Strategies for

MATHEMATICS INSTRUCTION AND INTERVENTION

K–5

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Reproducible pages are in italics.

Introduction

The Rationale for 21st Century Mathematics

In today's world, economic access and full citizenship depend crucially on mathematics and science literacy.

—Robert Moses, Civil Rights Leader

While schools have embraced the response to intervention (RTI) model for reading and behavior, implementation of RTI for mathematics continues to lag (Buffum, Mattos, & Weber, 2009, 2010, 2012). Several factors may contribute to this lag in implementation for numeracy.

First, we have valued written and spoken language abilities over mathematics. It is also not uncommon or unacceptable for adults, including elementary educators, to say, "I never liked mathematics as a student" or "I'm not really good at mathematics." It is less likely, however, that an educator would comfortably state, "I never liked reading" or "I've never been a good reader."

In addition, schools' hesitation with the implementation of tiered instruction for mathematics may be impacted by educators' levels of confidence with mathematics, mathematics instruction, and intervention. Often, the teachers with whom we partner freely express feeling less confident teaching mathematics than they do teaching language arts, and they often tell us they feel less professionally satisfied with the mathematics instruction in their classrooms. This may result not only from teachers' lack of confidence in their own conceptual understanding but also from lower levels of confidence in instructional and intervention practices for mathematics. When we ask educators to reflect on their own mathematical learning, their memories include extensive experiences with worksheets, textbook pages, timed assessments, and round-robin competitive games designed to practice automaticity. Story or word problems are often omitted. The reality is that many of us experienced mathematics instruction that was abstract, procedural, and computational. While elements of

Chapter 1

Prioritized Content in Mathematics

There is, perhaps, no greater obstacle to all students learning at the levels of depth and complexity necessary to graduate from high school ready for college or a skilled career than the overwhelmingly and inappropriately large number of standards that students are expected to master—so numerous, in fact, that teachers cannot even adequately *cover* them, let alone effectively teach them to mastery. Moreover, students are too often diagnosed with a learning disability because we have proceeded through the curriculum (or pacing guide or textbook) too quickly; we do not build in time for the remediation and reteaching that we know some students require. We do not focus our efforts on the most highly prioritized standards and ensure that students learn deeply, enduringly, and meaningfully (Lyon et al., 2011). In short, we move too quickly trying to cover too much.

Prioritized Standards

We distinguish between prioritized standards and supporting standards. We must focus our content and curriculum, collaboratively determining which standards are *must-knows* (prioritized) and which standards are *nice-to-knows* (supporting). This does not mean that we won't teach all standards; rather, it guarantees that all students will learn the prioritized, must-know standards. To those who suggest that *all* standards are important or that nonteachers can and should prioritize standards, we respectfully ask, "Have teachers not been prioritizing their favorite standards in isolation for decades? Has prioritization of content not clumsily occurred as school years conclude without reaching the ends of textbooks?" Other colleagues contend that curricular frameworks and district curriculum maps should suffice. But we ask, "Will teachers feel a sense of ownership if they do not participate in this process? Will they understand *why* standards were prioritized? Will they stay faithful to first

Finding the time within the existing number of school days for both conceptual and procedural teaching and learning will require us to be focused on the most highly prioritized mathematics content within a grade level—what the CCSS call critical areas of focus. Finding that time will also require that we plan instruction that makes explicit connections between mathematical ideas.

Concepts Versus Applications?

Many educators use the terms *concepts* and *applications* synonymously; however, we see them as distinct components of a balanced, comprehensive understanding of mathematics. Simply stated, a conceptual understanding addresses why the mathematics works; we can apply conceptual understandings, as well as procedural knowledge, to real-world, problem-solving situations.

The NMAP (2008) states that "to prepare students for Algebra, the curriculum must simultaneously develop conceptual understanding, computational fluency, and problem-solving skills" (p. xix). Students must be able to *apply* mathematics and model and solve real-world problems through mathematical thinking. The most authentic and important applications or problem-solving tasks will be those that can be solved using multiple approaches and that may have multiple solutions. To help classrooms reach this level of depth, we will likely need to begin by teaching students to persevere, think critically, and work collaboratively.

We might visualize the interconnectedness of these three realms with the schematic shown in figure 1.1.

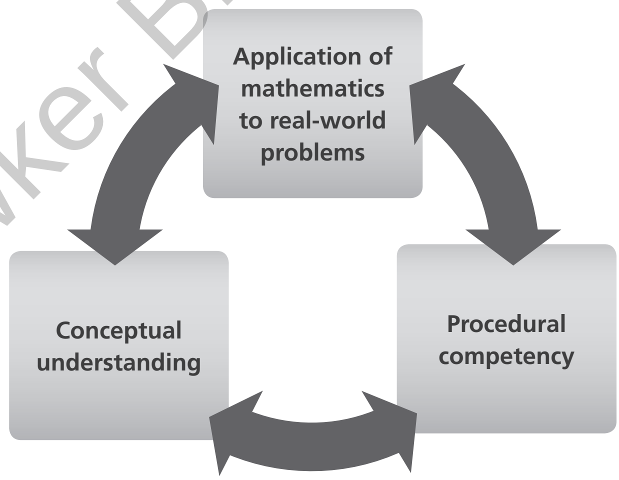
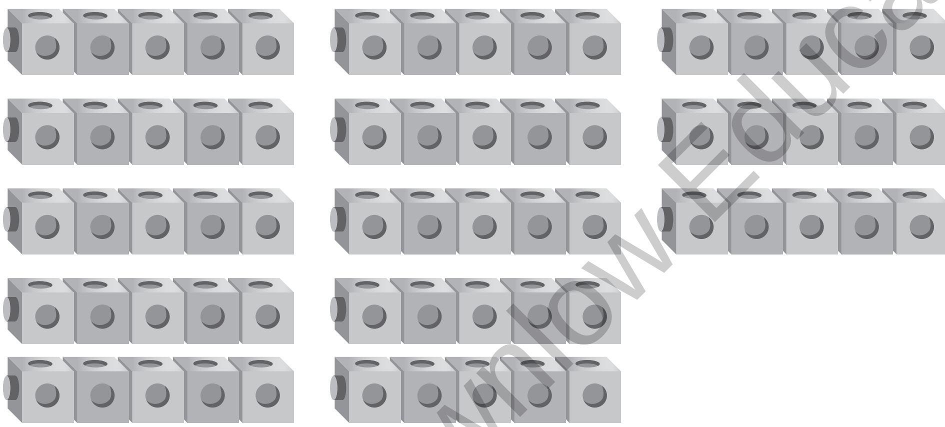


Figure 1.1: Conceptual understanding and procedural competency applied to real-world problems.

In the remainder of this chapter, we will provide examples of units of instruction in kindergarten through fifth grade during which authentic, balanced approaches to mathematics can occur. In figure 1.2, we show how concept, procedure, and application complement each other. The development of procedural and conceptual understanding is mutually reinforcing. Opportunities must be embedded into balanced mathematics instruction for application of both.

Let's take multidigit multiplication as an example. Teacher and students may first represent multiplication as repeated addition, then build on this understanding to represent multiplication as an array, first with concrete objects, such as blocks.



Next, teachers and students may represent and display the product representationally, as repeated addition and as an array.
Repeated addition: 13 × 5 = 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5

Array:

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* * * * *   * * * * *   * * * * *
* * * * *   * * * * *   * * * * *
* * * * *   * * * * *   * * * * *
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Rectangular models and grid paper may be used to help visualize multiplication.

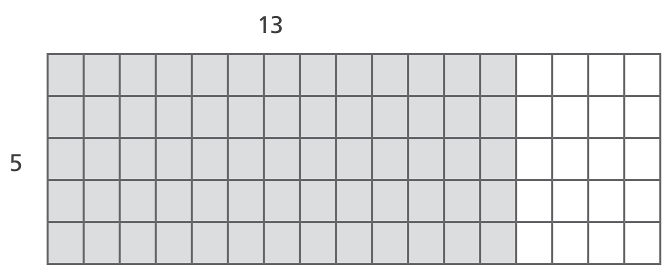

Figure 1.2: The interaction between concept, procedure, and application.

continued →

Table 3.1: Prioritized Understandings for Numbers and Operations K–2

Complex ← Simple

Concept	Description
Conceptual Understanding → Procedural Understanding → Application	
Correspondence	One-to-one relationships between two groups, often demonstrated by touching individual concrete objects with or without using the counting sequence, or by matching two objects. Earliest application can include real-life activities like setting the table and putting on gloves.
Count	Ability to recite numbers or to identify the number of objects, often demonstrated by rote oral counting or identifying the number of objects in a group. Instruction includes cardinal numbers (one, two, three . . .) and ordinal numbers (first, second, third . . .).
Subitize	Ability to see and know the number of objects without counting. As students master the relationship between numbers, five and ten serve as benchmarks. Five-frames and ten-frames are appropriate instructional tools to strengthen this skill.
Conservation	Ability to retain in memory the number of objects without recounting and to recognize that the quantity of a group does not change even if appearance or order changes. A critical skill for using the counting-on strategy.
Cardinality	The principle that when counting, the last number used is the total number of objects.
Compare and Contrast (Magnitude)	Ability to identify if a group or number is greater than or less than another group or number. Learning should include comparison language and requires one-to-one correspondence.
Place Value and Numeration System	The number system that allows students to move from individual counting to more efficient groupings of tens, hundreds, thousands, ten thousands, and so on. The position that a number occupies identifies its value.
Compose and Decompose Whole Numbers	Within the place-value system, the ability to recognize that a series of numbers maintain their individual values and the resulting combined (or added together) value. In the earliest grades, this should focus on the power of ten. Beginning in kindergarten, groups of two and five become the basis of composing and decomposing. By grade 1, learners are composing and decomposing based on sets of ten. In grade 2, students work toward using expanded form to compose and decompose (400 + 20 + 5 = 425). Numbers can be taken apart based on the values of the numerals (425 = 400 + 20 + 5). Flexible application would include the use of words and sets (4 hundreds + 2 tens + 5 ones = 425).
Addition and Subtraction	Addition: An operation that joins groups to make larger groups (I have five brown bunnies, and a friend gives me two more bunnies. 5 + 2 = 7) or represents part + part = whole (I have seven bunnies: five brown and two white. 5 + 2 = 7). Subtraction: An operation that removes a part of a group to create a smaller group (I have seven bunnies, and I give two to a friend: 7 – 2 = 5), or identifies a comparative amount (I have seven bunnies, and you have two bunnies. How many more bunnies do I have? 7 – 2 = 5). It also identifies a missing addend (I have five bunnies, and then my friend gives me some bunnies. Now I have seven bunnies. How many did my friend give me? 7 – 5 = 2).