

Strategies for

MATHEMATICS INSTRUCTION AND INTERVENTION

6–8

Chris Weber

Darlene Crane

Tom Hierck

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Prescribed Learning Outcomes) represent a broad set of learning targets for students to master at an appropriate level of depth and complexity—a fact the authors of the standards recognize and communicate. A learning target is any achievement expectation for students on the path toward mastery of a standard. Targets clearly state what students should know and be able to do. Within the CCSS, for example, an introduction that describes the topics that should represent critical areas of focus where instructional time should be prioritized precedes each grade level's standards. An agreed-on set of prioritized standards is critical to ensuring a guaranteed, viable curriculum, but the biggest impediment to implementing standards is the *number* of standards (Marzano, 2001). Prioritizing standards is therefore a must for teams. A number of steps can assist teams in collaboratively identifying the standards and learning targets that should be prioritized. These steps are an extension of the work of Reeves (2004) and Ainsworth (2003a).

1. List the standard in a standards column.
2. If necessary and desired, list more specific learning targets.
3. Determine whether the standard is a prerequisite for the next grade level.
4. Determine whether the standard is important within other content areas.
5. Determine whether the standard is a critical life skill for college and skilled-career readiness.
6. Determine whether the standard is heavily weighted on high-stakes tests (if applicable).
7. Determine whether the standard is an area of need for your students.

Then use the following six levels of Bloom's Revised Taxonomy (Anderson & Krathwohl, 2001) to identify the level of this standard.

1. **Remembering:** Recalling information (recognizing, listing, describing, retrieving, naming, finding)
2. **Understanding:** Explaining ideas or concepts (interpreting, summarizing, paraphrasing, classifying, explaining)
3. **Applying:** Using information in another familiar situation (implementing, carrying out, using, executing)
4. **Analyzing:** Breaking information into parts to explore understandings and relationships (comparing, organizing, deconstructing, interrogating, finding)
5. **Evaluating:** Justifying a decision or course of action (checking, hypothesizing, critiquing, experimenting, judging)

6. **Creating:** Generating new ideas, products, or ways of viewing things (designing, constructing, planning, producing, inventing)

Finally, use the following four levels of Webb's (1997) Depth of Knowledge to identify the level of this standard.

1. **Recall:** Requires recall or recognition of a fact, information, concept, or procedure
2. **Basic application of skill or concept:** Involves use of information and conceptual knowledge, selecting appropriate procedures, following two or more steps with decision points along the way, solving routine problems, and organizing and displaying data
3. **Strategic thinking:** Requires reasoning and developing a plan or sequence of steps to approach a problem and some decision making and justification; is abstract and complex, often involving more than one possible answer
4. **Extended thinking:** Requires an investigation or application to the real world; requires time to research, think, and process multiple conditions of the problem or task and nonroutine manipulations across disciplines and content areas and from multiple sources

Examples of Prioritized Standards, Grades 6–8

In this section, we recommend prioritized standards in each grade span. We will first summarize next-generation standards from the beginning of the 21st century from various states and provinces. Next, we will share our recommendations based on the actual work of teacher teams with whom we have worked. These teams have committed to making sense of and designing focused sets of standards that represent a guaranteed, viable curriculum. Furthermore, these recommendations strive to include coherence both within (horizontal) and between (vertical) grade levels. Our recommendations are meant only to generate dialogue. We strongly encourage each local school system to independently analyze, interpret, and prioritize next-generation standards. Such decisions are then contextualized to local realities, and the process of collaboratively prioritizing standards will lead to a significantly deeper level of understanding and a greater sense of ownership. First, we share the prioritized standards determined by the NGA and CCSSO (2010c). They use language that suggests that instructional time should focus on critical areas. While there are approximately three to four dozen standards in each grade level, NGA and CCSSO have prioritized a more focused set of topics or critical areas. Table 1.1 lists these for grades 6–8.

Table 1.1: CCSS Prioritized 6–8 Topics

Grade 6	Grade 7	Grade 8
<ol style="list-style-type: none"> 1. Interpret ratios and solve rate problems. 2. Multiply and divide with fractions. 3. Compute with rational numbers (except integers). 4. Make sense of integers on a number line and in the coordinate plane. 5. Evaluate and interpret expressions. 6. Solve and interpret simple equations and inequalities. 7. Find and interpret measure of center or variation. 8. Compute surface areas using nets. 9. Compute volumes of rectangular prisms. 	<ol style="list-style-type: none"> 1. Solve proportion and percent problems. 2. Find and interpret unit rates, including those that describe linear relationships in the first quadrant of the coordinate plane. 3. Represent quantities in various forms—decimals, fractions, and percents. 4. Compute with all rational numbers. 5. Solve and interpret multistep equations and inequalities. 6. Compute the area and circumference of circles. 7. Compute surface areas and volumes of prisms. 8. Generate and draw inferences from data sets. 	<ol style="list-style-type: none"> 1. Recognize, generate, interpret, and analyze linear equations and their graphs. 2. Solve and interpret equations and systems of linear equations. 3. Understand and describe functions. 4. Interpret and generate transformations. 5. Solve problems involving transversals and parallel lines. 6. Interpret and apply the Pythagorean theorem.

The State of New York (EngageNY, 2014), among other states, has proposed prioritized scopes and sequences for next-generation standards. These guides can inform the work of teacher teams in prioritizing standards. Table 1.2 lists these for grades 6–8.

Table 1.2: State of New York Prioritized 6–8 Scopes and Sequences

Grade 6	Grade 7	Grade 8
<ol style="list-style-type: none"> 1. Ratios and unit rates 2. Operations with rational numbers (except integers) 3. Expressions and equations 4. Surface area and volume 5. Statistics 	<ol style="list-style-type: none"> 1. Proportional relationships and percents 2. Operations with rational numbers 3. Expressions and equations 4. Statistics 5. Geometry 	<ol style="list-style-type: none"> 1. Scientific notation 2. Congruency and similarity 3. Linear equations 4. Functions 5. Pythagorean theorem and irrational numbers

In grades 3 and above, next-generation assessments will measure student learning of the next-generation standards. The organizations that are creating these assessments have prepared frameworks that define the relative weight of various standards at each grade level. Table 1.3 details how one of these organizations, the Partnership for Assessment of Readiness for College and Careers (PARCC), prioritizes clusters of standards for grades 6–8.

Table 1.3: PARCC’s Prioritized Clusters of 6–8 Standards

Grade 6	Grade 7	Grade 8
<ol style="list-style-type: none"> 1. Interpret ratios and solve rate problems. 2. Multiply and divide fractions. 3. Compute with rational numbers (except integers). 4. Evaluate expressions. 5. Interpret and solve one-variable equations and inequalities. 6. Interpret and write equations that directly relate two variables. 	<ol style="list-style-type: none"> 1. Interpret and solve proportions. 2. Compute with rational numbers. 3. Manipulate expressions. 4. Evaluate expressions and solve equations. 	<ol style="list-style-type: none"> 1. Evaluate radicals and expressions with integer exponents. 2. Interpret, produce, and solve linear equations and systems of linear equations. 3. Interpret, compare, and model with functions. 4. Interpret and produce congruent and similar shapes. 5. Interpret and apply the Pythagorean theorem.

Now that we have provided examples of prioritized 6–8 standards from the CCSS and policy groups, we share our own prioritized learning targets in table 1.4. We have several objectives in referencing the prioritized lists of topics, domains, and clusters of other organizations when crafting our own lists.

- We want to communicate priorities in a language that teachers will understand, leading to a smoother transition from standard to classroom learning target.
- We want to incorporate our knowledge of student needs and learning progressions while recognizing that our contexts may not match the realities of others.
- We want to complete the process ourselves to gain a deeper understanding of the standards and to feel a sense of ownership over our decisions.

Table 1.4: Weber, Crane, and Hierck Prioritized 6–8 Standards

Grade 6	Grade 7	Grade 8
<ol style="list-style-type: none"> 1. Interpret and solve rate problems. 2. Compute with rational numbers (except integers). 3. Make sense of integers and other rational numbers on a number line and integers in the coordinate plane. 4. Evaluate, solve, and interpret expressions and simple equations and inequalities. 5. Compute surface areas and volumes of rectangular prisms. 	<ol style="list-style-type: none"> 1. Solve proportion and percent problems. 2. Represent quantities in various forms—decimals, fractions, and percents. 3. Compute with all rational numbers. 4. Solve and interpret multistep equations and inequalities. 5. Compute the area and circumference of circles and the surface areas and volumes of prisms. 	<ol style="list-style-type: none"> 1. Recognize, generate, interpret, solve, and analyze linear equations, functions, graphs, and scatter plots. 2. Interpret and generate transformations. 3. Solve problems involving transversals and parallel lines. 4. Interpret and apply the Pythagorean theorem.

Focus: A Foundational Prerequisite to RTI

Improving mathematics teaching and learning is a significant challenge facing all educators in grades 6–8. For some students, the rapid pace at which topics are covered inhibits their learning. For many, the sheer number of topics per grade level prevents topics from being mastered to the depth and complexity required for even basic retention of knowledge. Mastery of procedures of the overwhelming number of topics is eluding too many, and yet, even mastery of procedures is insufficient. Students must possess mastery of the concepts, procedures, and applications of mathematics topics to succeed as next-generation citizens. RTI is best understood as practices and processes that impact, and are impacted by, virtually everything we do in education (Buffum et al., 2009, 2010, 2012). We leverage the knowledge, skills, and attributes of all members of a learning organization to positively impact all students in a proactive effort to ensure that students receive the supports they need as soon as they show signs of struggle with academic or behavioral expectations. RTI rests on the premise that all students can learn and achieve at high levels and that educators will provide the necessary supports and time needed to ensure this happens. This starts with a culture of belief in every single student and rests on our ability to guide every student to high levels of learning. It also requires that we focus content. The foundation of success for all students rests on the culture of the school and the focus of the students and staff.