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Introduction

No matter the year level, no matter the lesson, maths teachers are often confronted with the question, 'When will I ever use this?' Of course, to varying extents, adults use a variety of skills acquired throughout their schooling. For whatever reason, however, the connection between learned mathematics skills and their usefulness in everyday living is not as obvious as it is in English, history or science.

Mathematics textbooks have begun to address this problem by including more 'situational' problems. Each problem set now usually contains an application problem or two relating the current lesson to a real-life situation. Entire books have been written to detail links between topics in mathematics and a variety of careers.

'But what if I don't want to be a civil engineer?' a student may comment. 'What if I don't ever need to break a mathematical code?' For teachers, these questions are difficult to address. Often these real-life problems are pulled and stretched so tightly to conform to the given topic that they become cumbersome to students. Teachers find themselves breaking these problems down for students so far that the real-life is gone and only the skill work remains. In the end, many problems are omitted from assignments, having been judged as 'more trouble than they're worth'.

As teachers lose interest in these situational problems, students surmise that maybe the application level of mathematics isn't that important after all. Students begin to see mathematics as a long pursuit of a series of skills, one after the other, with no connection to the bigger picture. This is a misleading perception about maths and its relevance to the curriculum and today's world. Most teachers chose this career path because someone or something along their academic journey sparked an interest, that led to a realisation that mathematics is powerful . . . that mathematics is useful. Teaching allows us to share those ideas with our students. And even if all your students don't delve into maths-related careers, they can experience and learn to respect the beauty, power and functionality of mathematics as it applies to their own lives.

The goal, then, becomes finding real-life problems or projects that can capture the interest of students and break down the myth that mathematics is for mathematicians and has no place in our daily lives.

About this book

Maths Investigations is a collection of seven applications of mathematics. The focus of each application is centred around themes that are familiar to students. Most importantly, they are built to be interesting and challenging and include a variety of twists and turns along the solution paths.

The applications take the form of investigations. They may last anywhere from part of a class period to several days. Each investigation starts with a short background scenario and a focal question. Questions are simply stated for a number of reasons. They are easier to focus on, easier to share with others, and easier to remember six weeks later. Do not, however, let the initial simplicity of the question fool you or your students.

The question can often be investigated in many ways, with added variables that challenge students to use logical thinking skills that lead to viable solutions.

A familiar example may be the 'birthday problem': How many people would you need to have in a room before the odds are that two of them share the same birthday? The problem itself is easily understood and does not contain any technical mathematical terms. However, the solution path, as you may know, is more complex than the typical student may realise at first glance. Have you worked with the 'birthday problem' before? If so, you probably remember when it was that you studied it. You probably remember the general solution path and can still explain the problem and solution path to anyone. You may even remember that the answer is 23 people. And, you probably recall the problem and its solution because it intrigued and challenged you enough to apply your maths skills. Just as you were motivated to solve the 'birthday problem', there are many ways to make mathematics interesting to students.

This book presents the following maths investigations to get your students started.

- **Investigation 1:** *Exposing the mole* (a logic problem presented in the form of a card game)
- **Investigation 2:** *The last-second shot* (using probability to determine strategies in the last seconds of a basketball game)
- **Investigation 3:** *Look to the biscuit* (an investigation in geometry presented in the context of sharing a biscuit)
- **Investigation 4:** *Only the weak survive* (a probability situation presented in the context of a popular reality show-type scenario)
- **Investigation 5:** *Pickin' chicken* (an investigation into the purchase of combinations of 'nuggets')
- **Investigation 6:** *Popcorn economics* (an investigation in number sense presented in the context of buying popcorn at a movie theatre)
- **Investigation 7:** *Red rover* (a logic problem to determine the most efficient method of winning a game)

About this book *(cont.)*

The following suggestions are provided to help you plan ways to use the investigations.

Don't try to follow a strict pace

As in any good experiment, the work here is open-ended and unsure at its beginning. Allowing time for students to fully comprehend problems and solutions will build confidence and give students a sense of ownership. The investigations are written on a number of levels; each has an opener followed by a series of follow-up questions and optional extensions. Depending on your class personality, year level, and other factors, your students will enjoy and achieve more on some topics than others; this may change from year to year or even from period to period. If you feel the need to end a certain investigation early, the format certainly allows you to do that. Conversely, you may find your students interested in moving forward with the investigation on a more in-depth level, or possibly investigating further based on their findings or some hypothesis that a single student develops. The work may, then, actually continue longer than you had planned.

For these reasons, you will not see anything on the investigations that says you must get through a certain amount of study on day one during a four-day block. Be flexible and consider the students' motivations and interests as you schedule the investigations.

Watch for new opportunities

One of the positive outcomes of using these investigations with students is that they begin to analyse the world around them in a mathematical context. A by-product of this outcome is that it will affect the teacher as well! As you begin to see more opportunities to branch out from this collection, take them! And do not be overly concerned with analysis and development before presenting them to students. Students will enjoy working through the details with you and will appreciate knowing that you are there working along with them.

Don't lose sight of the master plan

Unless you teach an elective class, these investigations are intended to be used sparingly. The fun of performing and developing investigations makes it easy to get drawn into overuse; remember, these investigations are, for the most part, applications based on prior knowledge. Students will need lots of direct instruction and skill work to develop good mathematical habits.

Remember the goal here: Performing an investigation gives you and your class a temporary leave from the everyday routine to explore the mathematical world. Realise that one of the most important lessons students can learn from working with you on an investigation has nothing to do with any specific mathematical concept; instead, it is a chance for you to model an attitude of excitement and awe for mathematics.

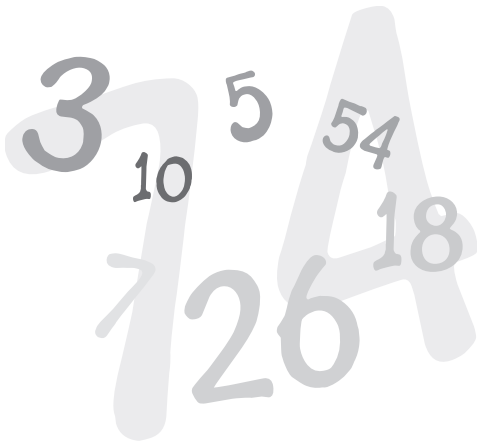
These investigations represent a chance for you to show students that mathematics is not just something used to get through textbook problems – it can be as exciting and purposeful for them as it is for you. They are watching you . . . take advantage of it.

Investigation 1

Exposing the mole

Concepts & skills

- arithmetic progression
- logical thinking
- problem-solving
- communication



Materials

- a deck of playing cards

Investigation overview

Exposing the mole is a logic problem presented in the form of a card game. To introduce the game to students, the teacher seats a group of four or five students in a circle. More advanced classes may eventually use larger groups, but four to five is a good number of students to use to demonstrate the game initially. If there are N students seated in the circle, the teacher shuffles a small pack of cards containing ace, 2, 3 . . . , N + 2 (i.e. two more cards than players).

The cards are dealt as follows: The teacher places one card against each student's forehead – students can hold the card in place with their index finger. Each card face should be exposed to the other students but never to the card's bearer. After dealing, two cards remain in the pack; one is pocketed by the teacher and, thus, removed from the game. The other, the 'mole card', is left face down in the centre of the group.

The teacher then informs the students of the value of the total. The value of the total is the sum of the values of all the visible cards minus the value of the mole card. Armed only with this information, the object of the game is for each student to deduce the value of the mole card without the assistance of any other student.

Inside the game

Let's take a look at an example of how this may play out. Imagine that you have selected a group of four students, and so, the playing deck includes ace, 2, 3, 4, 5, 6. Suppose your deal results in students receiving cards 2, 3, 5 and 6; the ace happens to have been pocketed and removed, and the 4 is left to be the mole card, face down in the centre.

You would report to students that the total for this round is 12 (2 + 3 + 5 + 6 – 4 = 12). With no other information, it would be up to each student to deduce the value of the mole card.

Student's perspective

Suppose the student is playing the game with the 2 card on his forehead. He can see the 3, 5 and 6; he can't see the ace, 2 or 4. He is told the Total is 12; the cards he can see have a sum of 14. From here, students can take different approaches.

Exposing the mole *(cont.)*

Solution by cases

Some students solve by trial and error on individual cases. Students may 'try on' each of the cards they cannot see until they find a winner. A common thought process here might emerge in this way:

If I have the ace, the visible cards would sum to 15. In order to make the total of 12, the mole card would need to be 3. Since I can already see the 3, that won't work.

If I have the 2, the visible cards would sum to 16. In order to make the total of 12, the mole card would need to be 4. Since I don't see the 4, that might be right.

If I have the 4, the visible cards would sum to 18. In order to make the total of 12, the mole card would need to be 6. Since I can already see the 6, that won't work.

Therefore, I must have the 2, making the mole card the 4.

Solution by algebra

A more sophisticated approach is to determine the value of the card algebraically. An equation, or relationship, with two variables can be formed based on the given information. Students would then test their solution set (the unknown cards) to fit the given relationship. The thought process for this type of solution might go something like this:

I know that the cards I can see add up to 14. The total is 12. So, 14 plus my card minus the mole card is 12.

or

$14 + ME - MOLE = 12$

or

$ME - MOLE = -2$

The cards I can't see are the ace, 2 and 4. To make my equation work, I must have the 2; the mole card must be 4.

Here is an example of the same basic method, just approached a bit more literally.

The cards I can see add up to 14. The Total is 12. So adding my card and subtracting the mole card results in a loss of two. My card must be two less than the mole card.

The cards I can't see are the ace, 2 and 4. To make the relationship true, I must have the 2; the mole card must be 4.

Exposing the mole *(cont.)*

Teaching notes

Although the solution paths described on the previous pages highlight the most common strategies, students may devise something completely different from either of them. Regardless of students' approaches or the problems they may encounter, keep the following ideas in mind.

Talk it through

Be sure to have your students verbalise their solution paths; it is a necessary process that can be helpful to you and the class in a number of ways. First, it will help students to clarify their own thinking. Some students may succeed without really knowing how they are doing so. Verbalising their solutions forces students to be specific in their processes and pinpoint strategies. Second, it will give possible strategies to students who may be struggling. Some children may not even know they are struggling until they hear how well a classmate is doing. Third, listening to students will help you know how they are thinking; if they struggle later in the investigation, perhaps in one of the extensions, you will know how to more efficiently relate new information to their existing structures. Finally, your students might just surprise you with a new solution method.

Work together

Have the entire group play together and confer as a single player; deal cards to three imaginary players as well. We'll call this the 'collective' version of the game. One facet of Exposing the mole that can be a challenge to manage as a teacher is that the game changes depending on where you are sitting; different students will see different cards. If you find that the students are in need of more direct instruction, play the collective version a few times so that the same cards are known and unknown to everyone. Students will build their skills together, and the teacher can speak to the entire group under the same conditions.

Discussion opportunity

Why is it important to remove a card from the game?

