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HIGH-QUALITY MATHEMATICS CONTENT

As schools and districts transition to the vision of the Common Core State Standards (CCSS), principals must consider what content issues to address in the mathematics curriculum. Looking at high-quality mathematics, they must also understand which essential elements and expectations are taught in their school, as well as how they know they're being taught.

This chapter focuses on issues involving mathematics content and, to a lesser extent, instructional practices as part of the overall content development. It begins with one of the most important content paradigm shifts of CCSS expectations: drilling deeper into content development and student understanding. At every grade level, faculty will teach fewer mathematics content standards. However, teachers will need to "dig deeper" into each standard as they teach for conceptual understanding and skill proficiency.

Research That Informs Content

As you consider the mathematical content needs and priorities at the school level, there are a number of resources that provide the research and best practices to guide and validate decisions related to the content priorities for your students, the development progressions of particular content domains, and the mathematical knowledge your teachers will need to ease your school's implementation of the CCSS.

As a first step, consider a review of the mathematics content, learning, and instructional recommendations from the *Principles and Standards for School Mathematics* (NCTM, 2000). Existing state mathematics standards were either developed or revised based on the *Principles and Standards for School Mathematics.* The more recent Curriculum Focal Points (NCTM, 2006) were intended for states, school districts, and local schools to begin a discussion around the focus topics in grades K–8. The Curriculum Focal Points provide excellent content insight, particularly since they are, in essence, the critical topics that introduce each grade level's content discussion within the CCSS.

An important research-based resource to help guide your transition and implementation of the CCSS is the National Research Council's *Adding It Up* (NRC, 2001). This well-respected resource examines the research related to K–8 mathematics teaching and learning and is a reliable source of authority for any school-based mathematics instructional leader.

As you consider the mathematical content expertise and needs of your faculty and staff, the Conference Board of the Mathematical Sciences' *Mathematical Education of Teachers* (CBMS, 2001) will help you understand the mathematical content background of teachers at all levels and will be a useful guide as you consider teacher background and their grade-level or cross-grade teaching assignments. A 2012 edition of this work is in press.

Implementation of the CCSS will require you and your teachers to carefully consider the pace and depth of particular content domains and standards within the domains, particularly those standards that require student understanding or expect the use of a variety of representations, from manipulative materials to drawings to technological tools. *How People Learn: Brain, Mind, Experience and School* (NRC, 1999), *Mathematics Learning in Early Childhood: Paths Toward Excellence and Equity* (Cross, Woods, & Schweingruber, 2009), the work of Clements, Sarama, Spitler, Lange, and Wolfe (2011), and others will help you when considering the developmental trajectory of the mathematics concepts and skills that promote understanding and lead to fluency at particular grade levels.

Visit **go.solution-tree.com/leadership** for further resources regarding mathematics content.

Less Is More

Mathematics content helps define what teachers teach and what students learn. The CCSS organizes mathematical content according to content domains (CCSSO, 2010a), and as mentioned previously (see table I.1, page 3), one of the talking points of the Common Core State Standards is the need to focus on fewer expectations and standards per grade level.

While this "less is more" story is a good one, the reality is that teaching for conceptual understanding using a variety of instructional tools is no longer something teachers should simply consider—rather, such teaching is now a daily expectation. To illustrate, consider the following fourth-grade standard excerpt from the CCSS content domain Measurement and Data with particular attention to angles and angle measures.

> **Geometric Measurement: Understand Concepts of Angle and Measure Angles**
>
> 5. Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement:
>
>   a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that

turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles.

b. An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees.

6. Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.

7. Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure. (CCSSO, 2010a, pp. 31–32)

This CCSS standard is deeper than many typical fourth-grade student encounters with angles, which tend to ask students to name types of angles—a low-level cognitive task. Also note that actual instructional tools and strategies are suggested as students acquire this level of understanding.

Table 1.1 shows the CCSS content domains for the K–6 mathematics curriculum. Note that the domains Operations and Algebraic Thinking, Number and Operations in Base Ten, Measurement and Data, and Geometry are elements of the elementary school mathematics curricula from kindergarten through grade 5. Also note the special attention given to the domain Counting and Cardinality at the kindergarten level and to Number and Operations—Fractions at grades 3–5.

Table 1.1: Grades K–6 Mathematics Content Domains

Kindergarten	Grades 1–2	Grades 3–5	Grade 6
Counting and Cardinality			
Operations and Algebraic Thinking	Operations and Algebraic Thinking	Operations and Algebraic Thinking	
Number and Operations in Base Ten	Number and Operations in Base Ten	Number and Operations in Base Ten	The Number System
		Number and Operations—Fractions	Ratios and Proportional Relationships
			Expressions and Equations
Measurement and Data	Measurement and Data	Measurement and Data	Statistics and Probability
Geometry	Geometry	Geometry	Geometry

*Visit **go.solution-tree.com/leadership** for a reproducible version of this table.*

Recognize, too, the departure of Operations and Algebraic Thinking, Number and Operations in Base Ten, Number and Operations—Fractions, and Measurement and Data as the mathematics "grows up" at the sixth-grade level. The content domains shift to much more of a pre-algebra focus at this level with attention to the number system, ratio and proportion, and expressions and equations. Similarly, the emphasis on data shifts from the link with

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HIGH-QUALITY MATHEMATICS INSTRUCTION

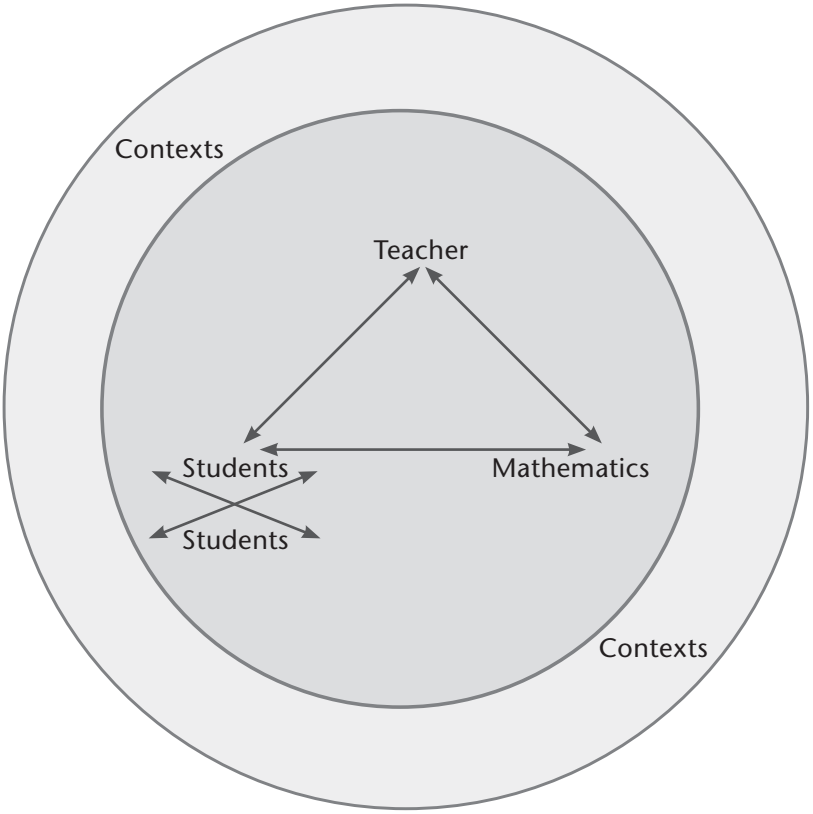
High-quality mathematics curriculum and instructional materials consistent with the Common Core State Standards are indeed important. How teachers implement the curriculum, however, determines what students will actually learn. Understanding the nature of high-quality mathematics instruction, and what teachers and students should be doing in the classroom, is essential for your role as instructional leader of your school. Such understanding is the basis for monitoring and evaluating instruction, assessing what teachers need to improve their instruction, and designing and supporting appropriate professional development. The Mathematical Practices (CCSSO, 2010a) and classroom indicators from chapter 1 provide an initial vision of what both students and teachers should be doing as part of the learning process.

This chapter first characterizes mathematics teaching and considers research about how students learn mathematics, then describes eight essential instructional practices that research supports as most effective in helping students become mathematically proficient, such as understand mathematical ideas, acquire skills, and develop fluency in the CCSS Mathematical Practices.

Research That Informs Instruction

Mathematics teaching can be defined as the "product of the interactions among the teacher, the students, and the mathematics in an instructional triangle" (NRC, 2001, p. 313). Figure 2.1 (page 22) illustrates this view.

While the teachers' knowledge, decisions, and actions are a central aspect of teaching, they are not the only factors that determine what is taught and learned. Figure 2.1 highlights a second component centered on student actions and learning experiences: how students engage with the instructional materials and teacher actions, and how they use the background



Source: NRC, 2001, p. 314. Used with permission.

Figure 2.1: The instructional triangle shows instruction as the interaction among teachers, students, and mathematics in contexts.

knowledge and beliefs they bring to learning. Student interactions with other students also play a crucial role in what is learned.

The third instructional component is the mathematics content, which refers to the mathematical ideas, concepts, and skills to be learned, as well as how the content is represented in the instructional tasks and curriculum materials used in the classroom. The interactions among all three of these components are what ultimately produce student learning.

As figure 2.1 shows, the interactions among teachers, students, and content occur in contexts. *Contexts* refers to the wide range of factors that influence instruction, including district and school policies, school organization, school leadership, and external assessments and related policies. Thus when characterizing mathematics instruction within your school, you must consider the influence of these contextual factors on the teacher-students-content interactions and your role as principal in creating or mitigating these factors.

Research from both cognitive science (Bransford, Brown, & Cocking, 2000) and mathematics education (Bransford & Donovan, 2005; Donovan & Bransford, 2005; Hiebert & Grouws, 2007) supports the characterization of mathematics learning as an active process in which students build their own mathematical knowledge from experience in conjunction with feedback from peers, teachers and other adults, and themselves. In addition, this research has identified a number of core principles of learning that are particularly important for teachers to understand and incorporate into their instruction. Specifically, learners should:

- Engage with challenging tasks that involve active meaning making
- Connect new learning with prior knowledge, and in the process, address preconceptions and misconceptions
- Acquire conceptual knowledge and skills to enable them to organize their knowledge, transfer knowledge to new situations, and acquire new knowledge
- Socially construct knowledge through talk, activity, and interaction around meaningful problems
- Receive timely feedback so they can revise their work, thinking, and understandings
- Develop metacognitive awareness of themselves as learners, thinkers, and problem solvers, and learn to monitor their learning and performance

These essential learning principles provide the research basis for the following eight strategies of effective mathematics instruction.

Visit **go.solution-tree.com/leadership** for further resources regarding instruction.

Instructional Strategies and the CCSS Mathematical Practices

Research on mathematics teaching, along with the learning principles described previously, has identified eight key instructional strategies that produce increased student learning (Bransford & Donovan, 2005; Dweck, 2007; Franke, Kazemi, & Battery, 2007; Fuson, Kalchman, & Bransford, 2005; Henningson & Stein, 1997; Hiebert & Grouws, 2007; NRC, 2001; Pashler et al., 2007; Resnick, 2006; Swan, 2005; Wiliam, 2007a, 2007b). Table 2.1 lists these strategies and highlights how their support for mathematics learning is explicitly connected to the CCSS Mathematical Practices (listed in appendix A, page 107). These strategies are *meta-strategies*—instructional practices that should be incorporated into lessons on a regular basis, regardless of the specific mathematics concepts or skills being taught. As such, these are important look-fors as you observe classrooms and discuss instruction with the teachers in your school.