use knowledge to solve problems in the personal, social, and economic realms, not just in textbooks. They need to know how to access, evaluate, and use information, all skills that are part of mathematics literacy.

More and more we see demand building for basing educational practice on empirical evidence and solid research. Why is education research an essential, yet insufficient, foundation on which to build student achievement? In the classroom, the critical factor in a lesson's success (that is, whether the students actually learn something that matters) is the professional knowledge and creative ability of the teacher—specifically the teacher's ability to know and understand research and to translate it into practical classroom experience. Research by itself will not result in effective teaching and learning, nor will practice alone result in positive student outcomes. Critical to student success is teachers' knowledge of subject content, skill in implementing appropriate instructional strategies, use of appropriate assessment tools, and commitment to ensure equal opportunity to learn for all students. Little of this can be accomplished unless teachers are knowledgeable about new research and determined to implement its findings. Effective teaching, therefore, involves the practical application of new research and theory in a classroom environment.

Principles and Standards for School Mathematics (*PSSM*; also known as the National Council of Teachers of Mathematics, or NCTM, Standards), published in 2000, describes a vision of a mathematically powerful student and offers a set of goals for mathematics instruction—the basic skills and understandings students need to function effectively in

the twenty-first century. It asserts that enhanced career opportunities exist for those who understand and can do mathematics. It further states that mathematics education should prepare all students, not just a select few, to use mathematics appropriately in their careers and their lives.

The National Mathematics Advisory Panel convened in 2006 to examine the best available scientific research and to recommend improvements in the mathematics education of U.S. children. The panel's final report, *Foundations for Success*, describes concrete steps that can be taken now to significantly improve mathematics education, but it views these steps only as a best start in a long process. The panel's recommendations included a long-range plan to improve the quality and quantity of research on effective mathematics education.

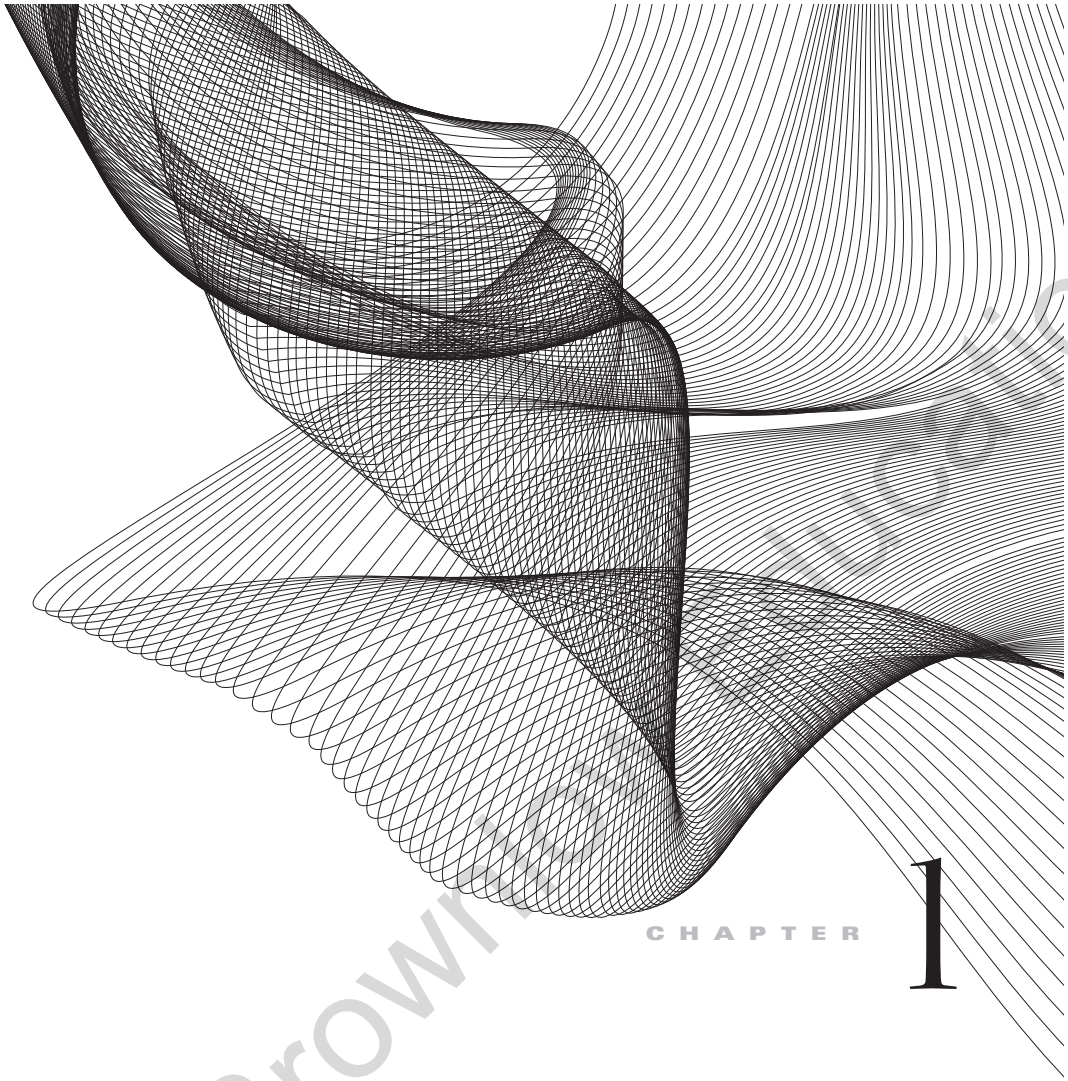
Our purpose in compiling this publication is to support reform of mathematics education and to bring the rich world of educational research and practice to preK–12 educators. Each question we address provides background information from the perspectives of research, followed by implications for improving classroom instruction. Each question concludes with a list of resources for further reading.

We recognize that both U.S. mathematics standards and *Foundations for Success* describe not only important curricular content, but also ways to reform all parts of the educational system to promote improved teaching and student achievement. Systemic reform purposefully revises and aligns all components of a system. The mathematics education system is complex, including components such as assessment, curriculum, equity, student outcome standards, teaching, professional development of teachers, stakeholder involvement, leadership, and policy. Although we do not address all of these topics in this publication, all are important in the context of systemic reform of mathematics education.

Because classroom teachers are mainly concerned with what works in their own classrooms, *What We Know About Mathematics Teaching and Learning* balances presenting research findings with drawing implications from the research. The background research and related documents for the questions we include are intentionally succinct. A full citation of all references, however, appears in the back of this resource. We encourage you to examine the primary source documents, delve into educational research, and apply the findings in your own classrooms.

Every person concerned with teaching and learning mathematics, whether a teacher, administrator, student of education, parent, community member, or member of the higher education community, will find useful information here. As the United States moves forward in mathematics education reform, it must apply lessons learned to achieve improved mathematics education for all students—a goal the nation cannot afford to ignore.

SOT0046 • 9781743300046 • © 2012 Hawker Brownlow Education



CHAPTER 1

Mathematics for All

All students can learn mathematics, and they deserve the opportunity to do so. The National Council of Teachers of Mathematics' *Principles and Standards for School Mathematics* sets forth mathematics literacy expectations for all students and describes what they are expected to learn. However, recognizing the diversity among American children, educators do not expect all students to learn the material in the same manner, with the same resources, or in the same time frame. The Equity Principle in *PSSM* states:

> All students, regardless of their personal characteristics, backgrounds, or physical challenges, must have opportunities to study—and support to learn—mathematics. Equity does not mean that every student should receive identical instruction; instead, it demands that reasonable and appropriate accommodations be made as needed to promote access and attainment for all students. (2000, p. 12)

To achieve "mathematics for all" will take a concerted effort from all stakeholders in our children's education. We must continue to make progress toward providing rich, well-supported learning environments that respond to the unique educational needs of every student. That is the goal of mathematics education reform.

What is equity, and how is it evident in mathematics classrooms?

Research and Ideas to Know About

An equitable mathematics program provides high-quality mathematics education for all students.

An equitable mathematics program provides high-quality mathematics education for all students, in which they have access not only to quality mathematics courses and instruction, but also to the support they need to succeed in those courses. Equitable school programs must ensure that student differences in achievement are not based on race, ethnicity, gender, class bias, or physical disability. Some research suggests that access to and success in higher mathematics leads to greater financial opportunities; therefore, it is becoming an important civil rights issue in education.

Differences in mathematics achievement among various gender, income, and ethnic groups have been widely reported. For instance, while the National Research Council did not find significant differences between male and female students who had taken the same mathematics coursework, the achievement gap for minority learners continues to widen. The number of English language learners (ELLs) in classrooms across the United States is increasing, even in localities that had no ELLs just a few years ago. Furthermore, the field of special education has moved steadily toward the goal of inclusive instruction for students with disabilities.

Achievement in higher-level mathematics is a gatekeeper to success in higher education and in twenty-first-century careers. Research by the U.S. Department of Education indicates that the intensity and quality of the academic content and performance a student brings from secondary school to higher education is a major predictor of postsecondary degree attainment. Group achievement differences in mathematics can often be attributed to enrollment patterns or instructional strategies, leading some to believe that these differences are more of an opportunity gap than an achievement gap. Low-socioeconomic status (SES) students and minority students are half as likely to enroll in higher-level mathematics courses as high-SES white students. Low-SES students and those belonging to minority groups who took high school algebra and geometry attended college in percentages approximately equal to high-SES white students who had enrolled in the same high school courses. Research indicates that when low-income and minority students experience greater success in high school mathematics and science courses, the overall achievement gap between students of differing ethnic and socioeconomic groups diminishes.

Research findings also indicate that younger and lower-ability students can learn to employ the same strategies and skills for mathematical reasoning and thinking as those used by older and higher-ability students. Because different students learn in different ways, equal treatment for all students does not guarantee equal success. Teachers and counselors need to facilitate equal access to algebra, geometry, and higher-level mathematics courses.

SOT0046 • 9781743300046 • © 2012 Hawker Brownlow Education

Implications to Think About

To create an equitable classroom, teachers use various strategies to reach all students with high-quality content. These strategies include:

- Clearly identifying the knowledge students need to master
- Addressing different student needs and learning styles
- Encouraging active participation by all students
- Challenging all students by communicating high expectations and a deep belief in their capabilities
- Diagnosing where students are struggling to learn and providing appropriate instruction
- Embedding various assessment types throughout units of study
- Engaging all students in higher-order thinking skills (such as data analysis, synthesis of results, and evaluation of potential solutions)
- Helping students make meaningful connections among related mathematics concepts, across other disciplines (such as science or social studies), and with everyday experiences
- Providing continual academic support for student learning
- Using inclusive language in all classroom communication
- Engaging parents in student learning

Teachers need adequate knowledge of mathematics content and pedagogy to effectively address the needs of a diverse group of students. Teachers should regularly take advantage of content-specific professional development opportunities to enrich their content knowledge and to stay abreast of the latest teaching techniques.

The physical environment of the classroom should be interesting and inclusive for all students, with visible displays of student work and materials that show diverse groups of people involved in mathematics activities and careers. The context for instruction (that is, small or large groups) should invite all students to participate regardless of their current achievement levels.

The focus of an equitable mathematics program must be on student outcomes. Teachers and principals are responsible for all students' achievement; consequences for lack of student success fall not only on students, but also on teachers, principals, the school, and the family.

Resources for Learning More

Adelman, C. (1999). *Answers in the tool box: Academic intensity, attendance patterns, and bachelor's degree attainment.*

Evan, A., Gray, T., & Olchefske, J. (2006). *The gateway to student success in mathematics and science: A call for middle school reform—the research and its implications.*

Flores, A. (2007, November). "Examining disparities in mathematics education: Achievement gap or opportunity gap?"

Hambrick, A., & Svedkauskaite, A. (2005). *Critical issue: Remembering the child: On equity and inclusion in mathematics and science classrooms.*

Moses, R. P., & Cobb, C. E., Jr. (2001). *Radical equations: Math literacy and civil rights.*

National Council of Supervisors of Mathematics. (2008b, Spring). *Improving student achievement by leading the pursuit of a vision for equity.*

National Mathematics Advisory Panel. (2008). *Foundations for success: The final report of the National Mathematics Advisory Panel.*

National Research Council. (1989). *Everybody counts: A report to the nation on the future of mathematics education.*

Payne, R. K. (2005). *A framework for understanding poverty.*

Quiroz, P. A., & Secada, W. G. (2003). "Responding to diversity."

Schoenfeld, A. H. (2002, January–February). "Making mathematics work for all children: Issues of standards, testing, and equity."

Tennison, A. D. (2007, August). "Promoting equity in mathematics: One teacher's journey."

How do ability grouping and tracking affect student learning?

Research and Ideas to Know About

When addressing diverse student needs, teachers should consider the implications of assigning students to ability groups or tracks for mathematics instruction. Research suggests that these practices do not provide the same educational experience for all students. Equity for the student should be a foremost reference point for teacher reflection in addressing the complex issue of who should learn what mathematics and when.

Students deemed less capable experience less depth and breadth in school mathematics.

Studies suggest that student expectations vary according to assigned ability groups or tracks. Students deemed less capable experience less depth and breadth in school mathematics. Research indicates that the most experienced teachers are assigned to teach high-level classes, whereas teachers with the least experience and mathematical background are assigned to teach the lowest-performing mathematics students. Studies also reveal crucial differences in the kinds of instruction offered in different tracks. Instruction in the lower tracks tends to be fragmented, often requiring mostly memorization of basic facts and algorithms as well as worksheet completion. Although some higher-track classes share these traits, they are more likely to offer opportunities for making sense of mathematics, including discussion, writing, and applying mathematics to real-life situations.

Ability grouping and tracking rarely allow for upward movement when a student makes a developmental leap. Research shows that due to course filtering in the eighth grade, many students do not have access to rigorous high school mathematics classes. As a result, a conflict exists between the structure of academic ability groups or tracks and the potential academic and intellectual growth of struggling students who may be late bloomers.

An alternative to homogeneous strategies of ability grouping or tracking is mixed-ability or heterogeneous grouping for instruction. Heterogeneous instruction emphasizes a differentiated classroom approach in which teachers diagnose students' needs and design instruction based on their understanding of mathematics content by using various instructional strategies that focus on essential concepts, principles, and skills. Inherent in this practice is the opportunity for all students to receive quality mathematics instruction. As the demand for a more mathematically literate society continues, schools need to respond to this challenge and provide meaningful mathematics to all students all of the time.

SOT0046 • 9781743300046 • © 2012 Hawker Brownlow Education

Implications to Think About

To effectively teach students coming from a variety of previous mathematics learning experiences and successes, teachers should thoughtfully choose instructional strategies for working with de-tracked or heterogeneous groups. Each teacher must believe that all students can learn, although in different ways and at different rates.

These instructional elements have been shown to be effective for mixed-ability mathematics classes:

- *A meaningful mathematics curriculum*—This means providing contexts that give facts meaning, teaching concepts that matter, and framing lessons as complex problems.
- *An emphasis on interactive endeavors that promote divergent thinking within a classroom*—Students need to construct knowledge with peers (including safe opportunities to take risks on a regular basis), exchange ideas, and revise their understanding of mathematics.
- *Diversified instructional strategies that address the needs of all types of learners*—Embracing multiple intelligences means presenting information in various ways.
- *Assessment that is varied, ongoing, and embedded in instruction*—Performance assessments, a portfolio of growth and achievements, projects demonstrating the accompanying mathematics, and the opportunity to solve and report on complex problems in varied contexts will provide evidence of student learning.
- *Focused lesson planning*—Instead of emphasizing what the classroom teacher wants to teach, a focused lesson plan begins by understanding what students need to learn and assessing what they already know.

Employing these techniques will provide a rich classroom experience and an effective way to enhance mathematics learning for all students.

Resources for Learning More

Battista, M. T. (1994). "Teacher beliefs and the reform movement of mathematics education."

Lubienski, S. T. (2006). "Examining instruction, achievement, and equity with NAEP mathematics data."

Moses, R. P., & Cobb, C. E., Jr. (2001). *Radical equations: Math literacy and civil rights.*

Oakes, J. (2005). *Keeping track: How schools structure inequality.*

Rousseau, C., & Tate, W. F. (2003, Summer). "No time like the present: Reflecting on equity in school mathematics."

Tennison, A. D. (2007, August). "Promoting equity in mathematics: One teacher's journey."

Tomlinson, C. A. (1999). *The differentiated classroom: Responding to the needs of all learners.*

Tomlinson, C. A. (2003). *Fulfilling the promise of the differentiated classroom: Strategies and tools for responsive teaching.*

Tomlinson, C. A., & McTighe, J. (2006). *Integrating differentiated instruction and understanding by design: Connecting content and kids.*